11. If $\omega \neq 1$ is a cube root of unity, then

$$A = \begin{bmatrix} 1+2\omega^{100}+\omega^{200} & \omega^2 & 1 \\ 1 & 1+2\omega^{100} & \omega \\ \omega & \omega^2 & 2+\omega^{100}+2\omega^{200} \end{bmatrix}$$

(A) A is singular (B) $|A| \neq 0$

(C) A is symmetric (D) None of the above

Solution: (A)

Given matrix can be rewritten as

$$A = \begin{vmatrix} 1+2\omega+\omega^2 & \omega^2 & 1 \\ 1 & 1+\omega^2+2\omega & \omega \\ \omega & \omega^2 & 2+\omega+2\omega^2 \end{vmatrix}$$

$$\therefore \quad |A| = \begin{vmatrix} \omega & \omega^2 & 1 \\ 1 & \omega & \omega \\ \omega & \omega^2 & 1+\omega^2 \end{vmatrix} \qquad (\because \quad 1+\omega+\omega^2 = 0)$$

$$= \omega \begin{vmatrix} \omega & \omega & 1 \\ 1 & 1 & \omega \\ \omega & \omega & -\omega \end{vmatrix} \qquad (\because \quad 1+\omega+\omega^2 = 0)$$

$$\Rightarrow \quad \omega(0) = 0$$

12. $\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$ equals to

(A) 1 (B) 2 (C) 0 (D) 3

Solution: (B)

$$\lim_{x \to \tan^{-1} 3} \frac{\tan^2 x - 2\tan x - 3}{\tan^2 x - 4\tan x + 3}$$

$$= \lim_{\tan x \to 3} \frac{(\tan x - 3)\,(\tan x + 1)}{(\tan x - 3)\,(\tan x - 1)}$$

$$= \lim_{\tan x \to 3} \frac{\tan x + 1}{\tan x - 1} = \frac{3+1}{3-1} = \frac{4}{2} = 2$$

13. The locus of the points of intersection of the tangents at the extremities of the chords of the ellipse $x^2 + 2y^2 = 6$ which touches the ellipse $x^2 + 4y^2 = 4$, is

(A) $x^2 + y^2 = 4$ (B) $x^2 + y^2 = 6$

(C) $x^2 + y^2 = 9$ (D) None of these

Solution: (C)

The given equation of second ellipse can be rewritten as

$$\frac{x^2}{4} + \frac{y^2}{1} = 1$$

Equation of tangent to this ellipse is

$\frac{x}{2}\cos\theta + y\sin\theta = 1$ …..(i)

Equation of the first ellipse can be rewritten as

$\frac{x^2}{6} + \frac{y^2}{3} = 1$ ……(ii)

Let ellipse (i) meets the first ellipse at P and Q and the tangents at P and Q to the second ellipse intersected at (h, k), then equation (i) is the chord of contact of (h, k) with respect to the ellipse (ii) and thus, its equation is

$\frac{hx}{6} + \frac{ky}{3} = 1$ …….(iii)

Since, equations (i) and (iii) represent the same line

$$\frac{\frac{h}{6}}{\cos\frac{\theta}{2}} = \frac{\frac{ky}{3}}{\sin\theta} = 1$$

$h = 3\cos\theta$ and $k = 3\sin\theta$

Hence, locus is $x^2 + y^2 = 9$.

14. Number of roots of the equation $|\sin x \cdot \cos x| + \sqrt{2 + \tan^2 x + \cot^2 x} = \sqrt{3}$, where $x \in [0, 4\pi]$, are

(A) 1 (B) 2 (C) 3 (D) None of these

Solution: (D)

We have, $|\sin x \cos x| + |\tan x + \cot x| = \sqrt{3}$

$$\Rightarrow \quad |\sin x \cos x| + \frac{1}{|\sin x \cdot \cos x|} = \sqrt{3}$$

But $|\sin x \cdot \cos x| + \frac{1}{|\sin x \cdot \cos x|} \geq 2$

Hence, there is no solution.

15. If $f(x) = \begin{cases} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}}, & x \neq 0 \\ k, & x = 0 \end{cases}$

For what value of $'k', f(x)$ is continuous at $x = 0$?

(A) 1 (B) 0 (C) e (D) e^2

Solution: (D)

$$\lim_{x \to 0} \left[\tan\left(\frac{\pi}{4} + x\right)\right]^{\frac{1}{x}} = \lim_{x \to 0} \left[\frac{1 + \tan x}{1 - \tan x}\right]^{\frac{1}{x}}$$

$$= \lim_{x \to 0} \left[(1 + \tan x)^{\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}} \times \lim_{x \to 0} \left[(1 - \tan x)^{\frac{1}{\tan x}}\right]^{\frac{\tan x}{x}}$$

$$= e \cdot e = e^2$$

16. The period of $\sin^2 \theta$ is

(A) π^2 (B) π (C) 2π (D) $\frac{\pi}{2}$

Solution: (B)

Since, $\sin^2 \theta = \frac{1 - \cos 2\theta}{2} = \frac{1}{2} - \frac{1}{2} \cos 2\theta$

$\therefore$ Period of $\sin^2 \theta = \frac{2\pi}{2} = \pi$

17. Five persons, A, B, C, D and E are in queue of a shop. The probability that A and E are always together, is

(A) $\frac{1}{4}$ (B) $\frac{2}{3}$ (C) $\frac{2}{5}$ (D) $\frac{3}{5}$

Solution: (C)

$\because$ Total number of ways = 5! and favourable number of ways $= 2 \cdot 4!$

$\therefore$ Required probability $= \frac{2 \cdot 4!}{5!} = \frac{2}{5}$

18. $\lim_{x\to\infty} \frac{x^4 \cdot \sin\left(\frac{1}{x}\right) + x^2}{1+|x|^3}$ equal to

(A) 0 (B) -1 (C) 2 (D) 1

Solution: (B)

$$\lim_{x\to\infty} \frac{x^4 \cdot \sin\left[\frac{1}{x}\right] + x^2}{1+|x|^3} = \lim_{x\to\infty}\left[\frac{x\sin\left(\frac{1}{x}\right)+\frac{1}{x}}{\frac{1}{x^3}+\frac{|x|^3}{x^3}}\right]$$

[Dividing numerator and denominator by x^3]

$$= \frac{\lim_{x\to\infty}\frac{\sin\left(\frac{1}{x}\right)}{\frac{1}{x}} + \lim_{x\to\infty}\frac{1}{x}}{\lim_{x\to\infty}\frac{1}{x^3} + \lim_{x\to\infty}\frac{|x|^3}{x^3}} = \frac{1-0}{0-1} = -1$$

19. The sum of the $\log_4 2 - \log_8 2 + \log_{16} 2$ .... Is

(A) e^2 (B) $\log_e 2 + 1$

(C) $\log_e 3 - 2$ (D) $1 - \log_e 2$

Solution: (D)

Given that, $\log_4 2 - \log_8 2 + \log_{16} 2 - \cdots$

$$= \frac{1}{\log_2 4} - \frac{1}{\log_2 8} + \frac{1}{\log_2 16} - \cdots$$

$$= \frac{1}{2} - \frac{1}{3} + \frac{1}{4} - \cdots$$

$$= 1 - \left[1 - \frac{1}{2} + \frac{1}{3} - \frac{1}{4} + \frac{1}{5} - \cdots\right]$$

$= 1 - \log_e 2$

20. The mean of n terms is $\bar{x}$. If the first term is increased by 1, second by 2 and so on, then the new mean is

(A) $\bar{x} + n$ (B) $\bar{x} + \frac{n}{2}$

(C) $\bar{x} + \frac{n+1}{2}$ (D) None of these

Solution: (C)

Let the observation be $x_1, x_2. x_3 \ldots\ldots, x_n$.

Now, mean $(\bar{x}) = \frac{x_1+x_2+\cdots+x_n}{n}$

When first term increased by 1, second by 2 and so on, then observations will be

$(x_1 + 1), (x_1 + 2), (x_1 + 3), \ldots\ldots, (x_n + n)$

Then, new mean

$$(\bar{x}_1) = \frac{(x_1 + 1) + (x_2 + 2) + \cdots + (x_n + n)}{n}$$

$$= \frac{(x_1 + x_2 + \cdots x_n) + (1 + 2 + 3 + \cdots + n)}{n}$$

$$\Rightarrow \quad \bar{x}_1 = \frac{x_1 + x_2 + \cdots + x_n}{n} + \frac{1 + 2 + 3 + \cdots + n}{n}$$

$$\Rightarrow \quad \bar{x}_1 = \bar{x} + \frac{n(n+1)}{2n} \qquad \left[\because \quad 1 + 2 + 3 + \cdots + n = \frac{n(n+1)}{2}\right]$$

$$\Rightarrow \quad \bar{x}_1 = \bar{x} + \frac{n+1}{2}$$

21. The greatest and least values of $(\sin^{-1} x)^2 + (\cos^{-1} x)^2$ are respectively

(A) $\frac{\pi^2}{4}$ and 0 (B) $\frac{\pi}{2}$ and $\frac{-\pi}{2}$

(C) $\frac{5\pi^2}{4}$ and $\frac{\pi^2}{8}$ (D) $\frac{\pi^2}{4}$ and $\frac{-\pi^2}{4}$

Solution: (C)

We have,

$$(\sin^{-1} x)^2 + (\cos^{-1} x)^2$$

$$= (\sin^{-1} x + \cos^{-1} x)^2 - 2\sin^{-1} x \cdot \cos^{-1} x$$

$$= \frac{\pi^2}{4} - 2\sin^{-1} x\left(\frac{\pi}{2} - \sin^{-1} x\right)$$

$$= \frac{\pi^2}{4} - \pi \sin^{-1} x + 2(\sin^{-1} x)^2$$

$$= 2\left[(\sin^{-1} x)^2 - \frac{\pi}{2}\sin^{-1} x + \frac{\pi^2}{8}\right]$$

$$= 2\left[\left(\sin^{-1} x - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$$

Thus, the least value is $2\left(\frac{\pi^2}{16}\right)$ i.e., $\frac{\pi^2}{8}$ and the greatest value is $2\left[\left(\frac{-\pi}{2} - \frac{\pi}{4}\right)^2 + \frac{\pi^2}{16}\right]$ i.e., $\frac{5\pi^2}{4}$.

22. The probability of simultaneous occurrence of atleast one of two events A and B is p. If the probability that exactly one of A, B occurs is q, then $P(A') + P(B')$ is equal to

(A) $2 - 2p + q$ (B) $2 + 2p - q$

(C) $3 - 3p + q$ (D) $2 - 4p + q$

Solution: (A)

Since, P (exactly one of A, B occurs) = q (given),

We get

$$P(A \cup B) - P(A \cap B) = q$$

$$\Rightarrow \quad p - P(A \cap B) = q$$

$$\Rightarrow \quad P(A \cap B) = p = q$$

$$\Rightarrow \quad 1 - P(A' \cup B') = p - q$$

$$\Rightarrow \quad P(A' \cup B') = 1 - p + q$$

$\Rightarrow \quad P(A') + P(B') - P(A' \cap B') = 1 + q - p$

$\Rightarrow \quad P(A') + P(B') = (1 - p + q) + [1 - P(A \cup B)]$

$= (1 - p + q) + (1 - p)$

$= 2 - 2p + q$

23. The solution of the differential equation $x = 1 + xy\frac{dy}{dx} + \frac{x^2y^2}{2!}\left(\frac{dy}{dx}\right)^2 + \frac{x^3\,y^3}{3!}\left(\frac{dy}{dx}\right)^3 + \cdots$ is

(A) $y = \log x + C$ (B) $y^2 = (\log x)^2 + C$

(C) $y = \log x + xy$ (D) $xy = x^y + C$

Solution: (B)

The given equation is reduced to $x = e^{xy\left(\frac{dy}{dx}\right)}$

$\Rightarrow \quad \log x = xy\frac{dy}{dx}$

$\Rightarrow \quad y\,dx = \frac{\log x}{x}\,dx$

$\Rightarrow \quad \int y\,dx = \int \frac{1}{x}\log x\,\,dx$

$\Rightarrow \quad \frac{y^2}{2} = \frac{(\log x)^2}{2} + C'$

$\Rightarrow \quad y^2 = (\log x)^2 + 2C'$

$\Rightarrow \quad y^2 = (\log x)^2 + C$ (where, $C = 2C'$)

24. If n is a positive integer, then $n^3 + 2n$ is divisible by

(A) 2 (B) 6 (C) 15 (D) 3

Solution: (D)

Let $P(n) = n^3 + 2n$

$P(1) = 1 + 2 = 3$

$P(2) = 8 + 4 = 12$

$P(3) = 27 + 6 = 33$

Clearly, we see that all these numbers are divisible by.

25. The sum of the coefficients in the expansion of $(5x - 4y)^n$, where n is a positive integer, is

(A) 0 (B) n (C) 1 (D) -1

Solution: (C)

$\because$ Using Binomial theorem,

$$(5x - 4y)^n = {}^nC_0(5x)^n + {}^nC_1(5x)^{n-1}(-4y) + {}^nC_2(5x)^{n-2}(-4y)^2 + \cdots + {}^nC_n(-4y)^n$$

Sum of coefficients

$= {}^nC_0\, 5^n + {}^nC_1\, 5^{n-1}(-4) + {}^nC_2\, 5^{n-2}.(-4)^2 + \cdots + {}^nC_n(-4)^n$

$= (5 - 4)^n$

$= 1^n = 1$

26. The distance of the point $(1, -5, 9)$ from the plane $x + y + z = 5$ measured along a straight line $x = y = z$ is $2\sqrt{3}k$, then the value of k is

(A) 5 (B) 6 (C) $\sqrt{3}$ (D) 4

Solution: (A)

Given equation of plane is $x + y + z = 5$.

The distance measured along the line $x = y = z$.

Direction ratio's of the given line is (1, 1, 1).

So, the equation of line PQ is

$$\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}$$

Now, let $\frac{x-1}{1}=\frac{y+5}{1}=\frac{z-9}{1}=\lambda$

$x=\lambda+1, y=\lambda-5, z=\lambda+9$

Lies on the plane $x+y+z=5$

$\lambda+1-\lambda+5+\lambda+9=5$

$\Rightarrow \quad \lambda=-10$

The coordinate of Q is $(-99,-15,-1)$ and the coordinate of P is $(1,-5,9)$.

$PQ=\sqrt{(10)^2+(10)^2+(10)^2}=10\sqrt{3}$

$\therefore \quad 2\sqrt{3}k=10\sqrt{3} \quad \Rightarrow \quad k=5$

27. $\lim_{x\to 1}\frac{x^m-1}{x^n-1}$ is equal to

(A) $\frac{n}{m}$ (B) $\frac{m}{n}$ (C) $\frac{2m}{n}$ (D) $\frac{2n}{m}$

Solution: (B)

$\lim_{x\to 1}\frac{x^m-1}{x^n-1}= \lim_{x\to 1}\frac{mx^{m-1}}{nx^{n-1}}$ (by L'Hospital rule)

$=\frac{m}{n}$

28. The value of $x>1$ satisfying the equation $\int_1^x t\log t dt=\frac{1}{4}$, is

(A) $\sqrt{e}$ (B) $e^{\frac{3}{2}}$ (C) e^2 (D) $2e-1$

Solution: (A)

Consider that, $I=\int_1^x t\log t\ dt$

$$= \left[\log t \cdot \frac{t^2}{2}\right]_1^x - \int_1^x \frac{1}{t} \cdot \frac{t^2}{2} dt$$

$$= \frac{x^2}{2} \log x - \frac{1}{2}\left[\frac{t^2}{2}\right]_1^x$$

$$= \frac{x^2}{2} \log x - \frac{1}{2}\left[\frac{x^2}{2} - \frac{1}{2}\right]$$

$\Rightarrow \quad \frac{1}{4} = \frac{x^2}{2} \log x - \frac{1}{4}(x^2 - 1)$

$\Rightarrow \quad \frac{1}{2}x^2 \log x - \frac{1}{4}x^2 = 0$ (as $x > 1$)

$\Rightarrow \quad x^2(2\log x - 1) = 0$

$\Rightarrow \quad 2\log x - 1 = 0$

$\Rightarrow \quad \log x = \frac{1}{2}$

$\Rightarrow \quad x = e^{\frac{1}{2}}$

$\Rightarrow \quad x = \sqrt{e}$

29. $a = \sum_{n=0}^{\infty} \frac{x^{3n}}{3n!}, b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!}$ And $c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$, then the value of $a^3 + b^3 + c^3 - 3abc$ is

(A) 1 (B) 0 (C) -1 (D) -2

Solution: (A)

We have,

$$a = \sum_{n=0}^{\infty} \frac{x^{3n}}{(3n)!}, b = \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!} \text{ and } c = \sum_{n=1}^{\infty} \frac{x^{3n-1}}{(3n-1)!}$$

Now, $a + b + c = \sum_{n=0}^{\infty} \frac{x^{3n}}{3n!} + \sum_{n=1}^{\infty} \frac{x^{3n-2}}{(3n-2)!} + \sum_{n=1}^{\infty} \frac{x^3 n-1}{(3n-1)!}$

$$= 1 + x + \frac{x^2}{2!} + \frac{x^3}{3!} + \cdots = e^x$$

$$a + b\omega + c\omega^2 = 1 + \omega x + \frac{\omega^2 x^2}{2!} + \frac{\omega^3 x^3}{3!} + \cdots = e^{\omega x}$$

and $a + b\omega^2 + c\omega = e^{\omega^2 x}, \omega$ is imaginary cube root of unity.

Now, $a^3 + b^3 + c^3 - 3abc$

$= (a + b + c)\,(a + b\omega + c\omega^2)\,(a + b\omega^2 + c\omega)$

$= e^x \,.\, e^{\omega x} \,.\, e^{\omega^2 x} = e^{x(1+\omega+\omega^2)} = e^{0.x} = 1$

30. The unit vector perpendicular to the vectors $\hat{\imath} - \hat{\jmath}$ and $\hat{\imath} + \hat{\jmath}$ forming a right handed system is

(A) $\hat{k}$ (B) $-\hat{k}$

(C) $\frac{\hat{\imath} - \hat{\jmath}}{\sqrt{2}}$ (D) $\frac{\hat{\imath} + \hat{\jmath}}{\sqrt{2}}$

Solution: (A)

Required unit vector is $\frac{(\hat{\imath} - \hat{\jmath}) \times (\hat{\imath} + \hat{\jmath})}{|(\hat{\imath} - \hat{\jmath}) \times (\hat{\imath} + \hat{\jmath})|}$

$$= \frac{\hat{k} + \hat{k}}{2} = \frac{2\hat{k}}{2} = \hat{k}$$

31. The number of solutions of the equation $x^3 + 2x^2 + 5x + 2\cos x = 0$ in $[0, 2\pi]$ are

(A) 0 (B) 1 (C) 2 (D) 3

Solution: (A)

$f(x) = x^3 + 2x^2 + 5x + 2\cos x$

$f'(x) = 3x^2 + 4x + 5 - 2\,.\sin x$

$$= 3\left(x + \frac{2}{5}\right)^2 + \frac{11}{3} - 2\,.\sin x$$

$\Rightarrow \quad f'(X) > 0, \forall\, x$

$f(x)$ is increasing for all $x \in R$.

Also, $f(0) = 2 \Rightarrow \quad f(x) = 0$

So, $f(x)$ has no solution.

32. $\lim_{x\to 0}\frac{(1+x)^8-1}{(1+x)^2-1}$ is equal to

(A) 8 (B) 6 (C) 4 (D) 2

Solution: (C)

$$\lim_{x\to 0}\frac{(1+x)^8 - 1}{(1+x)^2-1}$$

$$= \lim_{x\to 0}\frac{[(1+x)^4+1]\,[(1+x)^2+1]\,[(1+x)^2-1]}{(1+x)^2-1}$$

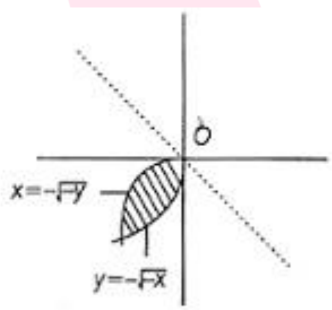
$$= 2\times 2 = 4$$

33. The area bounded by the curves $y=-\sqrt{-x}$ and $x= -\sqrt{-y}$, where $x, y \le 0$, is

(A) $\frac{1}{3}$ (B) $\frac{1}{4}$ (C) $\frac{1}{5}$ (D) $\frac{1}{2}$

Solution: (A)

Given that, $y= -\sqrt{-x}$



$\Rightarrow \quad y^2=-x$, where x and y both negative.

Now, $x=-\sqrt{-y}$

$\Rightarrow \quad x^2=-y$, where x and y both negative.

$\therefore \quad Area = \left| \int_{-1}^{0} -\sqrt{-x}\, dx - \int_{-1}^{0} -x^2\, dx \right|$

$= \frac{1}{3}$

34. If α and β are the roots of the equation $x^2 - px + q = 0$, then the value of $(\alpha + \beta)x - \left(\frac{\alpha^2+\beta^2}{2}\right)x^2 + \left(\frac{\alpha^3+\beta^3}{3}\right)x^3 + \cdots$, is

(A) $\log(1 - px + qx^2)$ (B) $\log(1 + px - qx^2)$

(C) $\log(1 + px + qx^2)$ (D) None of these

Solution: (A)

Given series

$$\left[\alpha x - \frac{1}{2}(\alpha x)^2 + \frac{1}{3}(\alpha x)^3 - \cdots\right] + \left[\beta x - \frac{1}{2}(\beta x)^2 + \frac{1}{3}(\beta x)^3 - \cdots\right]$$

$= \log(1 - \alpha x) + \log(1 - \beta x)$

$= \log[1 - (\alpha + \beta)\, x + \alpha\beta x^2]$

Now, $\alpha + \beta = p$ and $\alpha\beta = q$

Given series $= \log(1 - px + qx^2)$

35. If $\cos^{-1} x > \sin^{-1} x$, then

(A) $\frac{1}{\sqrt{2}} < x \leq 1$ (B) $0 \leq x < \frac{1}{\sqrt{2}}$

(C) $-1 \leq x < \frac{1}{\sqrt{2}}$ (D) $x > 0$

Solution: (C)

We know that,

$\sin^{-1} x + \cos^{-1} x = \frac{\pi}{2}, \forall x \in [-1, 1]$

$\therefore \quad \cos^{-1} x > \sin^{-1} x$

$$\Rightarrow \quad \frac{\pi}{2} - \sin^{-1} x > \sin^{-1} x$$

$$\Rightarrow \quad \frac{\pi}{2} > 2\sin^{-1} x$$

$$\Rightarrow \quad \sin^{-1} x < \frac{\pi}{4}$$

$$\Rightarrow \quad x < \frac{1}{\sqrt{2}}$$

$$\therefore \quad -1 \le x < \frac{1}{\sqrt{2}}$$

36. A student is allowed to select atmost $n$books from a collection of $(2n+1)$ books. If the number of ways in which he can do this, is 64, then the value of n is

(A) 6 (B) n (C) 3 (D) None of these

Solution: (C)

Number of ways

$$= {}^{2n+1}C_0 + {}^{2n+1}C_1 + {}^{2n+1}C_2 + \cdots + {}^{2n+1}C_n$$

$$= \frac{1}{2}(2^{2n+1}) = 2^{2n}$$

Thus, $2^{2n} = 64$

i.e., $2^{2n} = 2^6$

On comparing, $2n = 6 \Rightarrow \ n = 3$

37. Let $R = \{(3,\ 3), (6,6), (9,9), (12,12), (6,12), (3,9), (3,12), (3,6)\}$ be a relation on the set $A = \{3,6,9,12\}$. The relation is

(A) An equivalence relation

(B) Reflexive and symmetric

(C) Reflexive and transitive

(D) Only reflexive

Solution: (C)

R is reflexive as (3, 3), (6, 6), (9, 9), (12, 12) ∈ R. R is not symmetric as $(6, 12) \in R$ but $(12, 6) \notin R$. R is transitive as the only pair which needs verification is (3, 6) and (6, 12) ∈ R

$\Rightarrow \quad (3, 12) \in R$.

38. The value of a, so that the sum of squares of the roots of the equation $x^2 - (a-2)x - a + 1 = 0$ assume the least value, is

(A) 2 (B) 0 (C) 3 (D) 1

Solution: (D)

Let α and β be the roots of the equation

$x^2 - (a-2)\,x - a + 1 = 0$

$\therefore \quad \alpha + \beta = a - 2$ and $\alpha\beta = -(a-1)$

$s = \alpha^2 + \beta^2 = (\alpha + \beta)^2 - 2\alpha\beta = (a-2)^2 + 2(a-1)$

$= a^2 - 4a + 4 + 2a - 2 = a^2 - 2a + 2$

Now, $\frac{ds}{da} = 2a - 2$

For maximum and minimum, $\frac{ds}{da} = 0$

$\Rightarrow \quad 2a - 2 = 0$

$\Rightarrow \quad a = 1$

Also, $\frac{d^2s}{da^2} = 2 > 0$

Hence, at $a = 1, s$ will have minimum value.

39. The form of the differential equation of the central conics $ax^2 + by^2 = 1$ is

(A) $x = y\frac{dy}{dx}$

(B) $x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2} - y\frac{dy}{dx} = 0$

(C) $x + y\frac{d^2y}{dx^2} = 0$

(D) None of the above

Solution: (D)

We have, $ax^2 + by^2 = 1$

On differentiating both sides w.r.t. x, we get

$$2ax + 2by\frac{dy}{dx} = 0$$

$\Rightarrow \quad ax + by\frac{dy}{dx} = 0$ .....(i)

$\Rightarrow \quad \frac{-a}{b} = \frac{y}{x}\frac{dy}{dx}$ ......(ii)

Again, differentiating equation (i) w.r.t x, we get

$$a + b\left(\frac{dy}{dx}\right)^2 + by\frac{d^2y}{dx^2} = 0$$

$\Rightarrow \quad -\frac{a}{b} = \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}$ ....(iii)

From equations (ii) and (iii), we get

$$\frac{y}{x}\frac{dy}{dx} = \left(\frac{dy}{dx}\right)^2 + y\frac{d^2y}{dx^2}$$

$$\Rightarrow \quad y\left(\frac{dy}{dx}\right) = x\left(\frac{dy}{dx}\right)^2 + xy\frac{d^2y}{dx^2}$$

40. A furniture dealer deals in only two items namely tables and chairs. He has Rs. 5000 to invest and space to store at the most 60 pieces. A table cost him rs 250 and a chair rs 60. He can sell a table at a profit of rs 15. Assume that the can sell all the items that he produced. The number of constraints in the problem are

(A) 2 (B) 3 (C) 4 (D) 5

Solution: (C)

If x tables and y chairs are purchased for maximum profit.

Then, $x + y \leq 60$

$5x + \frac{6y}{5} \leq 100$

$x \geq 0, y \geq 0$

So, number of constraints are four.

41. $10^n + 3(4^{n+2}) + 5$ is divisible by $(n \in N)$

(A) 7 (B) 5 (C) 9 (D) 17

Solution: (C)

For $n = 1, 10^n + 3 \cdot 4^{n+2} + 5 = 10 + 3 \cdot 4^3 + 5$

$= 207$, which is divisible by 99

So, by induction, the result is divisible by 9.

42. The total number of subsets of a finite set A has 56 more elements than the total number of subsets of another finite set B. What is the number of elements in the set A?

(A) 5 (B) 6 (C) 7 (D) 8

Solution: (B)

Let set A and B have m and n elements, respectively.

$2^m - 2^n = 56$

$2^n(2^{m-n} - 1) = 56 = 8 \times 7 = 2^3 \times 7$

Comparing both sides, we get

$2^n = 2^3$ and $2^{m-n} = 7$

$\Rightarrow \quad n = 3$ and $2^{m-n} = 8$

$\Rightarrow \quad 2^{m-n} = 2^3 \Rightarrow \quad m - n = 3$

$\Rightarrow \quad m - 3 = 3 \Rightarrow \quad m = 6$

Number of the elements in A is 6.

43. A sample of 35 observation has the mean 80 and standard deviation as 4. A second sample of 65 observations from the same population has mean 70 and standard deviation 3. Then, the standard deviation of the combined sample is

(A) 5.85 (B) 5.58 (C) 34.2 (D) None of the above

Solution: (A)

Mean of the combined sample

$$\bar{X} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2}{n_1 + n_2} = \frac{35 \times 80 + 65 \times 70}{35 + 65}$$

$= 73.5$

Standard deviation of the combined sample is given by

$$\sigma^2 = \frac{n_1(\sigma_1^2 + d_1^2) + n_2(\sigma_2^2 + d_2^2)}{n_1 + n_2}$$

Where, $d_1^2 = (\bar{x}_1 - \ \bar{x})^2 = (80 - 73.5)^2 = 42.25$

$d_2^2 = (\bar{x}_2 - \ \bar{x})^2 = (70 - 73.5)^2 = 12.25$

$$\therefore \quad \sigma^2 = \frac{35(4^2 + 42.25) + 65(3^2 + 12.25)}{35 + 65}$$

$\Rightarrow \quad \sigma^2 = 34.2 \ \Rightarrow \ \sigma = 5.85$

44. Three straight lines $2x + 11y - 5 = 0, 24x + 7y - 20 = 0$and $4x - 3y - 2 = 0$

(A) From a triangle

(B) Are only concurrent

(C) Are concurrent with one line bisecting the angle between the other two

(D) None of the above

Solution: (C)

For the two lines $24x + 7y - 20 = 0$ and $4x - 3y - 2 = 0$, the angle bisectors are given by $\frac{24x+7y-20}{25} = \pm\frac{4x-3y-2}{5}$

Taking positive sign, we get $2x + 11y - 5 = 0$

So, the given three lines are concurrent with one line bisecting the angle between the other two.

45. If the plane $x + y + z = 1$ is rotated through an angle 90^o about its line of intersection with the plane $x - 2y + 3z = 0$, then the now position of the plane is

(A) $x - 5y + 4z = 1$

(B) $x - 5y + 4z = -1$

(C) $x - 8y + 7z = 2$

(D) $x - 8y + 7z = -2$

Solution: (D)

The new position of plane is

$(x - 2y + 33) + \lambda(x + y + z - 1) = 0$

$\Rightarrow \quad (1 + \lambda)x + (\lambda - 2)y + (3 + \lambda)z - \lambda = 0$

Given that this is perpendicular to $x + y + z = 1$

$\therefore \quad (1 + \lambda) \cdot 1 + (\lambda - 2) \cdot 1 + (3 + \lambda) \cdot 1 = 0$

$\Rightarrow \quad 1 + \lambda + \lambda - 2 + 3 + \lambda = 0$

$\Rightarrow \quad 3\lambda + 2 = 0$

$\Rightarrow \quad \lambda = \frac{-2}{3}$

Hence, the new position of the plane is $(x - 2y + 3z) - \frac{2}{3}(x + y + z - 1) = 0$

$\Rightarrow \quad 3x - 2x - 6y - 2y + 9z - 2z + 2 = 0$

$x - 8y + 7z = -2$

Physics

Single correct answer type:

1. In process of amplitude modulation of signal to be transmitted.

Signal to be modulated is given by $m(t) = A_m \sin \omega_m t$, carrier wave is given by $c(t) = A_c \sin \omega_c t$, modulated signal $c_m(t)$ is given by

(A) $C_m(t) = A_c \sin \omega_c t + A_m \sin \omega_m t$

(B) $C_m(t) = (A_c + A_m) \sin \omega_c t$

(C) $C_m(t) = [A_c + m(t)] \sin \omega_c t$

(D) None of the above

Solution: (C)

In amplitude modulation, amplitude of carrier wave varies with signal to be modulated so modulated signal $C_m(t)$ is given by

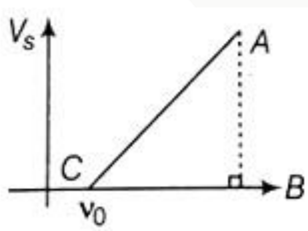
$C_m(t) = [A_c + m(t)] \sin \omega_c t$

$\Rightarrow \quad C_m(t) = (A_c + A_m \sin \omega_m t) \sin \omega_c t$

As $m(t) = A_m \sin \omega_m t$ (given)

2. Graph of stopping potential for most energetic emitted photoelectron (V_S) with frequency of incident radiation on metal is given below

Value of $\frac{AB}{BC}$, in graph is



(h = Planck's constant, e – electronic charge)

(A) h (B) e (C) $\frac{h}{e}$ (D) $\frac{e}{h}$

Solution: (C)

By Einstein's photoelectric effect equation

$KE_{max} = eV_S = h\mathrm{v} - h\mathrm{v}_0$

$\Rightarrow \quad V_S = \left(\frac{h}{e}\right)\mathrm{v} - \frac{h\mathrm{v}_0}{e}$

Graph of V_s with v is straight line whose slope $= \frac{h}{e}$

From given graph slope $= \frac{AB}{BC} \Rightarrow \frac{AB}{BC} = \frac{h}{e}$

3. A block of mass 0.18 kg is attached to a spring of force constant $2\,N/m$. The coefficient of friction between the block and the floor is 0.1. Initially the block is at rest and the spring is unstretched. An impulse is given to the block.

The block slides a distance of 0.06m and comes to rest for the first time the initial velocity of the block in m/s is $v = N/10$. Then N is

(A) 2 (B) 2 (C) 4 (D) 6

Solution: (C)

Here, $m = 0.18\,kg, K = 2\,N/m\,, \mu = 0.1,\ x = 0.06m$.

According to conservation of mechanical energy principle, we know

Decrease in mechanical energy = Work done against friction

$\frac{1}{2}mv^2 - \frac{1}{2}kx^2 = \mu mgx$

$\Rightarrow \quad v = \sqrt{\frac{2\mu mgx + Kx^2}{m}}$

Substituting the values of m, μ, g, x and K, we get

$v = \sqrt{\frac{2 \times 0.1 \times 0.18 \times 9.8 \times 0.06 + 2 \times 0.066}{0.18}}$

$v = \left(\frac{4}{10}\right)m/s.$

So, N = 4

4. Ohm's law says

(A) $V = IR$

(B) V/I = constant

(C) Both ($V = IR$) and (V/I = constant) are correct

(D) Both ($V = IR$) and (V/I = constant) are incorrect

Solution: (B)

Ohm's law says

$$\frac{V}{I} = R = constant$$

5. A train accelerating uniformly from rest attains a maximum speed of $40\, ms^{-1}$ in 20s. It travels at this speed for 20s and is brought to rest with uniform retardation in further 40s. What is the average velocity during this period?

(A) $80\, m/s$ (B) $25\, m/s$ (C) $40\, m/s$ (D) $30\, m/s$

Solution: (B)

(i) $v = u + at_1$

$40 = 0 + a \times 20$

$\therefore a = 2\ m/s^2$

$v^2 - u^2 = 2as$

$40^2 - 0 = 2 \times 2s_1$

$\therefore \quad s_1 = 400\, m$

(ii) $s_2 = v \times t_2 = 40 \times 20 = 800\, m$

(iii) $v = u + at$

$= 0 = 40 + a \times 40$

$\therefore \quad a = -1\ m/s^2$

$0^2 - 40^2 = 2(-1)s_3$

$\therefore \quad s_3 = 800\, m$

Total distance travelled

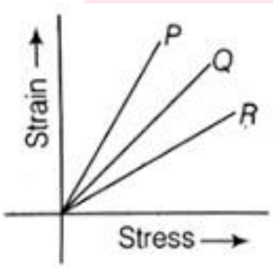
$= s_1 + s_2 + s_3$

$= 400 + 800 + 800 = 2000\, m$

Total time taken $= 20 + 40 = 80s$

$\therefore$ Average velocity $= \frac{2000}{80} = 25\, m/s$

6. The strain-stress curves of three wires of different material are shown in the figure. P, Q, R are the elastic limits of the wires, the figure shows that



(A) Elasticity of wire P is maximum

(B) Elasticity of wire Q is maximum

(C) Tensile strength of R is maximum

(D) None of the above

Solution: (C)

As stress is shown on x-axis and strain on y-axis.

So, we can say that $y = \cot\theta = \frac{1}{\tan\theta} = \frac{1}{slope}$

So, elasticity of wire P is minimum and R is maximum.

7. For the equation $F \propto A^a v^b d^c$, where F is the force, A is the area, v is the velocity and d is the density, the values of a, b and c are respectively.

(A) 1, 2, 1 (B) 2, 1, 1 (C) 1, 1, 2 (D) 0, 1, 1

Solution: (A)

$[MLT^{-2}] = [L^{2a}] \times [L^{b}T^{-b}]\ [M^{c}L^{-3c}]$

$= [M^{c}L^{2a+b-3c}T^{-b}]$

Comparing powers of M, L and T, we get

$c = 1, 2a + b - 3c = 1$

$-b = -2\ \ or\ \ b = 2$

Also, $2a + 2 - 3(1) = 1$

$\Rightarrow \quad 2a = 2\ \ or\ a = 1$

$\therefore$ This is 1, 2, 1

8. In hydrogen atom, an electron jumps from bigger orbit to smaller orbit so that radius of smaller orbit is one-fourth of radius of bigger orbit. If speed of electron in bigger orbit was v then speed in smaller orbit is

(A) $\frac{v}{4}$ (B) $\frac{v}{2}$ (C) v (D) $2v$

Solution: (D)

Radius of nth orbit $r_n \propto n^2$

$\frac{r_{n\ big}}{r_{n\ small}} = \frac{n_{big}^2}{n_{small}^2} = \frac{4}{1}$ (given)

$\Rightarrow \quad \frac{n_{big}}{n_{small}} = 2 \quad \Rightarrow \quad \frac{n_{small}}{n_{big}} = \frac{1}{2}$

Velocity of electron in nth orbit

$v_n \propto \frac{1}{n}$

$\frac{v_{n\ big}}{v_{n\ small}} = \frac{n_{small}}{n_{big}} = \frac{1}{2}$

$v_{n\ small} = 2(v_{n\ big)}\ = 2v$

9. A steel wire of length 4.7m and cross-section $3.0 \times 10^{-5}\ m^2$ stretches by the same amount as a copper wire of length 3.5m and cross-section $4.0 \times 10^{-5}\ m^2$ under a given load. What is the ratio of the Young's modulus of steel so that of copper?

(A) 1.5 : 2 (B) 1.8 : 2 (C) 1.5 : 1 (D) 1. 8 : 1

Solution: (D)

As given for steel wire

$A_1 = 3 \times 10^{-5}\ m^2, l_1 = 4.7m, \Delta l_1 = \Delta l; F_1 = F$

For copper wire,

$A_2 = 4 \times 10^{-5} m^2, l_2 = 3.5m, \Delta l_2 = \Delta l, F_2 = F$

Let Y_1 and Y_2 be the Young's modulus of steel wire and copper wire respectively.

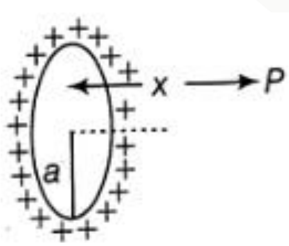
So, $Y_1 = \frac{F_1}{A_1} \times \frac{l_1}{\Delta l_1} = \frac{F}{3\times10^{-5}} \times \frac{4.7}{\Delta l}$

and $Y_2 = \frac{F_2 \times l_2}{A_2 \times \Delta l_2} = \frac{F\times3.5}{4\times10^{-5}\times\Delta l}$

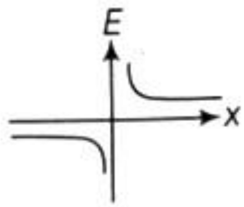
$$\frac{Y_1}{Y_2} = \frac{4.7 \times 4 \times 10^{-5}}{3.5 \times 3.0 \times 10^{-5}} = 1.8$$

So, $Y_1 : Y_2 = 1.8 : 1$

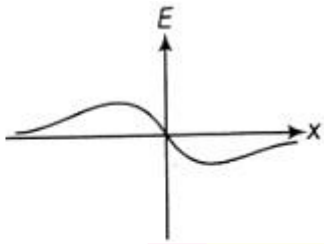
10. A ring shaped conductor with radius a carries a net positive charge q uniformly distributed on it as shown in figure. A point P is situated at a distance x from its centre. Which of following graph shows the correct variation of electric field (E) with distance (x)?



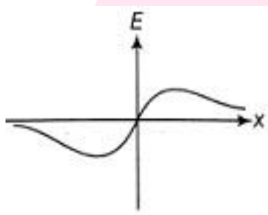
(A)



(B)



(C)



(D) None of the above

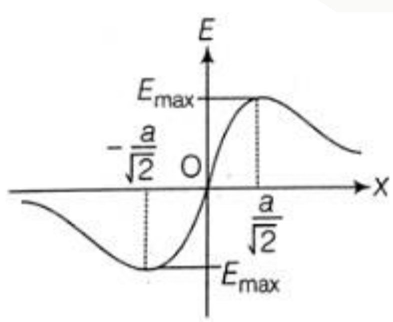
Solution: (B)

The net electric field at point P is given by

$$E = \frac{qx}{4\pi\varepsilon_0 (x^2 + a^2)^{\frac{3}{2}}}$$

∴ At centre of ring $x = 0 \Rightarrow E_0$

IT $x \gg a,\ E = \frac{1}{4\pi\varepsilon_0} \frac{q}{x^2}$



and E will be maximum where,

$$\frac{dE}{dx} = 0$$

or $x = \pm \frac{a}{\sqrt{2}}$

and $E_{max} = \frac{1}{4\pi\varepsilon_0} \frac{2q}{3\sqrt{3}a^2}$

11. ......A..... is the essential condition for coherent sources. Here A refers to

(A) Constant phase difference

(B) Equal amplitude

(C) Both (Constant phase difference) and (Equal amplitude) are correct

(D) Both (Constant phase difference) and (Equal amplitude) are incorrect

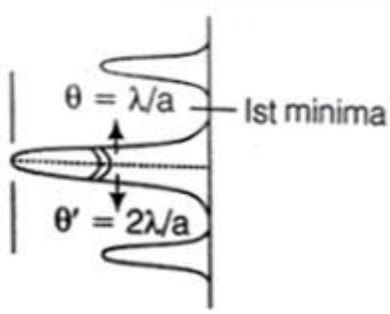
Solution: (A)

Constant phase difference is the essential condition for coherent sources.

12. The angular size of the central maxima due to a single slit diffraction is (a → slit width)

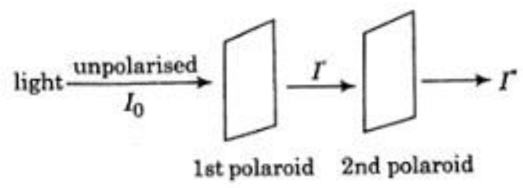
(A) $\frac{\lambda}{a}$ (B) $\frac{2\lambda}{a}$ (C) $\frac{3\lambda}{2a}$ (D) $\frac{\lambda}{2a}$

Solution: (B)



So, angular size of central maxima is $= 2\left(\frac{\lambda}{a}\right) = \frac{2\lambda}{a}$

13.



Find the final intensity of light (I''), if the angle between the axes of two polaroids is 60^o.

(A) $\frac{3I_0}{2}$ (B) $\frac{I_0}{2}$ (C) $\frac{I_0}{4}$ (D) $\frac{I_0}{8}$

Solution: (D)

From first polaroid the unpolarised light will become polarized with half the intensity.

So, $I' = \frac{I_0}{2}$

From second polaroid

$$I'' = I' \cos^2\theta = \frac{I_0}{2}\cos^2(60) = \frac{I_0}{2}\frac{1}{4} = \frac{I_0}{8}$$

14. Resolving power of a telescope will be more, if the diameter (a) of the objective is

(A) Larger (B) Smaller

(C) Resolving power does not depend on a (D) None of the above

Solution: (C)

$RP \propto a$

15. Let binding energy per nucleon of nucleus is denoted as E_{bn} and radius of nucleus is denoted as r. If mass number of nuclei A, B are 64 and 125 respectively, then

(A) $r_A < r_B, E_{bnA} < E_{bnB}$ (B) $r_A > r_B, E_{bnA} > E_{bnB}$

(C) $r_A = \frac{4}{5} r_B, E_{bnA} < E_{bnB}$ (D) $r_A < r_B, E_{bnA} > E_{bnB}$

Solution: (D)

$r = r_0(A)^{\frac{1}{3}}$

r increases with increasing A = mass number So, $r_A < r_B$ as mass number of A is smaller E_{bn} decreases with increasing A for $A > 56$. ^{56}Fe has highest E_{bn} value.

So, E_{bn} for A = 64 is larger as compared to E_{bn} for nucleus with A = 125

$E_{bnA} > E_{bnB}$

16. The heat energy

(A) Is a state variable

(B) Does not depend on the state of the system

(C) Is equal to internal energy of the system

(D) None of the above

Solution: (B)

Heat energy is not a state variable, it just energy in transition.

17. A current 4.0 A exist in a wire of cross-sectional area 2.0 mm^2. If each cubic metre of the wire contains 12.0×10^{28} free electrons then the drift speed is

(A) $2 \times 10^{-8}\ m/s$ (B) $0.5 \times 10^{-3}\ m/s$

(C) $1.04 \times 10^{-4}\ m/s$ (D) None of these

Solution: (C)

The current density in the wire is

$$J = \frac{i}{A} = \frac{4}{2 \times 10^{-6}} = 2 \times 10^{6}\ Am^{-2}$$

The drift speed is

$$v = \frac{j}{hc} = \frac{2 \times 10^{6}}{12 \times 10^{28} \times 1.6 \times 10^{-19}}$$

$$= \frac{10^{6}}{6 \times 1.6 \times 10^{9}} = 1.04 \times 10^{-4}\ m/s$$

18. In an experiment on the specific heat of a metal a 0.20 kg block of the metal at $150^{o}C$ is dropped in a copper calorimeter (of water equivalent 0.025kg) containing 150 cc of water at $27^{o}C$. The final temperature is $40^{o}C$. Calculate the specific heat of the metal. If heat losses to the surroundings are not negligible, is our answer greater or smaller than the actual value of specific heat of the metal?

(A) 0.02 (B) 0.02 (C) 0.01 (D) 0.1

Solution: (D)

Mass of metal $m = 0.2\ kg = 200g$

Fall in temperature of metal

$\Delta T = 150 - 40 = 11^{o}C$

If s is specific heat of metal, the heat lost by the metal

$\Delta Q = ms\ \Delta T = 200 \times 110 \times s$

Volume of water = 150 cc

Mass of water $m' = 150g$

Water equivalent of calorimeter

$w = 0.025kg = 25g$

Rise in temperature of water in calorimeter

$\Delta T' = 40 - 27 = 13^{o}C$

Heat gained by water and calorimeter

$\Delta Q' = (m' + w)\Delta T'$

$= (150 + 25) \times 13$

$\Delta Q' = 175 \times 13 = \Delta Q$

So, $200 \times s \times 100 = 175 \times 13$

$\Rightarrow \quad s = \dfrac{175 \times 13}{200 \times 100} \approx 0.1$

19. Electric field in a region is given by $E = \left(\frac{M}{x^3}\right)\hat{\imath}$, then the correct expression for the potential in the region is (assume potential at infinity is zero).

(A) $\frac{M}{2x^2}$ (B) Mx^2 (C) $\frac{M}{3x^4}$ (D) None of these

Solution: (A)

$$v(x, y, z) = -\int_{\infty}^{(x,y,z)} E \cdot dr$$

$$= -\int_{\infty}^{(x,y,z)} \frac{Mdx}{x^3} = \frac{M}{2x^2}$$

20. A ball is projected upwards from the top of tower with a velocity $50\,m/s$ making an angle 30^o with the horizontal. The height of tower is 70m. After how many seconds from the instant of throwing will the ball reach the ground?

(A) 2 s (B) 5 s (C) 7 s (D) 9 s

Solution: (C)

Taking vertical downward motion of projectile from point of projection to ground we have,

$u = -50\sin 30^o = -25\,m/s$

$a = +10\,m/s^2, s = 70m, t = ?$

$s = ut + \frac{1}{2}at^2$

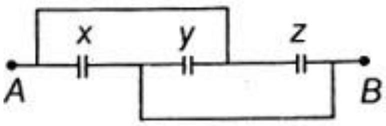
So, $70 = -25 \times t + \frac{1}{2} \times 10 \times t^2$

or $5t^2 - 25t - 70 = 0$

or $t^2 - 5t - 14 = 0$

On solving $t = 7s$

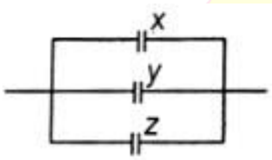
21. Three capacitors $X = 1\mu F, Y = 2\mu F$ and $Z = 3\mu F$ are connected as shown in figure, then the equivalent capacitance between points A and B is



(A) $6\ \mu F$ (B) $12\ \mu F$ (C) $3\ \mu F$ (D) None of these

Solution: (A)

The equivalent circuit of the following figure is as follow.



$\therefore \quad C_{eq} = X + Y + Z$

$= 1 + 2 + 3$

$= 6\mu F$

22. The work done in blowing a soap bubble of surface tension 0.06 Nm^{-1} from 2cm radius to 5cm radius is

(A) 0.004168 J (B) 0.003168 J (C) 0.003158 J (D) 0.004568 J

Solution: (B)

As given $s = 0.06\ Nm^{-1}$,

$r_1 = 2cm = 0.02m, r_2 = 5cm = 0.05m$

Since, bubble has two surface

Initial surface area of the bubble $= 2 \times 4\pi\ r_2^2$

$= 2 \times 4\pi \times (0.02)^2 = 32\pi \times 10^{-4} m^2$

Final surface area of the bubble $= 2 \times 4\pi\ r_2^2$

$= 2 \times 4 \times \pi \times (0.05)^2$

$= 200 \times \pi \times 10^{-4} m^2$

So, work done $= s \times$ increase in surface

$= 0.06 \times (200\pi \times 10^{-4} - 32\,\pi \times 10^{-4})$

$= 0.06 \times 168\pi \times 10^{-4}$

$= 0.003168\, J$

23. The sum of the magnitudes of two forces acting at a point is 16N. The resultant of these forces is perpendicular to the smaller force which has a magnitude of 8N. If the smaller force is magnitude x, then the value of x is

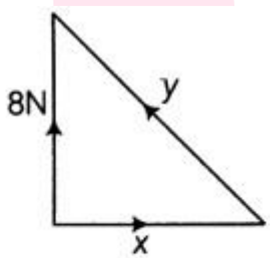
(A) 2N (B) 4N (C) 6N (D) 7N

Solution: (C)

$x + y = 16,$

Also, $y^2 = 8^2 + x^2$

or $y^2 = 64 + (16 - y)^2$



$(\because \quad x = 16 - y)$

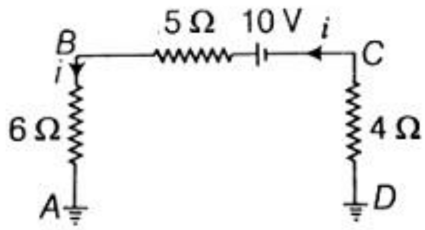
or $y^2 = 64 + 256 + y^2 - 32y$

or $32y = 320$

$Y = 10\, N$

$\therefore \quad x + 10 = 16 \Rightarrow \quad x = 6N$

24. What will be the value of current i in the circuit shown?



(A) 0.67A (B) 1A (C) 0.32 A (D) None of these

Solution: (A)

$V_A - V_D = -6i - 5i + 10 - 4i$

Here, $V_A = V_D$

Since, points A and D are centred.

$-6i - 5i + 10 - 4i = 0 \Rightarrow \quad 15i = 10$

$$\Rightarrow \quad i = \frac{10}{15} = 0.67A$$

25. The average depth of Indian Ocean is about 300m. Bulk modulus of water is $2.2 \times 10^4\ N/m^2$, $g = 10\, m/s^2$, then fractional compression $\frac{\Delta V}{V}$ of water at the bottom of the Indian Ocean will be

(A) 1.36% (B) 20.6% (C) 13.9% (D) 0.52%

Solution: (A)

The pressure exerted by a 3000m column of water on the bottom layer.

$p = h\rho g = 3000 \times 1000 \times 10$

$= 3 \times 10^7\ kg\, m^{-1} s^{-2} = 3 \times 10^7\ N/m^2$

Fractional compression $\left(\frac{\Delta V}{V}\right)$

$$= \frac{Stress}{B} = \frac{3 \times 10^7}{2.2 \times 10^9}$$

$= 1.36 \times 10^{-2}$

$$\frac{\Delta V}{V} \times 100 = 1.36\%$$

26. If A and B denote the sides of parallelogram and its area is $\frac{1}{2}AB$ (A and B are magnitude of A and B respectively), the angle between A and B is

(A) 30^o (B) 45^o (C) 60^o (D) 90^o

Solution: (A)

Area of parallelogram $= |A \times B|$

$$AB \sin\theta = \frac{1}{2}AB$$

$$\sin\theta = \frac{1}{2}, \theta = 30^o$$

27. If edge length of a cuboid are measured to be 1.2cm, 1.5cm and 1.8cm, then volume of the cuboid is

(A) $3.240\ cm^3$ (B) $3.24\ cm^3$ (C) $3.2\ cm^3$ (D) $3.0\ cm^3$

Solution: (C)

Volume of cuboid $= l \times b \times h$

$$= 1.8 \times 1.5 \times 1.2\ cm^3$$

$$= 2.70 \times 1.2$$

$$= 3.240\ cm^3$$

Using concept of significant figures, product is reported in number of significant figures present in measurement which has least number of significant figures, here all measurement have 2 significant figures.

So, volume $= 3.2\ cm^3$

(keeping 2 significant figures only.)

28. A ball thrown upward from the top of a tower with speed v reaches the ground in t_1 sec. If this ball is thrown downward from the top of the same tower with speed v it

reaches the ground in t_2 sec. In what time the ball shall reach the ground, if it is allowed to fall freely under gravity from the top of the tower?

(A) $\frac{t_1+t_2}{2}$ (B) $\frac{t_1-t_2}{2}$ (C) $\sqrt{t_1t_2}$ (D) t_1+t_2

Solution: (C)

$h=-vt_1+\frac{1}{2}gt_1^2 \ \ or \ \frac{h}{t_1}=v+\frac{1}{2}gt_1$ ......(i)

$h=vt_2+\frac{1}{2}gt_2^2 \ \ or \ \frac{-h}{t_2}= -v+\frac{1}{2}gt_2$ ......(ii)

$\therefore \quad \frac{h}{t_1}+\frac{h}{t_2}=\frac{1}{2}g\,(t_1+t_2)$

or $\quad h=\frac{1}{2}gt_1t_2$

For falls under gravity from the top of the tower $h=\frac{1}{2}gt^2$

$\therefore \quad \frac{1}{2}gt_1t_2=\frac{1}{2}gt^2$

$\Rightarrow \qquad t=\sqrt{t_1t_2}$

29. One end of steel wire is fixed to ceiling of an elevator moving up with an acceleration $2\ m/s^2$ and a load of 10kg hands from other end. Area of cross-section of the wire is $2\ cm^2$. The longitudinal strain in the wire is $(g=10\,m/s^2$ and $y=2\times 10^{11}\ Nm^{-2})$

(A) 4×10^{11} (B) 3×10^{-6} (C) 8×10^{-6} (D) 2×10^{-6}

Solution: (B)

As $T=m(g+a_0)=10(10+2)=120\ N$

Stress $=\frac{T}{A}=\frac{120}{2\times10^{-4}}=60\times10^4Nm^{-2}$

and $\quad Y=\frac{\text{Stress}}{\text{Strain}},\text{Strain}=\frac{\text{Stress}}{Y}$

$=\frac{60\times10^4}{2\times10^{11}}$

$=30\times10^{-7}=3\times10^{-6}$

30. the electrostatic force of repulsion between two positively charged ions carrying equal charge is $3.7 \times 10^{-9}N,$ when they are separated by a distance of 5 Å. How much electrons are missing from each ion?

(A) 10 (B) 8 (C) 2 (D) 1

Solution: (C)

Here, $F = 3.7 \times 10^{-9}N$

Let, $q_1 = q_2 = q$

$r = 5\text{Å} = 5 \times 10^{-10}m$

$\because \quad F = \frac{1}{4\pi\varepsilon_0} \frac{q_1\, q_2}{r^2}$

$\Rightarrow \quad 3.7 \times 10^{-9} = 9 \times 10^9 \frac{q \times q}{(5 \times 10^{-10})^2}$

$q^{22} = \frac{3.7 \times 10^{-9} \times 25 \times 10^{-20}}{9 \times 10^9}$

$= 10.28 \times 10^{-38}$

or $q = 3.2 \times 10^{-19}C$

Now, $q = ne$

$\therefore \quad n = \frac{q}{e} = \frac{3.2 \times 10^{-19}}{1.6 \times 10^{-19}} = 2$

31. A narrow beam of protons and deuterons, each having the same momentum, enters a region of uniform magnetic field directed perpendicular to their direction of momentum. The ratio of the radii of the circular paths described by them is

(A) $1:2$ (B) $1:1$ (C) $2:1$ (D) $1:3$

Solution: (B)

Since the radius of circular path of charge particle of in magnetic field $r = \frac{mv}{qB} = \frac{p}{qB}$

Now, the radius of circular path of charge particle of given momentum p and magnetic field B is given by

$$r \propto \frac{1}{q}$$

But charge on both charge particles protons and deuterons, is same. Therefore,

$$\frac{r_p}{r_D} = \frac{q_D}{q_p} = \frac{1}{1}$$

32. A solenoid of length 1.0m has a radius of 1cm and has a total of 1000 turns wound on it. It carries a current of 5A. If an electron were to move with a speed of $10^4\ ms^{-1}$ along the axis of this current carrying solenoid the force experienced by this electron is

(A) 2N (B) 1.2N (C) zero (D) 2.5N

Solution: (C)

Here, $L = 1m, N = 1000$

The number of turn per unit length $n = N/L = 1000\ \ turn/m$

Magnetic field inside the solenoid

$$B = \mu_0 nI = \mu_0 \times 1000 \times 5 = 2\pi \times 10^{-3} T$$

The direction of magnetic field is along the solenoid.

For electron $q = -e, v = 10^4\ ms^{-1}$

Magnetic Lorentz force $F = -evB \sin 0^o = 0$ as the angle between B and v is 0^o.

33. Magnetic field

(A) Can increase the speed of charge particle

(B) Can accelerate a charge particle

(C) Both (can increase the speed of charge particle) and (can accelerate a charge particle) are correct

(D) Both (can increase the speed of charge particle) and (can accelerate a charge particle) are incorrect

Solution: (B)

Magnetic field can accelerate a charge particle by charging the direction of its velocity but it cannot change the speed of charged particle as magnetic force always acts perpendicular to the velocity of charged particle.

34. A square coil of side 10cm has 20 turn and carries a current of 12A. The coil is suspended vertically and the normal to the plane of the coil, makes an angle θ with the direction of a uniform horizontal magnetic field of 0.80 T. If the torque, experienced by the coil, equals 0.96N-m, the value of θ is

(A) 0^o (B) $\frac{\pi}{2}$ radian (C) $\frac{\pi}{3}$ radian (D) $\frac{\pi}{6}$ radian

Solution: (D)

Area of coil $A = side^2 = (0.1)^2 - 0.01m^2,$

Number of turns N = 20, current $I = 12A,$

Normal to the coil make an angle θ with the direction of B, magnetic field $B = 0.80T$
Torque, experienced by the coil, $\tau = 0.96\ N - m$

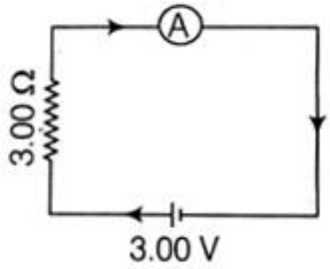
Since, total torque on the coil $\tau = (NIA)B \sin\theta$

Substituting the values in above formula $0.96\ N - m = 20 \times 12\ A \times 0.01m^2 \times 0.80T \times \sin\theta$

$$\sin\theta = \frac{0.96}{1.92} = \frac{1}{2}$$

$$\theta = \frac{\pi}{6} \text{ radian}$$

35. In the circuit (figure) the current is to be measured. The ammeter shown is a galvanometer with a resistance $R_G = 60.00\Omega$ converted to an ammeter by a shunt resistance $r_s = 0.02\Omega$. The value of the current is



(A) 0.79 A (B) 0.29 A (C) 0.99 A (D) 0.8 A

Solution: (C)

$R_g = 60.00\Omega$ shunt resistance $r_s = 0.02\Omega$

Total resistance in the circuit is,

$R_G + 3 = 63\Omega$. Hence, $I = \frac{3}{63} = 0.048\ A$

Resistance of the galvanometer converted to an ammeter is,

$$\frac{R_G r_s}{R_G + r_s} = \frac{60\Omega \times 0.02\Omega}{(60 + 0.02)\Omega} = 0.02\Omega$$

Total resistance in the circuit is,

$= 0.02 + 3 = 3.02\Omega$

Hence, $l = \frac{3}{3.02} = 0.99\ A$

36. The susceptibility of a magnetism at 300K is 1.2×10^{-5}. The temperature at which the susceptibility increases to 1.8×10^{-5} is

(A) 150 K (B) 200 K (C) 250 K (D) 20 K

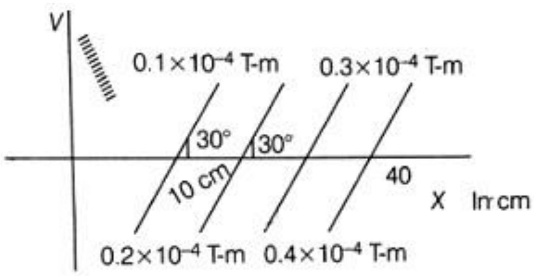
Solution: (B)

$$x = \frac{C}{T} \Rightarrow \frac{x_1}{x_2} = \frac{T_2}{T_1}$$

$$\Rightarrow \quad \frac{1.2 \times 10^{-5}}{1.8 \times 10^{-5}} = \frac{T_2}{300}$$

$$\Rightarrow \quad T_2 = \frac{12}{18} \times 300 = 200K$$

37. Figure shows some of the equipotential surface of the magnetic scalar potential. The magnetic field B at a point in the region is



(A) $1 \times 10^{-4} T$ (B) $2 \times 10^{-4} T$ (C) $3 \times 10^{-4} T$ (D) $4 \times 10^{-4} T$

Solution: (B)

Since perpendicular distance between two equipotential is

$dx = 10 \sin 30^o \ cm = 5cm$

Since the potential gradient gives magnetic field

$(B)\ as$

$$\frac{dV}{dx} = B$$

Substituting the values

$$\frac{dV}{dx} = B = \frac{0.1 \times 10^{-4} T - m}{5 \times 10^{-2} m}$$

$B = 2 \times 10^{-4} T$

Since B is perpendicular to equipotential surface. Here, it is at angle 120^o with (+ ve) x-axis.

38. In a thermodynamic process, the pressure of a fixed mass of a gas is changed in such a manner that the gas release 20J of heat and 8J of work is done on the gas. If the initial internal energy of the gas was 30J, then the final internal energy will be

(A) 2 J (B) 42 J (C) 18 J (D) 58 J

Solution: (C)

Given, $Q = -20\,J, W = -8\,J$

Using I^{st} law

$Q = \Delta U + W$

$\Rightarrow \quad -20 = \Delta U + (-8)$

$\Rightarrow \quad \Delta U = -12\,J$

$\Rightarrow \quad U_f - U - i = -12\ J$

$\Rightarrow \quad U_f = 30 = -12\,J$

$\Rightarrow \quad U_f = 18\,J$

39. Two balloons are filled one with pure He gas and other with air respectively. If the pressure and temperature of these balloons are same, then the number of molecules per unit volume is

(A) More in He filled balloon

(B) Same in both balloons

(C) More in air filled balloon

(D) In the ratio 1 : 4

Solution: (B)

Assuming the balloons have the same volume, as $pV = nRT$. If p, V and T are the same, n the number of moles present will be the same, whether it is He or air. Hence, number of molecules per unit volume will be same in both the balloons.

40. The equation of a simple harmonic motion is given by $y = 3\sin\frac{\pi}{2}\,(50t - x)$, where x and y are in metres and t is in seconds, the ratio of maximum particle velocity to the wave velocity is

(A) 2π (B) $\frac{3}{2}\pi$ (C) 3π (D) $\frac{2}{3}\pi$

Solution: (B)

The given wave equation is

$y = 3 \sin \frac{\pi}{2}\ (50t - x)$

$\Rightarrow \quad y = 3 \sin \left(25\pi t - \frac{\pi}{2} x\right)$ .....(i)

Comparing with standard equation

$y = a \sin(\omega t - kx)$ .....(ii)

$\omega = 25\pi, k = \frac{\pi}{2}$

Wave velocity $v = \frac{\omega}{K} = \frac{25\pi}{\frac{\pi}{2}} = 50\ m/s$