TEST PAPER 10

Total Questions: 60 — Time allotted 75 minutes

Q.1) The coordinates of a point equidistant from four distinct points (0, 0, 0), (*a*, 0, 0), (0, *b*, 0) and (0, 0, *c*) are

(a) $\left(\frac{a+b+c}{3}, \frac{a+b+c}{3}, \frac{a+b+c}{3}\right)$ (b) (*a*, *b*, *c*)

(c) $\left(\frac{a}{2}, \frac{b}{2}, \frac{c}{2}\right)$ (d) $\left(\frac{a}{3}, \frac{b}{3}, \frac{c}{3}\right)$

Q.2) A straight line with direction cosines (0, 1, 0) is

(a) parallel to the *x*-axis (b) parallel to the *y*-axis

(c) parallel to the *z*-axis (d) equality inclined to all the axes

Q.3) The equation of the set of all points equidistant from the point (4, 2) and the *x*-axis is

(a) $x^2 - 6x + 4y + 10 = 0$ (b) $x^2 - 6x + 4y - 10 = 0$

(c) $x^2 - 8x - 4y + 20 = 0$ (d) $y = 3$

Q.4) The equation $ax^2 + 2hxy + by^2 = 0$ represents a pair of real and distinct straight lines passing through the origin if

(a) $h^2 \geq ab$ (b) $h^2 > ab$

(c) $h^2 = ab$ (d) $h^2 < ab$

Q.5) The line passing through the points (1, 2, -1) and (3, -1, 2) meets the *yz*-plane at the points

(a) $\left(0, \frac{-7}{2}, \frac{5}{2}\right)$ (b) $\left(0, \frac{7}{2}, \frac{5}{2}\right)$

(c) $\left(0, \frac{-7}{2}, \frac{-5}{2}\right)$ (d) $\left(0, \frac{7}{2}, \frac{-5}{2}\right)$

Q.6) The equation of the sphere whose centre is (1, 1, 1) and which passes through (3, 3, 2), is

(a) $x^2 + y^2 + z^2 + 2x + 2y + 2z = 6$ (b) $x^2 + y^2 + z^2 - 2x - 2y - 2z = 0$

(c) $x^2 + y^2 + z^2 - 2x - 2y - 2z = 6$ (d) $x^2 + y^2 + z^2 + 2x + 2y - 2z = 38$

Q.7) If $f(x) = \sqrt{x^2 + 4x - 5}$, then the domain of the real valued function *f*(*x*) is

(a) set of all real numbers (b) set of all integers

(c) [-5, 1] (d) $(-\infty, -5] \cup [1, \infty)$

Q.8) The inverse f^1 (*x*) of the function *f*: *IR*[1]⌈*R*/{1}, defined by $f(x) = \frac{x+1}{x-1}$, is

(a) $\frac{x+1}{x-1}$ (b) $\frac{x-1}{x+1}$

(c) $\frac{1}{x}$ (d) *x*

Q.9) $\lim_{x \to 3} \frac{|x-3|}{x-3}$

(a) is equal to 1 (b) is equal to 0

(c) is equal to -1 (d) does not exist

Q.10) $\lim_{\theta \to 0} \frac{\sqrt{1-\cos\theta}}{\theta}$ is equal to

(a) $\sqrt{2}$ (b) $2\sqrt{2}$

(c) $\frac{1}{\sqrt{2}}$ (d) $\frac{1}{2\sqrt{2}}$

Q.11) For the parabola $y^2 - 8y - x + 19 = 0$; the focus and directrix are

(a) $\left(\frac{13}{4}, 4\right)$ and $x = \frac{11}{4}$ (b) $\left(\frac{19}{7}, 8\right)$ and $y = 7$

(c) $\left(\frac{7}{2}, 3\right)$ and $y = 9$ (d) (6, 3) and $x = 7$

Q.12) $f(x) = \cos(|x|)$ is a continuous function because

(a) composition of continuous function is a continuous function

(b) product of a continuous function is a continuous function

(c) cosine is an even function

(d) sum of a continuous function is continuous

Q.13) If $y = \text{cosec}^{-1}\frac{\sqrt{x}+1}{\sqrt{x}-1} + \cos^{-1}\frac{\sqrt{x}-1}{\sqrt{x}+1}$, then $\frac{dy}{dx}$ is equal to

(a) $\frac{2}{\left(\sqrt{x}-1\right)^2}$ (b) $\frac{2}{\left(\sqrt{x}+1\right)^2}$

(c) $\frac{1}{x-1}$ (d) 0

Q.14) The derivative of $\sin^{-1}\left(\frac{2x}{1+x^2}\right)$ with respect to $\tan^{-1}\left(\frac{2x}{1-x^2}\right)$ is

(a) -2 (b) -1

(c) 1 (d) 2

Q.15) If $f(x) = \sin(\cos x)$, then $f'(x)$ is

(a) $\cos(\cos x)$ (b) $\sin(-\sin x)$

(c) $-\sin(\cos x)$ (d) $-\sin x \,.\, \cos(\cos x)$

Q.16) The second derivative of $f(e^x)$ with respect to x, where f is a polynomial is

(a) $f''(e^x)e^x + f'(e^x)$ (b) $f''(e^x)e^{2x} + f''(e^x)e^x$

(c) $f''(e^x)$ (d) $f''(e^x)e^{2x} + f'(e^x)e^x$

Q.17) A real function f of a real variable x is defined as $f(x) = x - [x]$, where $[x]$ denotes the greatest integer $\leq x$. On which one of the following sets f is monotonically increasing?

(a) R, the set of reals (b) [0, 1)

(c) [0, 11] (d) (0, 1]

Q.18) The maximum value of sin x + cos x is equal to

(a) $-\sqrt{2}$ (b) $\sqrt{2}$

(c) 2 (d) $\sqrt{3}$

Q.19) The value of k for which the integral of $\frac{3x^3+7x^2-2}{x}+\frac{3kx+1}{x^2},(x \neq 0)$; may be a rational function is

(a) $\frac{3}{2}$ (b) $\frac{2}{3}$

(c) $\frac{-3}{2}$ (d) $\frac{-2}{3}$

Q.20) Match List 1 (Equality/Inequality) with List II (Inference) and select the correct answer using the codes given below the lists:

	List I (*Equality / Inequality)*)		List II (*Inference*)
A.	$P(E_1)+P(E_2)=1$	1.	E_1, E_2 are mutually exclusive events
B.	$P(E_1)+P(E_2)=0$	2.	E_1, E_2 are mutually exhaustive events
C.	$P(E_1)+P(E_2)\leq 0$	3.	E_1, E_2 are sure events
D.	$P(E_1).P(E_2)=1$	4.	E_1, E_2 are impossible events
		5.	E_1, E_2 are not equally likely events

Codes:

	A	B	C	D
(a)	2	1	4	3
(b)	3	1	5	2
(c)	2	4	1	3
(d)	1	3	2	5

Q.21) The value of the integral $\int\left(\frac{2+\sin 2x}{1+\cos 2x}\right)e^x dx$ is equal

(a) $e^x \sin x$ (b) $e^x \cos x$

(c) $e^x \tan x$ (d) $e^x \cot x$

Q.22) The value of $\int \frac{x^{e-1}+e^{x-1}}{x^2+e^x} dx$ is equal to

(a) x (b) $\log(x+e)$

(c) $\log(e^x+x^e)$ (d) $\log\left[\left(x^e+e^x\right)^{1/e}\right]$

Q.23) If $I_1=\int_e^{e^2}\frac{dx}{\log x}$ and $I_2=\int_1^2\frac{e^x}{x}dx$, then

(a) $I_1 = I_2$ (b) $2I_1 = I_2$

(c) $I_1 = I_2 = 0$ (d) $I_1 = 2I_2$

Q.24) The value of the integral $\int\frac{1}{x\left(x^7+1\right)}dx$ is equal to

(a) $\frac{1}{2}\log\left|\frac{x^7-7}{x^7+1}\right|+c$ (b) $\frac{1}{7}\log\left|\frac{x^7}{x^7+1}\right|+c$

(c) $\log\left|\frac{x^7+1}{7x}\right|+c$ (d) $7\log\left|\frac{x^7}{x^7+1}\right|+c$

Q.25) The value of the integral $\int\frac{dx}{e^x-1}$ is equal to

(a) $\log\left(e^x-1\right)+c$ (b) $\log\left(1-e^{-x}\right)+c$

(c) $\log\left(1-e^x\right)+c$ (d) $\log\left(e^{-x}-1\right)+c$

Q.26) The value of the integral $\frac{1}{8}\int\left[\operatorname{cosec}h^2x+\frac{x^3\left(\tan^{-1}x^4\right)}{1+8x}\right]dx$ is equal to

(a) $-\frac{1}{8}\cot h\ x+\frac{1}{8}\tan^{-1}\ x^4+c$ (b) $\frac{1}{8}\cot h\ x+\frac{1}{8}\tan^{-1}\ x^4+c$

(c) $\frac{1}{8}\cot h\ x+\frac{1}{64}\left(\tan^{-1}\ x^4\right)^2+c$ (d) $-\frac{1}{8}\cot h\ x+\frac{1}{64}\left(\tan^{-1}\ x^4\right)^2+c$

Q.27) The value of the integral $\int_0^{\sqrt{2}}\left[x^2\right]dx$, where [*x*] is the greatest integer ≤ *x*, is given by

(a) $\sqrt{2}-1$ (b) $1-\sqrt{2}$

(c) $\frac{\left(\sqrt{2}\right)^3}{3}$ (d) $2\left(\sqrt{2}-1\right)$

Q.28) Let $f:\mathbb{R}\to\mathbb{R}$ and $g:\mathbb{R}\to\mathbb{R}$ be two continuous functions, then the value of $\int_{-a}^{a}\left[f(x)g(-x)-f(-x)g(x)\right]dx$ is equal to

(a) e^x (b) $2\int_0^a f(x)\ g(x)\ dx$

(c) 1 (d) 0^0

Q.29) The area bounded by the curves $y = x^2$ and $y = 2|x|$ is equal to

(a) $\frac{4}{3}$ (b) $\frac{8}{3}$

(c) $\frac{2}{3}$ (d) $\frac{1}{3}$

Q.30) If *A* is the area between the curve *y* = sin *x* and *x*-axis in the interval [0, π/4], then in the same interval, area between the curve *y* = cos *x* and *x*-axis, is equal to

(a) *A* (b) $\frac{\pi}{2}-A$

(c) 1 – *A* (d) 2*A*

Q.31) The degree and order of the differential equation $\left\{1+\left(\frac{dy}{dx}\right)^2\right\}^{3/2}=\frac{d^2y}{dx^2}$, respectively are

(a) 3, 1 (b) 2, 2/3
(c) 1, 2 (d) 2, 2

Q.32) Which one of the following differential equations is satisfied by the family of curves $y=2+c.e^{-2x^2}$?

(a) $\frac{dy}{dx}=8x$ (b) $\frac{dy}{dx}+4xy=8x$

(c) $\frac{dy}{dx}+4xy=0$ (d) $\frac{dy}{dx}-4xy=0$

Q.33) The general solution of the differential equation $\frac{dy}{dx}+\frac{x}{y}=0$ is given by

(a) $x^2+y^2=a^2$ (b) $(x-a)^2+(y-a)^2=1$

(c) $x^2+y^2=axy$ (d) $x+y=a$

Q.34) The solution of the differential equation $\left(2x\cos y+3x^2y\right)dx+\left(x^3-x^2\sin y-y\right)dy=0$, is given by

(a) $x^2\cos y+x^3y-\frac{y^2}{2}=\text{constant}$ (b) $x^2\sin y+x^3y-\frac{y^2}{2}=\text{constant}$

(c) $x^2\sin y-x^3y-\frac{y^2}{2}=\text{constant}$ (d) $x^2\cos y-x^3y-\frac{y^2}{2}=\text{constant}$

Q.35) The solution of the differential equation $\frac{dy}{dx}=\frac{y^2}{1-3xy}$ is given by

(a) $y^3x=\frac{y^2}{2}+c$ (b) $y^3=\frac{xy^2}{2}+c$

(c) $x=\frac{(1+2cy)}{y^3}$ (d) $x=\frac{c}{y^3}$

Q.36) The probability of solving a problem by three students, *X*, *Y* and *Z* is $\frac{1}{2},\frac{1}{3}$ and $\frac{1}{4}$ respectively. The probability that the problem will be solved is

(a) $\frac{1}{4}$ (b) $\frac{1}{2}$

(c) $\frac{3}{4}$ (d) $\frac{1}{3}$

Q.37) The projection of the vector $\vec{a}=\hat{i}-2\hat{j}+\hat{k}=0$ on the vector $\vec{b}=4\hat{i}-4\hat{j}+7\hat{k}$ is equal to

(a) $\frac{\sqrt{6}}{9}$ (b) $\frac{19}{9}$

(c) $\frac{9}{19}$ (d) $\frac{\sqrt{6}}{19}$

Q.38) Position vector of a point P is $\vec{r}$ from origin of coordinate axis. A force $\vec{F}$ passes through the point P. The moment of the force about the origin is

(a) $\vec{r} \times \vec{F}$ (b) $\vec{r}.\vec{F}$

(c) $\vec{F} \times \vec{x}$ (d) Zero

Q.39) If $|\vec{a}| = 3, |\vec{b}| = 4$ and $|\vec{a} - \vec{b}| = 5$, then the value of $|\vec{a} + \vec{b}|$, is equal to

(a) 6 (b) $5\sqrt{2}$

(c) 5 (d) 4

Q.40) If $\vec{a}, \vec{b}$ are two unit vectors and θ is the angle between them, then the value of $\cos\frac{\theta}{2}$ is equal to

(a) $\frac{1}{2}|\vec{a} - \vec{b}|$ (b) $\frac{1}{2}(\vec{a}.\vec{b})$

(c) $\frac{|\vec{a} - \vec{b}|}{2|\vec{a}||\vec{b}|}$ (d) $\frac{1}{2}|\vec{a} + \vec{b}|$

Q.41) If the sides of parallelogram are $2\hat{i} + 4\hat{j} - 5\hat{k}$ and $\hat{i} + 2\hat{j} + 3\hat{k}$, then the unit vector parallel to one of the diagonals is equal to

(a) $\frac{1}{7}(2\hat{i} + 6\hat{j} - 2\hat{k})$ (b) $\frac{1}{7}(3\hat{i} - 6\hat{j} - 2\hat{k})$

(c) $\frac{1}{7}(-3\hat{i} + 6\hat{j} - 2\hat{k})$ (d) $\frac{1}{7}(3\hat{i} + 6\hat{j} + 2\hat{k})$

Q.42) If $\vec{a} + \vec{b} + \vec{c} = 0$ and $|\vec{a}| = 6, |\vec{b}| = 8$ and $|\vec{c}| = 10$, then the value of $\vec{a}.\vec{b} + \vec{b}.\vec{c} + \vec{c}.\vec{a}$ is equal to

(a) 100 (b) – 100

(c) 200 (d) – 200

Q.43) The magnitude of the displacement vector from position (2, 4, 2) to position (6, 1, 12) is

(a) $5\sqrt{5}$ (b) $2\sqrt{3}$

(c) $3\sqrt{7}$ (d) $3\sqrt{8}$

Q.44) If $\vec{a} + 2\vec{b} + 3\vec{c} = 0$ and $(\vec{a} \times \vec{b}) + (\vec{b} \times \vec{c}) + (\vec{c} \times \vec{a}) = \lambda(\vec{b} \times \vec{c})$ then the value of λ is equal to

(a) 2 (b) 3

(c) 4 (d) 6

Q.45) A particle moves along a circular path of radius r in xy-plane. The position vector $\vec{R}$ of this particle as a function of its y coordinate is

(a) $\sqrt{x^2 - y^2}\,\hat{i} + y\hat{j}$ (b) $\sqrt{r^2 - y^2}\,\hat{i} + y\hat{j}$

(c) $\sqrt{y^2 - r^2}\,\hat{i} - y\hat{j}$ (d) $\sqrt{r^2 - y^2}\,\hat{i} - y\hat{j}$

Q.46) A ticket is drawn at random from the ticket align numbered 1 to 10. The probability that the ticket has a number which is multiple either of 2 or 3 is

(a) $\frac{1}{10}$ (b) $\frac{7}{10}$

(c) $\frac{2}{7}$ (d) $\frac{3}{5}$

Q.47) A card is drawn from a well-shuffled pack of 52 cards. The probability of its being a spade or a queen is

(a) $\frac{1}{13}$ (b) $\frac{1}{4}$

(c) $\frac{17}{52}$ (d) $\frac{4}{13}$

Q.48) A dice is thrown twice. The probability that at least one of the two throws comes up with the number 3 is

(a) $\frac{5}{12}$ (b) $\frac{11}{36}$

(c) $\frac{1}{12}$ (d) $\frac{7}{24}$

Q.49) The statistical data regarding production of food grains in India, available in a Government of India publication, is

(a) primary (b) secondary

(c) primary as well as secondary (d) neither primary nor secondary

Q.50) In a frequency distribution, class marks are 37, 47, 57, 67. Its class boundaries are given

(a) 34.5-44.5, 44.5-54.5, 54.5-64.5…

(b) 31.5-41.5, 41.5-51.5, 51.5-61.5, …

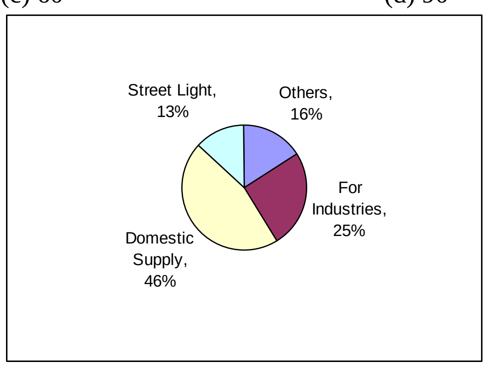
(c) 32-42, 42-52, 52-62, …

(d) 35-45, 45-55, 55-65, …

Q.51) The adjoining diagram gives a Pie-chard representing the units of electricity sold to different classes in a month. The angle of the sector corresponding to supply for industries is

(a) 25^0 (b) 46^0

(c) 60^0 (d) 90^0



Q.52) The mean of 30 given number, when it is given that the mean of 10 of them is 12 and the mean of the remaining 20 is 9, is equal to

(a) 11 (b) 10

(c) 9 (d) 5

Q.53) If $n = 20$, $\bar{x} = 50$ and $\Sigma x^2 = 84000$, then the variance is equal to

(a) 1500 (b) 1700

(c) 1750 (d) 1800

Q.54) If for the variables x and y, the two regression lines are $3x + 2y - 25 = 0$ and $6x + y - 30 = 0$, then the coefficient of correlation r is equal to

(a) 0.5 (b) – 0.5

(c) 0.6 (d) – 0.6

Q.55) When the number of classes is increased indefinitely and the width of the classes is decreased indefinitely, then the frequency polygon becomes

(a) histogram (b) frequency curve

(c) pie-chart (d) line graph

Directions : *The following five items consist of two statements, one labeled the 'Assertion A' and the other labeled as 'Reason R'. You are to examine these two statements carefully and decide if the Assertion A and the Reason R are individually true and if so, whether the reason is a correct explanation of the assertion. Select your answer to these items using the codes below and mark your answer sheet accordingly:*

Codes :

(a) Both (A) and (R) are true and (R) is the correct explanation of (A).

(b) Both (A) and (R) are true but (R) is NOT a correct explanation of (A).

(c) (A) is true but (R) is false.

(d) (A) is false but (R) is true.

Q.56) Assertion : For $b = -5$, $x + 3$ is a factor of $x^3 + 2x^2 + bx - 6$

Reason : If $f(x)$ is a polynomial and $f(a) = 0$, then $x - a$ is a factor of $f(x)$.

Q.57) Assertion : If A is an $n \times n$ matrix, then $\det(mA) = m^m \det A$, where m is any scalar.

Reason : If U is a matrix obtained from V by multiplying any row of column by a scalar m, then $\det U = m \det V$.

Q.58) Assertion : If A is any event and $P(B) = 1$, then A and B are independent.

Reason : $P\left(A \cap B\right) = P\left(A\right) . P(B)$, if A and B are independent.

Q.59) Assertion : The modulus function : $f(x) = | \times |$ is NOT one-one.

Reason : The negative real numbers are NOT the images of any real numbers.

Q.60) Assertion : The elimination of two arbitrary constants in $y = (a + b)\, x$ results into a differential equation of the first order $x\dfrac{dy}{dx} = y$.

Reason : Elimination of n arbitrary constants requires, in general, a differential equation of the nth order.

ANSWER KEYS

1.	(c)	13.	(d)	25.	(b)	37.	(b)	49.	(b)
2.	(b)	14.	(d)	26.	(c)	38.	(a)	50.	(c)
3.	(c)	15.	(a)	27.	(c)	39.	(c)	51.	(d)
4.	(b)	16.	(d)	28.	(d)	40.	(b)	52.	(b)
5.	(d)	17.	(d)	29.	(a)	41.	(a)	53.	(b)
6.	(c)	18.	(c)	30.	(c)	42.	(b)	54.	(b)
7.	(d)	19.	(d)	31.	(d)	43.	(a)	55.	(d)
8.	(b)	20.	(c)	32.	(b)	44.	(d)	56.	(a)
9.	(a)	21.	(c)	33.	(a)	45.	(b)	57.	(a)
10.	(c)	22.	(d)	34.	(a)	46.	(b)	58.	(a)
11.	(a)	23.	(a)	35.	(a)	47.	(d)	59.	(c)
12.	(a)	24.	(b)	36.	(c)	48.	(b)	60.	(a)