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(B) Description of Module

Items	Description of Module
Subject Name	Geography
Paper Name	Quantitative Techniques in Geography
Module Name	Measurement of Inequality:Lorenz Curve and Gini's Concentration Ratio
Module ID	**QT-8**
Pre- Requisites	Coordinate geometry and elementary ariithamatics
Objectives	To understand the statistical technique to measure the inequality in the distributions.
Key words	Inequality, line of equal distribution, concentration ratio,

Module 8

Measurement of Inequality:Lorenz Curve and Gini's Concentration Ratio

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(1) E- Text

Distribution of resources over individuals or over space is seldom uniform. Sometimes it is geographical variation which causes inequality in the distributions of resources and sometimes these inequalities are caused by the manmade policies also. Measurement of inequality in the distribution of income or any other variable like distribution of land among people and population or location of various facilities over space has always been the concern of scholars.. Measures like standard deviation, skewness, location quotient index of dissimilarity, Lorenz

Curve and Ginni's Concentration Ratio etc. are few well known and useful indices serving this purpose.

Lorenz curve has since long been used to measure the inequality in the distribution of wealth or income in societies. It has also been used to show the inequalities in the distribution of several social facilities over space, concentration of population and other demographic attributes etc. The basis of Lorenz Curve is to compare cumulative percentage distribution of one variable in relation to the cumulative percentage distribution of another variable at different stages. Theory behind Lorenz Curve is that if at different stages cumulative percentage of one variable is equal to cumulative percentage of another variable, the distribution is said to be uniform and has no inequality. On the contrary, if cumulative distribution of one variable does not match with the cumulative distribution of another variable, the distribution is not equal. One variable is found to be distributed among the observations not according to its share. Some observations have higher share than the other, showing the inequality in its distribution.

Graphically if we plot cumulative percentages of one variable of equal distribution at different levels on the X axis and the corresponding cumulative percentages of another variable at the same levels on Y- axis, all these points will fall on a diagonal line of the square of 100 x 100 grid. This diagonal line is known as line of equal distribution. In reality, when we plot cumulative distribution of any two variables in the manner described above the points in most of the cases will not fall on the line of equal distribution. Joining of these points by a freehand will form the shape of a curve which is called "Lorenz Curve". The curve deviates from the line of equal distribution, showing the level of inequality in the distribution. The extent of deviation from the line of equal distribution will be proportional to the level of inequality in the distribution. Lorenz curve has been introduced first by Lorenz in 1905 to study the concentration of wealth. Since then it has found huge popularity to show the inequality of wealth and other characteristics all over the World. Coefficient of variation, Location Quotient, Sopher's Index etc. are other measures of inequality sometimes used in place of Lorenz curve to show the levels of inequality in any distribution.

Example

To show the spatial concentration of population distribution of population and the area of the 13 districts of the State of Uttrakhand is given in Table 1 below for 2011:

Table 1

Area, population and density of population in districts of Uttrakhand2011

	District (1)	Population (As of 2011)[2]	Area (km²)[3]	Density (/km²) (4)
1	Almora	621,972	3,083	202
2	Bageshwar	259,840	2,302	113
3	Chamoli	391,114	7,613	51
4	Champawat	259,315	1,781	146
5	Dehradun	1,695,860	3,088	550
6	Haridwar	1,927,029	2,360	817
7	Nainital	955,128	3,860	247
8	PauriGarhwal	686,572	5,399	127
9	Pithoragarh	485,993	7,100	68
10	RudraPrayag	236,857	1,890	125
11	TehriGarhwal	616,409	4,080	151
12	Udham Singh Nagar	1,648,367	2,908	567
13	Uttarkashi	329,686	8,016	41

Column number 1 in table 1 gives the name of the district, column 2 gives the total population of each district, column 3 gives the area and column 4 gives the density of population per square km. of area of each district.

For showing the inequality in the distribution of population over area, we find the percentage share of both, population and area of each district in the state and compare the cumulative distribution of both together. However, to bring smoothness in the distribution first we arrange the districts in either from low to high or from high to low density of population.

In Table 2, therefore, distribution of population and area of each of the districts is given after arranging them from low density to high density of population.

Table 2

Area and population of the districts of Uttrakhand arranged in ascending order of population density

	District	Population (As of 2011)[2]	Area (km²)[3]	Density (/km²)
1	Uttarkashi	329,686	8,016	41
2	Chamoli	391,114	7,613	51
3	Pithoragarh	485,993	7,100	68
4	Bageshwar	259,840	2,302	113
5	RudraPrayag	236,857	1,890	125
6	PauriGarhwal	686,572	5,399	127

7	Champawat	259,315	1,781	146
8	TehriGarhwal	616,409	4,080	151
9	Almora	621,972	3,083	202
10	Nainital	955,128	3,860	247
11	Dehradun	1,695,860	3,088	550
12	Udham Singh Nagar	1,648,367	2,908	567
13	Haridwar	1,927,029	2,360	817
	Total	10,114,142	53,480	

In table 3 given below the percentage share of each district is calculated both for population and for area.

Table 3

Percentage share of area and population of the districts of Uttrakhand

afterarranging them in ascending order of population density

	District	% shar of population	% share of area
1	Uttarkashi	3.260	14.99
2	Chamoli	3.870	14.24
3	pithoragargh	4.810	13.28
4	Bageshwar	2.570	4.3
5	RudraPrayag	2.340	3.53
6	Pauriarhwal	6.790	10.1
7	Champawat	2.560	3.33
8	Tehriarhwal	6.090	7.63
9	Almora	6.150	5.76
10	Nainital	9.440	7.22
11	Dehradun	16.770	5.76
12	UdhamS.Nagar	16.300	5.44
13	Haridwar	19.050	4.42

Now, we work out cumulative percentage share of population and area successively from first district to the last district as shown in Table 4 given below.

Table 4

Cumulative area and population of districts of Uttrakhand

arranged in ascending order of Density

S.N.	District (1)	Cum% Pop.(Xi) (2)	Cum % Area(Yi)(3)
1	Uttarkashi	3.26	14.99
2	Chamoli	7.13	29.23
3	pithoragargh	11.94	42.51
4	Bageshwar	14.51	46.81
5	RudraPrayag	16.85	50.34
6	PauriGarhwal	23.64	60.44
7	Champawat	26.2	63.77
8	TehriGarhwal	32.29	71.4
9	Almora	38.44	77.16
10	Nainital	47.88	84.38
11	Dehradun	64.65	90.14
12	UdhamS.Nagar	80.95	95.98
13	Haridwar	100.00	100.00

These cumulative values of the percentages are plotted on a graph paper choosing X-axis for population and Y-axis for the area. The graph is given in Figure 1 below. As two percentages do not go together, we do not get a straight line, instead we get a curve above the straight line (Line of equal distribution) showing the inequality in the distribution of population over area. The curve shows the inequality in the distribution of population in relation to the distribution of the area over all the districts. Such a curve is known as " Lorenz Curve". From the same data we can get the "Lorenz Curve" below the line of equal distribution also if we interchange the axes i.e. if we take area on the X-axis and population on the Y-axis.

The Lorenz curve given above shows sufficient inequality in the population distribution of tha state of Uttrakhand over the area. As the topography of the state is mainly hilly, population settlements are not always possible to every area. Some of the plain areas of the state are more suitable for human settlements than the hilly areas, causing the above mentioned inequality in the distribution of population over area.

Figure 1: Lorenz Curve

Distribution of population over area in districts of Uttrakhand 2011

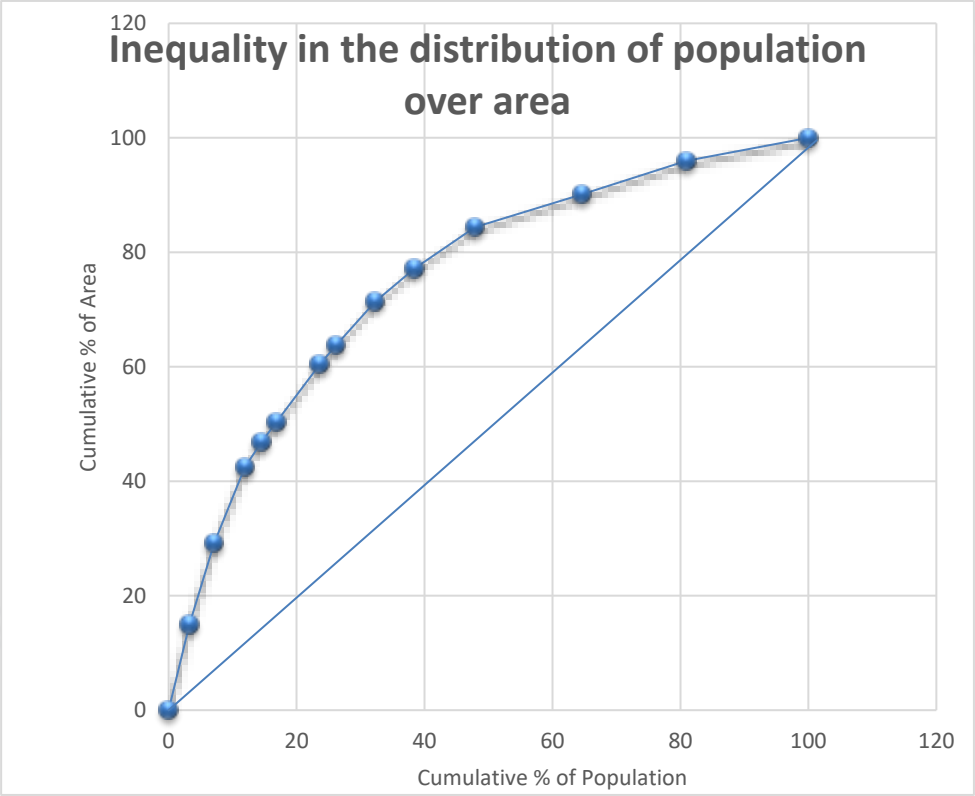


Table 5

Cumulative area and population of districts of Uttrakhand

arranged in ascending order of Density

S.N.	District (1)	XiYi+1 (2)	YiXi+1 (3)
1	Uttarkashi	-	-

2	Chamoli	95.29	106.88
3	pithoragargh	303.1	349.01
4	Bageshwar	558.91	616.82
5	RudraPrayag	730.43	788.75
6	PauriGarhwal	1018.41	1190.04
7	Champawat	1507.52	1583.53
8	TehriGarhwal	1870.68	2059.13
9	Almora	2491.5	2744.17
10	Nainital	3243.57	3694.42
11	Dehradun	4315.9	5455.17
12	UdhamS.Nagar	6205.11	7296.83
13	Haridwar	8095.95	9598.00
	Total	30436.37	35482.75

The deviation of the curve from the straight line (above it or below) shows the extent of inequality in the distribution. In a "Lorenz Curve" the deviation is only assessed visually, a comparison of the deviations in two curves, therefore, will not be accurate. Ginni's Concentration ratio "G" has provided a more accurate quantitative measure of this deviation. It measures the area between the line of equal distribution and a given Lorenz Curve and divide it by its maximum value which is 10000x1/2 (=100x100x1/2). The value of Ginni's Concentration ratio "G" varies from a maximum of 1 (showing extreme inequality) to zero(showing no inequality or uniformity in the distribution). Thus a lower value of "G" will show low inequality and a higher value will show a higher level of inequality in the distribution. Since "G" is a quantitative measure even a small difference between two values of "G" will be accurately measured and noticed.

Measuring the area between the Lorenz Curve and the line of equal distribution is possible by overlying a grid on the paper and adding the small area of each grid. To avoid this lengthy process we use the method of numerical analysis to get the value of "G". The advantage of the numerical method is that we can calculate it by using a small computer programme also. The value of "G" given by this method is as given below:

$$G = \frac{1}{100x100} I\left(\sum_{i=1}^{n} XiYi + 1 - \sum_{i=1}^{n} YiXi + 1\right) I$$

Where Xi and Yi are the cumulative percentages of population and area at each district and vertical bar before the brackets indicates that the absolute value of the sum total in the brackets has to be considered i.e. the value is to be taken without the plus or minus sign.

Example

After plotting the Lorenz Curve showing the inequality in the population distribution over area of the districts of Uttrakhand as given in Figure 1, from Table 4 and 5, we can also measure the inequality through Ginni's Concentration ratio "G". Colum 2 and 3 of the Table 4, give the cumulative percentages of population Xi and area Yi. In Table 5, column 2 give the values of XiYi+1, by multiplying each value of X with the value of Y following X i.e. X1 with Y2, X2 with Y3 and so on. In the next column 3 of the same table we do reverse of it i.e. multiply Y1 with X2, Y2 with X3 and so on. Total of column 2 and 3of Table5 will give the values of $\sum_{i=1}^{n}(\mathrm{XiYi}+1)$ and $\sum_{i=1}^{n}(\mathrm{YiXi}+1)$ and value of the "G" can be calculated as:

G = $\frac{1}{100x100}$ (I 30436.37 – 35482.75 I) =$\frac{1}{10000}$ 5046.38 = 0.504638

Thus the value of G = 0.504638 indicates fairly good amount of inequality in the distribution of population over area.

Example

Two compare the areal concentration of population of Uttrakhand with the same in Kerala, the inequality in the population distribution over area of the districts of Kerala for 2011 has also been shown below using the Lorenz curve and Ginni's concentration ratio.

Table 6 given below gives the population, area and the density of population of the districts of Kerala as given by the Census of India 2011.

Table 6

Area, population and density of population in districts of Kerala 2011

S.N.	Distrct	Area(KM2)	Population 2011	Population density
1	Kasaragod	1,989	1,307,375	657
2	Kannur	2,961	2,523,003	852
3	Wayanad	2,130	817,420	383
4	Kozhikode	2,345	3,086,293	1316
5	Malappuram	3,554	4,112,920	1157
6	Palakkad	4,482	2,809,934	626

7	Thrissur	3,027	3,121,200	1031
8	Ernakulam	3,063	3,282,388	1071
9	Idukki	4,356	1,108,974	254
10	Kottayam	2,206	1,974,551	895
11	Alappuzha	1,415	2,127,789	1503
12	Pathanamthitta	2,652	1,197,412	451
13	Kollam	2,483	2,635,375	1061
14	Thiruvananthapuram	2,189	3,301,427	1508
	Total	38852	33406061	859

Before working out cumulative percentages of area and population we arrange the districts according to ascending order of their population densities so that those districts which accommodate more population compare to areas come first and other districts follow in that order. This arrangement is necessary to get a smooth curve. Otherwise in some cases the curve is irregular and even cross the line of equal distribution. In Table 7 given below, therefore, we have given these percentages after rearranging the districts according to the increasing order of their population densities.

Table 7

Percentage share of area and population of the districts of Kerala arranged in ascending order of population density

S.N.	Distrct	Population density	%Area(KM2)	%Population 2011
1	Idukki	255	11.21	3.32
2	Wayanad	383	5.48	2.45
3	Pathanamthitta	451	6.82	3.58
4	Palakkad	626	11.54	8.41

5	Kasaragod	657	5.12	3.91
6	Kannur	852	7.62	7.55
7	Kottayam	895	5.68	5.91
8	Thrissur	1031	7.79	9.34
9	Kollam	1061	6.39	7.89
10	Ernakulam	1071	7.88	9.83
11	Malappuram	1157	9.15	12.31
12	Kozhikode	1316	6.04	9.24
13	Alappuzha	1503	3.64	6.37
14	Thiruvananthapuram	1508	5.64	9.89
	total	859	100	100

Table 8 given below gives the cumulative percentages of area and population from first district to the last district successively. The table also gives the cross products under the column heading XiYi+1 and YiXi+1 with their column total at the bottom. These values are needed for the calculation of Ginni's concentration ratio.

Cumulative percentages of area and population are plotted on a graph paper to give the graph of Lorenz curve for showing the inequality in the distribution of population over area. Lorenz Curve is given below in Figure 2.

Table 8

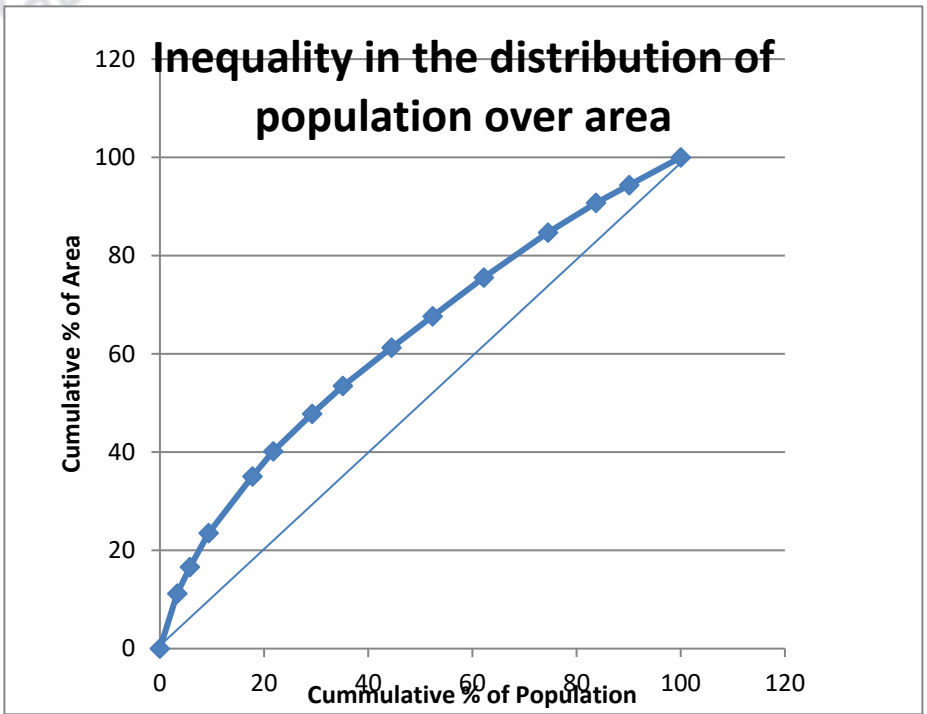
Cumulative percentage of area and population of districts of Kerala arranged in asscending order of population density

S.N.	Distrct	Cum.% Pop. Xi	Cum% Area Yi	XiYi+1	YiXi+1
1	Idukki	3.32	11.21		

2	Wayanad	5.77	16.59	55.0788	64.6817
3	Pathanamthitta	9.35	23.51	135.6527	155.1165
4	Palakkad	17.76	35.05	327.7175	417.5376
5	Kasaragod	21.76	40.17	713.4192	762.688
6	Kannur	29.22	47.79	1039.91	1173.767
7	Kottayam	35.13	53.47	1562.393	1678.863
8	Thrissur	44.47	61.26	2152.064	2377.811
9	Kollam	52.36	67.65	3008.396	3207.574
10	Ernakulam	62.19	75.53	3954.751	4207.154
11	Malappuram	74.5	84.68	5266.249	5626.985
12	Kozhikode	83.74	90.72	6758.64	7091.103
13	Alappuzha	90.11	94.36	7901.706	8174.779
14	Thiruvananthapuram	100	100	9011	9436
	total			41886.98	44374.06

Figure2: Lorenz Curve

Distribution of population over area in districts of Kerala 2011



It is to be noted that Lorenz curve shows less deviation from the line of equal deviation than Uttrakhand. It is known fact that the state of Uttrakhand is a hilly state and population can be settled only in few areas of moderate slopes. Kerala on the other hand is a plane area with sufficient water everywhere which helps a uniform settlement of population. This geographical influence on human settlement is effectively reflected in the Lorenz Curve of population distribution over area in two states. In the case of Uttrakhand shown above the Lorenz Curve showed larger deviation from the line of equal distribution. In the case of Kerala as the distribution is quite uniform, the deviation of the Lorenz curve from the line of equal distribution is quite low.

This visual difference in the deviation can also be shown quantitatively throughGinni's concentration ratio, which in the case of Kerala is :

$G = \frac{1}{100x100} (|41886.98 - 44374.06|) = \frac{1}{10000} 2487.08 = 0.248708$

Ginni's concentration ratio gives a quantitative value for the deviation equal to 0.248708 which can be compared accurately with the value of G= 0.504638 for Uttrakhand. Thus through G not only we conclude that the inequality is higher in Uttrakhand but also know that it is more than double in Uttrakhand compared to Kerala.