CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY

Second Semester of M. Sc. (Mathematics) Examination May 2018

MA726: Problems and Exercises in Mathematics- 2

May 12, 2018, Saturday    Time 10.00 a.m. to 10.45 a.m.    Maximum marks 20

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Multiple Choice Questions

***Instructions:***

1. Tick the correct answer only in this sheet.
2. You can use only non-programmable calculator. No other gadget is allowed in the examination hall.
3. Each question carries one mark.
4. In this question paper, $\mathbb{N}$ denotes the set of all natural numbers and $\mathbb{R}$ denotes the field of all real numbers. Other notations are standard.

Multiple Choice Questions (MA726 : Problems and Exercises in Mathematics - 1)

Q-I Choose the correct answer from the given options in the following: [20]

(1) A group of order 77 is ___________.

(a) simple (b) nonabelian (c) cyclic (d) abelian but non cyclic

(2) Let G be a group such that $a^2 = a$ for all $a \in G$. Then _________.

(a) $G \neq \{e\}$ (b) $G = \{e\}$ (c) G is not finite (d) none of these

(3) Let G be a group such that ____________, then G is abelian.

(a) $\varphi : G \to G$ defined by $\varphi(x) = x^2$, $x \in G$, is a homomorphism
(b) G has subgroup which is not normal
(c) G has no normal subgroup
(d) G has no Sylow subgroup

(4) The number of distinct congruent classes of the group $S_5$ is _________.

(a) 5 (b) 7 (c) 1 (d) none of these

(5) A Cartesian representation of the curve $\bar{\gamma}(t) = (\cos^3 t\ , \sin^3 t)$ is _________.

(a) $x + y = 1$ (b) $x^2 + y^2 = 1$
(c) $x - y = 1$ (d) $x^{\frac{2}{3}} + y^{\frac{2}{3}} = 1$

(6) Which of the following surface is **not** a smooth surface?

(a) $\{(x, y, z): x^2 + y^2 = z^2, z \geq 0\}$ (b) $\{(x, y, z): x^2 + y^2 = z^2, z > 0\}$
(c) $\{(x, y, z): x^2 + y^2 = z\}$ (d) None of these

(7) The first fundamental form on plane and cylinder is _________.

(a) $(du)^2 - (dv)^2$ (b) $(du)^2 + (dv)^2$
(c) $(du)^2 + 2dudv + (dv)^2$ (d) None of these

(8) If every point of connected surface $S$ is an *umbilic* then $S$ is _________.

(a) a part of plane (only) (b) a part of a sphere (only)
(c) (a) or (b) (d) (a) and (b)

(9) The rank of the linear transformation $T : \mathbb{R}^3 \to \mathbb{R}^3$ defined by $T(x_1, x_2, x_3) = (x_2, x_1, x_1)$, $(x_1, x_2, x_3) \in \mathbb{R}^3$.

(a) 1 (b) 0 (c) 3 (d) 2

(10) $W = \{(x_1, x_2, x_3) \in \mathbb{R}^3 : 2x_1 + 5x_2 = x_3\}$. Then $\dim(W^\circ)$ = __________ .

(a) 1 (b) 3 (c) 2 (d) 0

(11) Define $T : R^2 \to R^2$ by $T(x_1, x_2) = (x_2, x_1)$, $(x_1, x_2,) \in R^2$. Then T has characteristic roots ______________.

(a) 1 and 1 (b) 1 and –1 (c) 1 and 2 (d) 0 and 2

(12) Let A be a square matrix of order 3 with real entries such that $A \neq I$ and $A^2 = A$. Then det(A) = ___________.

(a) 3 (b) 1 (c) 2 (d) 0

(13) If $\varphi$ and $\psi$ are arbitrary functions, then the general solution of the partial differential equation $\frac{\partial^2 z}{\partial x^2} - \frac{\partial^2 z}{\partial x \partial y} = 0$ is ____________.

(a) $z = \phi(y) + \psi(x + y)$ (b) $z = \phi(x - y) + \psi(x + y)$
(c) $z = \phi(x + y) + \psi(x - y)$ (d) $z = \phi(x - y) + \psi(x)$

(14) The initial value problem $\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 1$, u(s, s) = sin(s), 0 ≤ s ≤ 1 has __________.

(a) two solutions (b) a unique solution
(c) no solution (d) infinitely many solutions

(15) Which of the following equation is elliptic?

(a) wave equation (b) heat equation (c) Laplace Equation (d) r + 4s+ 4t = 0

(16) Which of the following is characteristic of the partial differential equation $r + 2s + t = 0$?

(a) x – y = c (b) xy = c (c) x + y = c (d) $\frac{x}{y} = c$

(17) A sequence $\{f_n\}$ of measurable functions is said to ________ in measure to $f$ if given $\varepsilon > 0$ there is an $N$ such that for all $n \geq N$ we have $m\{x: |f(x) - f_n(x)| \geq \varepsilon\} < \epsilon$.

(a) diverge (b) converge (c) oscillate (d) none of these

(18) $\lim_{n \to \infty} (-1)^n \left(1 + \frac{1}{n}\right) =$ ________.

(a) 0 (b) 1 (c) –1 (d) none of these

(19) $m^*(\bigcup_{n=1}^{\infty} A_n)$ ________ , where $m^*$ is an outer measure.

(a) $\leq \sum_{n=1}^{\infty} m^*(A_n)$ (b) $\neq \sum_{n=1}^{\infty} m^*(A_n)$
(c) $\leq \prod_{n=1}^{\infty} m^*(A_n)$ (d) $< \prod_{n=1}^{\infty} m^*(A_n)$

(20) For the set $A = \{x\}, x \in \mathbb{R}$, ________ is not true.

(a) A is non Borel set (b) A is measurable
(c) $m^*(\{x\}) = 1$ (d) A is an open set

Candidate's seat number__________

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First Semester of M. Sc. (Mathematics) Examination May 2018

MA726: Problems and Exercises in Mathematics - 2

May 12, 2018, Saturday Time 10.45 a.m. to 01.00 p.m. Maximum marks 50

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***Instructions:***

1. Section I and Section II must be written in separate answer books.
2. You can use only non-programmable calculator.No other gadget is allowed in the examination hall.
3. Figures to the right indicate marks.
4. In this question paper, $\mathbb{N}$ denotes the set of all natural numbers and $\mathbb{R}$ denotes the field of all real numbers. Other notations are standard.

## SECTION I

Q-II(A) Attempt any TWO of the following: [8]

(1) Show that surface $S = \{(x, y, z): x^2 + y^2 = z\}$ is a smooth surface.

(2) For the Cauchy problem $\frac{\partial u}{\partial t} - u\frac{\partial u}{\partial x} = 0$, $x \in \mathbb{R}$, $t > 0$, $u(x, 0) = x$, $x \in \mathbb{R}$. Then show that the solution $u$ exist for $t < 1$ and breaks down at $t = 1$.

(3) Evaluate $\lim\limits_{n\to\infty} \int_0^1 \frac{\log(x+n)}{n} e^{-x} \cos x \, dx$

(B) Attempt any SIX of the following: [12]

(1) Let G be a group such that $(ab)^2 = a^2b^2$ for all $a, b \in G$. Show that G is abelian

(2) Find the number of 3- Sylow subgroups of a group of order 24.

(3) Find second fundamental form of surface patch $\sigma(u, v) = (\cos v, \sin v, u)$.

(4) Prove that the sum of interior angles of a regular $n - gon$ on a plane is $(n - 2)\pi$.

(5) Let V be a vector space over the field $\mathbb{R}$ of real numbers. Show that there do not exist two proper nontrivial subspaces U and W such that $V = W \cup U$.

(6) Let $A = (\alpha_{ij})$ be a square matrix with entries in R of order n such that for each $j = 1,2,\ldots,n$, $\sum_{i=1}^{n} \alpha_{ij} = 1$. Prove that 1 is a characteristic root of A.

(7) Show that the partial differential equations $\frac{\partial^2 u}{\partial y^2} - y\frac{\partial^2 u}{\partial x^2} = 0$ has vertical lines as a family of characteristic curves for $y = 0$.

(8) Write the Charpit's auxiliary equations for the partial differential equation $up^2 + q^2 + x + y = 0, p = \frac{\partial u}{\partial x}, q = \frac{\partial u}{\partial y}$.

(9) Using arc length show that the function $f(x) = e^x$ is of bounded variation over [a, b].

(10) Is the statement "If $\int_E f = 0$, then f = 0 a.e." true?. Justify.

## SECTION II

Q-III(A) Attempt any THREE of the following: [18]

(1) Let G be a group; H be a subgroup of G and N be a normal subgroup of G. Show that

(a) HN is a subgroup of G

(b) $H \cap N$ is a normal subgroup of H and

(c) HN / N is isomorphic to H / $(H \cap N)$.

(2) Find the vertices of $\bar{\gamma}(t) = (a\cos t\ , b\sin t)$, where $a, b > 0$ and $a \neq b$.

(3) Let V be a vector space over the field $\mathbb{R}$ and $T \in A(V)$ be such that $T^3 = T$. Show that $V = V_0 \oplus V_1 \oplus V_2$, where $V_0 = \{ v_0 \in V : Tv_0 = 0\}$; $V_1 = \{ v_1 \in V : Tv_1 = v_1\}$ and $V_2 = \{v_2 : Tv_2 = -v_2\}$.

(4) Solve the partial differential equation $\dfrac{\partial^2 z}{\partial x^2} - 3\dfrac{\partial^2 z}{\partial x \partial y} + 2\dfrac{\partial^2 z}{\partial y^2} = e^{x+y} + \cos(x+2y)$.

(5) If $f$ is differentiable on (a, b) and its derivative $f'$ is an increasing function, then prove that $f$ is convex.

(B) Attempt any THREE of the following: [12]

(1) Let G be a group and $a \in G$ be of order 5. If $b \in G$ is such that $aba^{-1} = b^2$, find order of b

(2) Compute the Christoffel's symbols for the circular cylinder $\{(x, y, z): x^2 + y^2 = 1\}$

(3) Let F be a field and $W_k = \{\alpha_0 + \alpha_1 x + \alpha_2 x^2 + \ldots + \alpha_k x^k \in F[x] : \alpha_0 + \alpha_1 + \alpha_2 + \ldots + \alpha_k = 0\}$. Show that $W_k$ is a vector space. Find a basis of $W_k$.

(4) If the initial value problem for the partial differential equation $\dfrac{\partial u}{\partial t} + \dfrac{\partial^2 u}{\partial x^2} = 0; u(x,0) = \sin(\pi x)$, has a solution of the form $u(x, t) = \psi(t)\sin(\pi x)$, then show that $\psi$ is always a positive and increasing function.

(5) Prove that $C'[a, b] \subset BV[a, b]$.

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