CODE:- AG-9-5544 पजियन क्रमांक **REG.NO:-TMC -D/79/89/36**

GENERAL INSTRUCTIONS :-

1. All question are compulsory.
2. The question paper consists of 29 questions divided into three sections A,B and C. Section – A comprises of 10 question of 1 mark each. Section – B comprises of 12 questions of 4 marks each and Section – C comprises of 7 questions of 6 marks each .
3. Question numbers 1 to 10 in Section – A are multiple choice questions where you are to select one correct option out of the given four.
4. There is no overall choice. However, internal choice has been provided in 4 question of four marks and 2 questions of six marks each. You have to attempt only one lf the alternatives in all such questions.
5. Use of calculator is not permitted.
6. Please check that this question paper contains 4 printed pages.
7. Code number given on the right hand side of the question paper should be written on the title page of the answer-book by the candidate.

सामान्य निर्देश :

1. सभी प्रश्न अनिवार्य हैं।
2. इस प्रश्न पत्र में 29 प्रश्न है, जो 3 खण्डों में अ, ब, व स है। खण्ड – अ में 10 प्रश्न हैं और प्रत्येक प्रश्न 1 अंक का है। खण्ड – ब में 12 प्रश्न हैं और प्रत्येक प्रश्न 4 अंको के हैं। खण्ड – स में 7 प्रश्न हैं और प्रत्येक प्रश्न 6 अंको का है।
3. प्रश्न संख्या 1 से 10 बहुविकल्पीय प्रश्न हैं। दिए गए चार विकल्पों में से एक सही विकल्प चुनें।
4. इसमें कोई भी सर्वोपरि विकल्प नहीं है, लेकिन आंतरिक विकल्प 4 प्रश्न 4 अंको में और 2 प्रश्न 6 अंको में दिए गए हैं। आप दिए गए विकल्पों में से एक विकल्प का चयन करें।
5. कैलकुलेटर का प्रयोग वर्जित हैं ।
6. कृपया जाँच कर लें कि इस प्रश्न–पत्र में मुद्रित पृष्ठ 4 हैं।
7. प्रश्न–पत्र में दाहिने हाथ की ओर दिए गए कोड नम्बर को छात्र उत्तर–पुस्तिका के मुख–पृष्ठ पर लिखें।

Pre-Board Examination 2011 -12

Time : 3 Hours	अधिकतम समय : 3
Maximum Marks : 100	अधिकतम अंक : 100
Total No. Of Pages :4	कुल पृष्ठों की संख्या : 4

CLASS – XII **CBSE** **MATHEMATICS**

SECTION A

Q.1	Check whether the relation R in R defined by R = $\{(a,b): a \le b^2\}$ is transitive. Ans = not $(4,-3)\in R \,\&\, (-3,1)\in R \Rightarrow (4,1)\notin R$
Q.2	Evaluate : $\int \frac{\cos 2x - \cos 2\alpha}{\cos x - \cos\alpha}\,dx.$ Ans : $2\sin x + 2x\cos\alpha$
Q.3	Find the value of k for which the matrix $\begin{pmatrix} k & 2 \\ 3 & 4 \end{pmatrix}$ has no inverse. Ans. K= $\frac{3}{2}$
Q.4	Write the principal branch of $\sec^{-1} x$. Ans $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$
Q.5	Find the value of x if the area of Δ is 35 square cms with vertices (x,4),(2, -6)and (5,4). Ans x = -2 or 12
Q.6	Evaluate : $\int [1 + 2\tan x(\tan x + \sec x)]^{1/2}\,dx.$ Ans $\log(\sec x + \tan x) + \log \sec$
Q.7	Write down a unit vector in XY-plane, making an angle of 30° with the positive direction of x-axis. Ans. $\frac{\sqrt{3}i + j}{2}$

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Q.8	If $\vec{a}$ and $\vec{b}$ are non-collinear vectors, find the value of x for which the vectors $\vec{I} = (2x+1)\vec{a} - \vec{b}$ and $\vec{m} = (x-2)\vec{a} + \vec{b}$ are collinear. Ans x = 1/3
Q.9	If $\vec{a} = \vec{b} + \vec{c}$, then is it true that $\lvert\vec{a}\rvert = \lvert\vec{b}\rvert + \lvert\vec{c}\rvert$? Justify your answer. Ans = not
Q.10	Find the perpendicular distance from (2,5,6) on XY plane . Ans : 6 unit
	SECTION B
Q.11	Solve the following equation : $3\sin^{-1}\frac{2x}{1+x^2} - 4\cos^{-1}\frac{1-x^2}{1+x^2} + 2\tan^{-1}\frac{2x}{1-x^2} = \frac{\pi}{3}$. Ans $6\tan^{-1}x - 8\tan^{-1}x + 4\tan^{-1}x = \frac{\pi}{3} \because 2\tan^{-1}x = \frac{\pi}{3} \Rightarrow \tan^{-1}x = \frac{\pi}{6} \therefore x = \tan\frac{\pi}{6} = \frac{1}{\sqrt{3}}$ OR Solve for x : $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x), 0 < x < \frac{\pi}{2}$. Ans $x = \frac{\pi}{4} \in \left(0, \frac{\pi}{2}\right)$
Q.12	If f(x) and g (x) be two invertible function defined as $f(x) = \frac{2x+1}{3x-5}$ be defined as $g(x) = \frac{3x+3}{7x-2}$. Prove that $(gof)^{-1} = f^{-1}og^{-1}$. Ans : $(gof)x = \frac{15x-12}{8x+17} \Rightarrow (gof)^{-1} = \frac{12+17y}{15-8y}$ $f^{-1} = \frac{1+5x}{3x-2}$ & $g^{-1} = \frac{2x+3}{7x-3} \Rightarrow f^{-1}og^{-1} = \frac{12+7y}{15-8y}$
Q.13	Using the properties of determinants, prove the following: $\begin{vmatrix} 1+a^2-b^2 & 2ab & -2b \\ 2ab & 1-a^2+b^2 & 2a \\ 2b & -2a & 1-a^2-b^2 \end{vmatrix} = (1+a^2+b^2)^3$.
Q.14	An air force plane is ascending vertically at the rate of 100 km/ h. If the radius of the earth is r km, how fast is the area of the earth , visible from the plane , increasing at 3 minutes after it started ascending ? Given that the visible area A at height h is given by $A = \frac{2\pi r^2 h}{r+h}$. Ans $\frac{dA}{dt} = \frac{200\pi r^3}{(r+5)^2}$
Q.15	If $y = \sin(m\sin^{-1}x)$, prove that $(1-x^2)\frac{d^2y}{dx^2} - x\frac{dy}{dx} + m^2y = 0$. OR If $x^y = e^{x-y}$, prove that $\frac{dy}{dx} = \frac{\log x}{(1+\log x)^2}$.
Q.16	Find the distance of the point (2,3,4) from the line $\frac{x+3}{3} = \frac{y-2}{6} = \frac{z}{2}$ measured parallel to the plane 3x + 2y + 2z - 5 = 0. Ans $distance = \sqrt{33}units$
Q.17	Find all the local maximum values and local minimum values of the function $f(x) = \sin 2x - x, -\frac{\pi}{2} < x < \frac{\pi}{2}$. Ans $f'(x) = 0 \therefore x = \pm\frac{\pi}{6}$. f(x) is maximum at $x = \frac{\pi}{6}$ and maximum value is $f\left(\frac{\pi}{6}\right) = \frac{\sqrt{3}}{2} - \frac{\pi}{6}$ and f(x) is minmum at $x = -\frac{\pi}{6}$ & minmum value is $f\left(-\frac{\pi}{6}\right) = -\frac{\sqrt{3}}{2} + \frac{\pi}{6}$
Q.18	Evaluate $\int \frac{\sin 4x - 2}{1-\cos 4x} e^{2x} dx$. Ans $\frac{1}{2}e^{2x}\cot 2x$
Q.19	Solve the differential equation : $(3xy + y^2)dx + (x^2 + xy)dy = 0$. Ans $\frac{1}{4}\log\left(\frac{4xy+2y}{x^2}\right) = \log c - \log x$ OR Solve the differential equation, $(1+y+x^2y)dx + (x+x^3)dy = 0$ where y = o when x = 1

Ans. $yx = -\tan^{-1} x + \frac{\pi}{4}$

Q.20 A girl walks 4 km towards west, then she walks 3 km in a direction 30° east of north and stops. Determine the girl's displacement from her initial point of departure. Ans. $= \frac{-5}{2} i + \frac{3\sqrt{3}}{2} j$

OR

If $\vec{a} = \hat{i} - 2\hat{j} + 3\hat{k}$ & $\vec{b} = 3\hat{i} + \hat{j} + 2\hat{k}$, find a unit vector which is linear combination of $\vec{a}$ & $\vec{b}$ and is also perpendicular to $\vec{a}$. Ans. $= \frac{-(5i + 4j + k)}{\sqrt{42}}$

Q.21 Form the differential equation of the family of curve $y = ae^{x} + be^{2x} + ce^{3x}$; where a, b, c are some arbitrary constants. Ans. $y_3 - 6y_2 + 11y_1 - 6y = 0$

Q.22 Evaluate : $\int \frac{x}{x^3 - 1} dx$. Ans. $\frac{1}{3}\log(3x-1) - \frac{1}{6}\log(x^2 + x + 1) + \frac{1}{\sqrt{3}} \tan^{-1}\left(\frac{2x+1}{\sqrt{3}}\right)$

SECTION C

Q.23 Let A be a square symmetric matrix, Show that : (i) $\frac{1}{2}(A + A')$ is a symmetric matrix. (ii) $\frac{1}{2}(A - A')$ is a skew symmetric matrix. Also prove that any square matrix can be uniquely expressed as the sum of a symmetric matrix and a skew-symmetric matrix.

OR

Find the matrix P satisfying the matrix equation $\begin{bmatrix} 2 & 1 \\ 3 & 2 \end{bmatrix} p \begin{bmatrix} -3 & 2 \\ 5 & -3 \end{bmatrix} = \begin{bmatrix} 1 & 2 \\ 2 & -1 \end{bmatrix}$.

Ans P= $\begin{bmatrix} 25 & 15 \\ -37 & -22 \end{bmatrix}$

Q.24 Reduce in symmetrical form , the equation of the line x – y + 2z = 5 , 3x+ y + z = 6 . ans : d . r . of line is – 3 , 5 , 4 & point $\left(\frac{11}{4}, -\frac{9}{4}, 0\right)$. Equation of required line

$\frac{x - \frac{11}{4}}{-3} = \frac{y + \frac{9}{4}}{5} = \frac{z - 0}{4}$ $\quad$ *eq of line* $\frac{4x - 11}{-3} = \frac{4y + 9}{5} = \frac{z}{1}$

Q.25 Assume that the chances of a patient having a heart attack is 40%. It is also assumed that a meditation and yoga course reduce the risk of heart attack by 30% and prescription of certain drug reduces its chances by 25%. At a time a patient can choose any one of the two options with equal probabilities. It is given that after going through one of the two options the patient selected at random suffers a heart attack. Find the probability that the patient followed a course of meditation and yoga? Ans.

$= P(E_1) = P(E_2) = \frac{1}{2}; P(A/E_1) = \frac{40}{100} \times (1 - \frac{30}{100}) = \frac{28}{100}; P(A/E_2) = \frac{40}{100} \times (1 - \frac{25}{100}) = \frac{30}{100}$

$= \frac{\frac{1}{2} \times \frac{28}{100}}{\frac{1}{2} \times \frac{30}{100} + \frac{1}{2} \times \frac{28}{100}} = \frac{28}{56} = \frac{14}{29}$

Q.26 Draw the rough sketch of the region enclosed between the circles $x^2 + y^2 = 4$ and $(x-2)^2 + y^2 = 1$. Using integration, find the area of the enclosed region .

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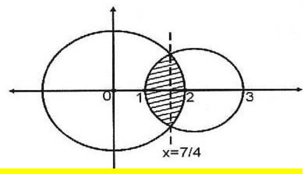
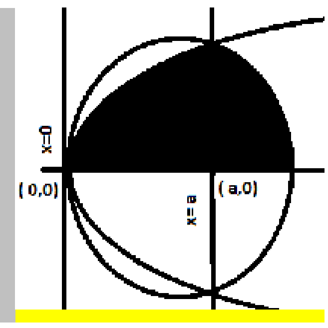
	Ans: Correct figure Solving $x^2+y^2 = 4$ and $(x-2)^2+y^2 = 1$ we get $x = \frac{7}{4}$  $\therefore$ Required area $= 2\left[\int_{7/4}^{2}\int\sqrt{4-x^2}\,dx + \int_{1}^{7/4}\sqrt{1-(x-2)^2}\,dx\right] = 2\left[\left[\frac{x}{2}\sqrt{4-x^2}+2\sin^{-1}\frac{x}{2}\right]_{\frac{7}{4}}^{2} + \left[\frac{x-2}{2}\sqrt{1-(x-2)^2}+\frac{1}{2}\sin^{-1}\cdot(x-2)\right]_{1}^{\frac{7}{4}}\right]$ $= 2\left[\pi - \frac{7}{8}\frac{\sqrt{15}}{4} - 2\sin^{-1}\frac{7}{8} + \left(-\frac{1}{8}\cdot\frac{\sqrt{15}}{4}+\frac{1}{2}\sin^{-1}\left(-\frac{1}{4}\right)+\frac{1}{2}\cdot\frac{\pi}{2}\right)\right] = \frac{5\pi}{2} - \frac{\sqrt{15}}{2} - \sin^{-1}\left(\frac{1}{4}\right) - 4\sin^{-1}\left(\frac{7}{8}\right)$ sq.u OR Find the area of the lying circle $x^2+y^2=2ax$ lying above x-axis and interior of the parabola $y^2 = ax$. Ans.  $A_1 = \int_0^a \sqrt{ax}\,dx = \frac{2a^2}{3}$ & $A_2 = \int_0^a \sqrt{a^2-(x-a)^2}$ $0r \int_a^{2a}\sqrt{a^2-(x-a)^2} = \frac{\pi a^2}{4}$ $\therefore A_1 + A_2 = a^2\left(\frac{\pi}{4}+\frac{2}{3}\right)$
Q.27	Prove that all normals to the curve $x = a\cos t + at\sin t,\ y = a\sin t - at\cos t$ are at a distance a from the origin. Ans;dy/dx= slope of tangent = tan t ; slope of normal = - cot t then equation of normal : x cos t + y sin t = a & distance from origin is = a
Q.28	Evaluate: $\int_0^{\pi} x \log \sin x\, dx$. $\frac{\pi^2}{2}\log\frac{1}{2}$
Q.29	A fruit grower can use two types of fertilizer in his garden, brand P and brand Q. The amounts (in kg) of nitrogen, phosphoric acid, potash, and chlorine in a bag of each brand are given in the table. Tests indicate that the garden needs at least 240 kg of phosphoric acid, at least 270 kg of potash and at most 310 kg of chlorine. If the grower wants to minimize the amount of nitrogen added to the garden, how many bags of each brand should be used? What is the minimum amount of nitrogen added in the garden?

kg per bag		
	Brand P	Brand Q
Nitrogen	3	3.5
Phosphoric acid	1	2
Potash	3	1.5
Chlorine	1.5	2

Ans. $(20,140),(40,100),(140,50)$, $P = 40, Q = 100$, minimam nitrogen = 470 kg

___________x___________

USE SOFT WORDS AND HARD ARGUMENTS.

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