CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY
Second Semester of M.Sc.(Mathematics) Examination April-May 2019
MA724 Partial Differential Equations

Date: 02/05/2019,Thursday **Time**: 10:00 a.m. to 10:45 a.m. **Maximum Marks**: 20

Multiple Choice Questions

Instructions:

(1) **Tick the correct answer only in this sheet.**
(2) **You can use only non-programmable calculator. No other gadget is allowed in the examination hall.**
(3) **Each question carries one mark.**

Q.1. Choose the correct answer for the following questions. [20]

1. The partial differential equation $3\frac{\partial z}{\partial x} - 2\frac{\partial z}{\partial y} - \left(\frac{\partial z}{\partial x}\right)\left(\frac{\partial z}{\partial y}\right) = 0$ is a_________.

   a. linear equation
   b. semi-linear equation
   c. quasi-linear equation
   d. non-linear equation.

2. A two-parameter family of solutions $z = F(x, y, a, b)$ is called a complete integral of a first order partial differential equation $f(x, y, z, p, q) = 0$, if in the region considered, the rank of the matrix $\begin{bmatrix} \frac{\partial F}{\partial a} & \frac{\partial^2 F}{\partial x \partial a} & \frac{\partial^2 F}{\partial y \partial a} \\ \frac{\partial F}{\partial b} & \frac{\partial^2 F}{\partial x \partial b} & \frac{\partial^2 F}{\partial y \partial b} \end{bmatrix}$ is _________.

   a. 0
   b. 1
   c. 2
   d. 3.

3. The partial differential equation obtained by eliminating arbitrary functions f and g from $\theta = f(x+it) + g(x-it)$, where $i = \sqrt{-1}$ is__________.

   a. $\frac{\partial^2\theta}{\partial x^2} - \frac{\partial^2\theta}{\partial t^2} = 0$
   b. $\frac{\partial^2\theta}{\partial x^2} + \frac{\partial^2\theta}{\partial t^2} = 0$
   c. $\frac{\partial\theta}{\partial x} + \frac{\partial^2\theta}{\partial t^2} = 0$
   d. $\frac{\partial\theta}{\partial x} - \frac{\partial\theta}{\partial t} = 0.$

4. The Pfaffian differential equation $\overrightarrow{X} \cdot \overrightarrow{dr} = P(x,y,z)dx + Q(x,y,z)dy + R(x,y,z)dz = 0, \overrightarrow{X} = (P,Q,R)$ is exact if and only if__________.

   a. $\nabla\overrightarrow{X} = 0$
   b. $\nabla \cdot \overrightarrow{X} = 0$
   c. $\nabla \times \overrightarrow{X} = 0$
   d. $\nabla^2\overrightarrow{X} = 0$

5. The complete integral of the partial differential equation $z = px + qy + \sqrt{p^2+q^2-5}$ is__________, where a and b are arbitrary constants.

   a. $z = ax + by + \sqrt{a^2+b^2-5}$
   b. $z = ax - by - \sqrt{a^2+b^2-5}$
   c. $z = -ax - by + \sqrt{a^2+b^2-5}$
   d. $z = ax - by - \sqrt{a^2-b^2-5}.$

6. The integral surface of the partial differential equation $xp + yq = 0$ satisfying the condition $u(1,y) = y$ is__________.

   a. $u(x,y) = \frac{y}{2-x}$
   b. $u(x,y) = \frac{y}{x}$
   c. $u(x,y) = \frac{2y}{x+1}$
   d. $u(x,y) = x + y - 1.$

7. __________ is the complete integral of the partial differential equation $p + q - pq = 0$, where a and b are arbitrary constants.

   a. $z = \frac{ax}{a-1} + ay + b$
   b. $z = ay - bx - ab$
   c. $z = ax + by - ab$
   d. $z = ay + bx - ab$

8. Which of the following operator is irreducible?

   a. $2D^2 - DD' - D'^2$
   b. $D^2 + 2DD' - 3D'^2$
   c. $5D^2 - 4DD' - D'^2$
   d. $D^2 - 5D - 6D'$

9. Which of the following equation is elliptic?

   a. wave equation
   b. heat equation
   c. Laplace equation
   d. $r + 2s - 4t = 0.$

10. __________ is useful to find solution of general linear hyperbolic partial differential equation.

a. Riemann's method
b. Charpit's method
c. Jacobi's method
d. none of these.

11. The partial differential equation $x^2\frac{\partial^2 u}{\partial x^2} - 4xy\frac{\partial^2 u}{\partial x \partial y} + 4y^2\frac{\partial^2 u}{\partial y^2}=0$ is__________.

a. hyperbolic
b. elliptic
c. parabolic
d. non-linear equation.

12. For a partial differential equation $y^2r - x^2t = 0$, we get__________.
a. only one family of characteristics
b. two distinct families of characteristics
c. two families of complex characteristics
d. no any family of characteristics

13. If $(aD + bD' + c)^2$ is a factor of $F(D, D')$, then the solution of partial differential equation $F(D, D')z = 0$ is__________, where Φ and Ψ are arbitrary functions.
a. $e^{-\frac{b}{c}x}(\Phi(bx - ay) + \Psi(bx - ay))$
b. $e^{-\frac{b}{c}x}(\Phi(bx - ay) + x\Psi(bx - ay))$
c. $e^{-\frac{c}{a}x}(\Phi(bx - ay) + \Psi(bx - ay))$
d. $e^{-\frac{c}{a}x}(\Phi(bx - ay) + x\Psi(bx - ay))$.

14. The particular integral of the partial differential equation $(D - 3D')(D - 4D')z = e^{2x+3y}$ is__________.

a. xe^{2x+3y}
b. $\frac{e^{2x+3y}}{x}$
c. $\frac{e^{2x+3y}}{70}$
d. $70e^{2x+3y}$.

15. $\frac{D'}{D}(x^2y^2)$=__________.

a. $\frac{x^3y^2}{3}$
b. $\frac{x^2y^3}{3}$
c. $\frac{2x^3y}{3}$
d. $\frac{2xy^3}{3}$.

16. Which of the following partial differential equation describe temperature distribution of the plate in the steady-state?
a. Laplace's equation
b. one dimensional heat equation
c. two dimensional heat equation
d. two dimensional wave equation.

17. An equation $\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2}$ is a__________.

   a. Laplace equation
   b. three dimensional wave equation
   c. two dimensional wave equation
   d. two dimensional heat equation.

18. Which of the following partial differential equation is the governing equation in modeling of vibrations of string?

   a. Laplace equation
   b. wave equation
   c. one dimensional heat diffusion equation
   d. two dimensional heat diffusion equation.

19. For solving one dimensional wave equation $\frac{\partial^2 u}{\partial t^2} = c^2 \frac{\partial^2 u}{\partial x^2}$, the number of initial and boundary conditions required are __________ respectively.

   a. 2 and 2   b. 2 and 1   c. 1 and 0   d. 1 and 3,

20. Consider the boundary value problem:

$$\text{Laplace equation: } \frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = 0, 0 \le x \le 5,\ 0 \le y \le 5$$

$$\text{Boundary conditions: } u_x(0,y) = 0,\ u_x(5,y) = 0,\ 0 \le y \le 5,$$

$$u_y(x,0) = 0 \text{ and } u_y(x,5) = f(x),\ 0 \le x \le 5.$$

   This problem is called__________.

   a. Dirichlet's problem
   b. Neumann's problem
   c. mixed problem
   d. none of these

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Second Semester of M.Sc.(Mathematics) Examination April-May 2019
MA724 Partial Differential Equations

Date: 02/05/2019,Thursday **Time**: 10:45 a.m. to 01:00 p.m. **Maximum Marks**: 50

Instructions:

(5) **Section I and II must be written in separate answer sheets**
(6) **You can use only non-programmable calculator. No other gadget is allowed in the examination halll.**
(7) **Figures to the right indicate marks.**
(8) **In this question paper, all notations are standard.**

SECTION-I

Q-2(a) **Attempt any two of the following:** [08]

(i) Using method of separation of variables, solve the partial differential equation $4\frac{\partial u}{\partial x} + \frac{\partial u}{\partial y} = 3u, u(0, y) = e^{-5y}$.

(ii) Find a complete integral of $(p^2 + q^2)y - qz = 0$ using Charpit's method.

(iii) Show that for the equation $\frac{\partial^2 z}{\partial y \partial x} + \frac{2}{x+y}\left(\frac{\partial z}{\partial x} + \frac{\partial z}{\partial y}\right) = 0$, the Green's function is $w(x, y; \xi, \eta) = \frac{(x+y)\{2xy+(\xi-\eta)(x-y)+2\xi\eta\}}{(\xi+\eta)^3}$.

(b) **Attempt any four of the following:** [12]

(i) Form the partial differential equation by eliminating the arbitrary function from $f(x + y + z, x^2 + y^2 + z^2) = 0$.

(ii) Find the general integral of the partial differential equation $y^2p - xyq = x(z - 2y)$.

(iii) What is boundary value problem? State Dirichlet boundary value problem in rectangle.

(iv) Find a complete integral of the equation $p^2x + q^2y = z$ using Jacobi's method.

(v) Prove that $F(D, D')e^{kx+ly} = F(k, l)e^{kx+ly}$, where k and l are constants.

(vi) Find the particular integral of the partial differential equation
$(D + D')(2D - 3D' + 5)z = e^{x-y}$.

SECTION II

Q-3(a) **Attempt any THREE of the following** [18]

(i) Show that the partial differential equations $xp - yq - x = 0$ and $x^2p + q - xz = 0$ are compatible and find a one-parameter family of common solutions.

(ii) Solve the partial differential equation $\frac{\partial^2 z}{\partial x^2} + 2\frac{\partial^2 z}{\partial x \partial y} - 8\frac{\partial^2 z}{\partial y^2} = \sqrt{x + 2y}$.

(iii) Using Monge's method, solve the partial differential equation
$r - t\cos^2 x + p\tan x = 0$.

(iv) Determine the solution of one dimensional heat equation $\frac{\partial u}{\partial t} = c^2\frac{\partial^2 u}{\partial x^2}$ subject to boundary conditions $u(0,t) = 0, u(10,t) = 0, t > 0$ and the initial condition $u(x,0) = x$.

(v) Find the steady state temperature distribution in a rectangular plate of sides 2 cm and 4 cm insulated at the lateral surface and satisfying boundary conditions $u(0,y) = u(2,y) = 0, 0 \le y \le 4$ and $u(x,0) = 0$ and $u(x,4) = f(x)$ for $0 \le x \le 2$.

(b) **Attempt any TWO of the following** [12]

(i) Show that a necessary and sufficient condition that the Pfaffian differential equation $\overrightarrow{X} \cdot \overrightarrow{dr} = P(x,y,z)\,dx + Q(x,y,z)\,dy + R(x,y,z)\,dz = 0$ be integrable is that $\overrightarrow{X} \cdot curl\overrightarrow{X} = 0$.

(ii) Reduce the partial differential equation $(n-1)^2r - y^{2n}t = ny^{2n-1}q$ to cannonical form.

(iii) A tightly stretched string with fixed end points $x = 0$ and $x = \pi$ is initially at rest in its equilibrium position. If it is set vibrating by giving to each of its points an initial velocity $\left(\frac{\partial y}{\partial t}\right)_{t=0} = 0.07\sin x + 0.09\sin 5x$, then find the displacement $y(x,t)$ at any point of string at any time t.

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