IITJEE MATHEMATICS SAMPLE PAPER – V

SOLUTIONS

SECTION – I
Straight Objective Type

1. **(d)**

2. **(b)** The total number of squares is

$$1^2+2^2+3^2+4^2+.....+14^2+[1\times 13^2+2\times 12^2+3\times 11^2......13\times 1^2]=4200$$

The total number of squares with length of side as integer is

$$(1^2+2^2+3^2+.....+14^2)+(2\times 8\times 8)+(2\times 1\times 1)=1145$$

Hence probability = $\dfrac{1145}{4200}=\dfrac{229}{840}$

3. **(a)**

4. **(a)** $(y+x^4y)dx+(x-x^5)dy=0$

$$\Rightarrow\ ydx+xdy+x^4(ydx-xdy)=0\ \Rightarrow\ \frac{ydx+xdy}{x^4y^4}+\frac{ydx-xdy}{y^4}=0$$

$$\Rightarrow\ \frac{2xy^2dx+2x^2ydy}{x^4y^4}+\frac{2xy^2dx-2x^2ydy}{y^4}=0$$

$$\Rightarrow\ -d\left(\frac{1}{x^2y^2}\right)+d\left(\frac{x^2}{y^2}\right)=0\ \Rightarrow\ -\frac{1}{x^2y^2}+\frac{x^2}{y^2}=c\ \Rightarrow\ cx^2y^2=x^4-1$$

$$\Rightarrow\ cx^2y^2-x^4+1=0$$

5. **(d)**

6. **(d)**

7. **(a)**

8. **(c)**

SECTION – II
Reasoning Type

9. **(a)**

10. **(a)**

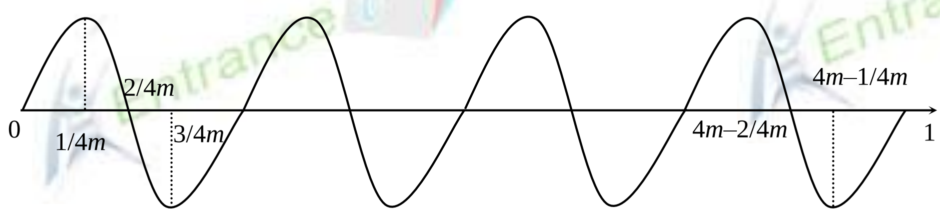
11. **(c)**

12. **(c)**

SECTION–III
Linked Comprehension Type

Passage-I

$f(x) = \sin(2\pi m x),\ x \in [0, 1]$



13. **(a)** $\because \int_a^b f(x)\,dx \ge 0,\ \forall\ b \in [a, 1]$

So 'a' must belongs to $\left\{0, \frac{4}{4m}, \frac{8}{4m}, \ldots\ldots\ldots\ldots \frac{4m-4}{4m}\right\}$

If $a = 0$, then 'b' can take $2m$ values

1st in $\left(0, \frac{2}{4m}\right)$, 2nd in $\left(\frac{2}{4m}, \frac{4}{4m}\right)$......$2m^{\text{th}}$ in $\left(\frac{4m-2}{4m}, 1\right)$

If $a = \frac{4}{4m}$, then 'b' can take $2m - 2$ values

1st in $\left(\frac{4}{4m}, \frac{6}{4m}\right)$, 2nd in $\left(\frac{6}{4m}, \frac{8}{4m}\right)$.......$(2m-2)^{\text{th}}$ in $\left(\frac{4m-2}{4m}, 1\right)$

Total pairs of (a, b) are $2m + (2m-2) + \ldots.. + 4 + 2 = m(m+1)$

14. **(b)** Because 'a' is root of equation $f(x) = 1$

So 'a' must belong to $\left(\frac{1}{4m}, \frac{5}{4m}, \ldots. \frac{4m-3}{4m}\right)$

If $a = \frac{1}{4m}$, then 'b' can take $(4m-1)$ values

1st lies in $\left(\frac{1}{4m}, \frac{2}{4m}\right)$; 2nd in $\left(\frac{2}{4m}, \frac{3}{4m}\right)$……..and $(4m-1)^{\text{th}}$ in $\left(\frac{4m-1}{4m}, 1\right)$

If $a = \frac{5}{4m}$, then 'b' can take $(4m-5)$ values

1^{st} in $\left(\frac{5}{4m}, \frac{6}{4m}\right)$; 2^{nd} in $\left(\frac{6}{4m}, \frac{7}{4m}\right)$......... and $(4m-5)^{th}$ in $\left(\frac{4m-1}{4m}, 1\right)$

Total pair of (a, b) are $(4m-1)+(4m-5)+......7+3 = m(2m+1)$

15. **(c)** Because $\int_a^b f(x)\,dx \le 0,\ \forall b \in [a, 1]$

So 'a' must belongs to $\left\{\frac{2}{4m}, \frac{6}{4m}, ...\frac{4m-2}{4m}\right\}$

If $a = \frac{2}{4m}$ then 'b' can take $(2m-1)$ values

1^{st} values lies in $\left(\frac{2}{4m}, \frac{4}{4m}\right)$; 2^{nd} lies in $\left(\frac{4}{4m}, \frac{6}{4m}\right)$........and $(2m-1)^{th}$ lies in $\left(\frac{4m-2}{4m}, 1\right)$

If $a = \frac{6}{4m}$, then 'b' can take $(2m-3)$ values

1^{st} values lies in $\left(\frac{6}{4m}, \frac{8}{4m}\right)$, 2^{nd} lies in $\left(\frac{8}{4m}, \frac{10}{4m}\right)$......... and $(2m-3)^{th}$ lies in $\left(\frac{4m-2}{4m}, 1\right)$

Total pairs of (a, b) are $(2m-1)+(2m-3)+....+3+1 = m^2$

Passage-II

16. **(a)**

17. **(b)**

18. **(c)**

SECTION–IV
Matrix–Match Type

1. **(A) – 2, 3, 4 ; (B) – 1, 2, 3, 4 ; (C) – 2 ; (D) – 1, 3**

2. **(A) – 1, 2, 3, 4 ; (B) – 1, 2 ; (C) – 1, 2, 4 ; (D) – 1, 3**

SECTION – V
Subjective or Numerical problem Type

1. nth term of 1, 3, 6, 10, ……….

$a_n = \frac{1}{2}\left(n^2 + n\right)$ at $n = 10$, $a_n = 55 = k$

So sum of number is nth brackets

$= k + k^2 + ...k^{2n+1}$

$A = \frac{k\left(k^{2n+1} - 1\right)}{k-1} \Rightarrow \frac{54A}{55} + 1 = 55^{111}$

$\Rightarrow B = 111$

2. $f(x) = 7e^{\sin^2 x} - e^{\cos^2 x} + 2$

Let $e^{\sin^2 x} = t \Rightarrow t \in [1, e]$

$g(t) = 7t - \frac{e}{t} + 2$

$g'(t) = 7 + \frac{e}{t^2} = 0 \Rightarrow$ no critical point

$g(1) = 9 - e =$ minimum value

$g(e) = 7e + 1 =$ maximum value

$\sqrt{7f\ \min + f\ \max} = 8$