CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY
Fourth Semester of M.Sc.(Mathematics) Examination April-May 2019
MA821 Mathematical Methods-II

Date: 02/05/2019,Thursday **Time**: 01:30 p.m. to 02:15 p.m. **Maximum Marks**: 20

Multiple Choice Questions

Instructions:

(1) **Tick the correct answer only in this sheet.**
(2) **You can use only non-programmable calculator. No other gadget is allowed in the examination hall.**
(3) **Each question carries one mark.**
(4) **In this question paper, $\mathbb{R}$ denotes the set of all real numbers and $\mathbb{Z}$ denotes the set of all integers. All notations are standard.**

Q.1. Choose the correct answer for the following questions. [20]

1. If the functional $I[y(x)] = \int\limits_{x_1}^{x_2} f(y, y')dx$ has the extremum value, then f satisfies __________, where c is constant.

   a. $\frac{\partial f}{\partial y'} = c$
   b. $f - y'\frac{\partial f}{\partial y'} = c$
   c. $\frac{\partial f}{\partial y} = 0$
   d. $y'' = 0$.

2. What is the curve passing through two points in a plane which is rotated about the X-axis gives a minimum surface area?

   a. circle
   b. straight line
   c. catenary
   d. parabola.

3. For a given surface area, what is the solid of revolution which has maximum volume?

a. Sphere
b. cylinder
c. cube
d. parallelepiped.

4. The length of the arc $y(x) = x$ between a and b is ________.

a. $\frac{b-a}{2}$
b. $\sqrt{2}(b-a)$
c. $\sqrt{2}(b+a)$
d. $\frac{b+a}{2}$.

5. A necessary condition for the functional $I[y(x)] = \int\limits_{x_1}^{x_2} f(x, y, y', y'')dx$ to be extremum is________, with boundary conditions $y(x_1) = c_1$, $y(x_2) = c_2, y'(x_1) = c_3$ and $y'(x_2) = c_4$.

a. $\frac{\partial f}{\partial y} - \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0$
b. $\frac{\partial f}{\partial y} + \frac{d}{dx}\left(\frac{\partial f}{\partial y'}\right) - \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0$
c. $\frac{\partial f}{\partial y} - \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0$
d. $\frac{\partial f}{\partial y} + \frac{d^2}{dx^2}\left(\frac{\partial f}{\partial y''}\right) = 0$

6. $\int\limits_5^x \int\limits_5^z \int\limits_5^y f(t)dtdydz =$________.

a. $\int\limits_5^x (x-t)f(t)dt$
b. $\frac{1}{2}\int\limits_5^x (x-t)f(t)dt$
c. $\int\limits_5^x (x-t)^2 f(t)dt$
d. $\frac{1}{2}\int\limits_5^x (x-t)^2 f(t)dt$

7. $\frac{d}{dx}\left(\int\limits_0^x (x+t)f(t)dt\right)=$________.

a. $\int\limits_0^x f(t)dt$
b. $\int\limits_0^x f(t)dt + 2xf(x)$
c. $\int\limits_0^x f(t)dt + f(x)$
d. $\int\limits_0^x f(t)dt - 2xf(x)$.

8. Which of the following is the Abel's integral equation?

a. $\int\limits_0^x \frac{\sin t}{x-t}dt = e^x$
b. $\int\limits_0^x \frac{\cos t}{(x-t)^{\frac{3}{2}}}dt = 0$
c. $\int\limits_0^x (\sqrt{x-t})dt = x^2 + x - 1$
d. $\int\limits_0^x \frac{t^3+t-1}{\sqrt[5]{x-t}}dt = \sin x$.

9. Which of the following integral equation is Voltera integral equation?

a. $y(x) - x = 5\int\limits_0^2 \sin(x-t)y(t)dt$

b. $y(x) = \int\limits_0^1 e^{x-t}y(t)dt$

c. $y(x) = 5x^3 - 7\int\limits_{-1}^2 \log(xt)y(t)dt$

d. $y(x) = \int\limits_0^x \cos(x+t)y(t)dt$

10. The initial value problem corresponding to the integral equation $y(x) = 1 - \int\limits_0^x ty(t)dt$ is__________.

a. $y' + xy = 0, y(0) = 0, y'(0) = 1$
b. $y' + xy = 0, y(0) = 1, y'(0) = 0$
c. $y' - xy = 0, y(0) = 1, y'(0) = 0$
d. $y' - xy = 0, y(0) = 0, y'(0) = 1$.

11. If $L^{-1}[\bar{f}(s)] = f(x)$ is the inverse Laplace transform of $\bar{f}(s)$, then $L^{-1}\left[s^{-\frac{5}{2}}\right] =$ __________.

a. $\dfrac{x^{\frac{3}{2}}}{\Gamma(\frac{3}{2})}$ b. $\dfrac{x^{\frac{5}{2}}}{\Gamma(\frac{3}{2})}$ c. $\dfrac{x^{\frac{3}{2}}}{\Gamma(\frac{5}{2})}$ d. $\dfrac{x^{\frac{5}{2}}}{\Gamma(\frac{5}{2})}$.

12. If $L[f(x)]$ is the Laplace transform of $f(x)$, then $L\left[\int\limits_0^x \sin(x-t)e^t dt\right] =$ __________.

a. $\frac{1}{(s-1)(s^2+1)}$ c. $\frac{s}{(s-1)(s^2+1)}$
b. $\frac{1}{(s+1)(s^2+1)}$ d. $\frac{s}{(s+1)(s^2+1)}$.

13. Which of the following relationship is not true?

a. $L^2[0,1] \subset L^1[0,1]$ c. $L^2[0,1] \subset L^\infty[0,1]$
b. $L^\infty[0,1] \subset L^1[0,1]$ d. $C[0,1] \subset L^1[0,1]$.

14. Which of the following is not a separable kernel?

a. $e^{(x+t)}$ c. $\log(x^t)$
b. $\cos(x-t)$ d. $\sin(xt^2)$.

15. Which of the following is a Fredholm integral equation of second category?

a. $y(x) = \lambda \int\limits_0^x \sin(x-t)y(t)dt$

b. $y(x) = x^2 - \lambda \int\limits_0^x \cos(x-t)y(t)dt$

c. $y(x) = \lambda \int\limits_0^2 e^{x-t}y(t)dt$

d. $y(x) = x^3 - \lambda \int\limits_0^5 \sin(x-t)y(t)dt.$

16. What is the eigen value of the Fredholm integral equation $y(x) = \lambda \int\limits_0^1 x^3 y(t)dt$?

a. 4 b. 3 c. 2 d. 1.

17. If $L[f(t)]$ is the Laplace transform of a function $f(t)$, then $L[\cos 5t] =$ __________.

a. $\frac{5}{s^2+25}$ b. $\frac{s}{s^2+25}$ c. $\frac{5}{s^2-25}$ d. $\frac{s}{s^2-25}$.

18. The differential equation $y'' - 2xy' + 100y = 0$ is__________.

a. Legendre's equation
b. Hermite's equation
c. Chebyshev's equation
d. Laguerre's equation.

19. Which of the following is Bessel's differential equation?

a. $x^2y'' + xy' + (x^2 - 16)y = 0$
b. $(1 - x^2)y'' - xy' + 25y = 0$
c. $xy'' + (1 - x)y' + 7y = 0$
d. $(1-x^2)y'' - 2xy' + 30y = 0.$

20. The differential equation $[e^x y']' + [\lambda \sin x + \cos x] = 0$ is__________, where $\lambda \in \mathbb{R}$.

a. Legendre's equation
b. Laguerre's equation
c. Hermite's equation
d. Strum-Liouville equation.

CHAROTAR UNIVERSITY OF SCIENCE & TECHNOLOGY

Fourth Semester of M.Sc.(Mathematics) Examination April-May 2019

MA821 Mathematical Methods-II

Date: 02/05/2019,Thursday **Time**: 02:15 p.m. to 04:30 p.m. **Maximum Marks**: 50

Instructions:

(5) **Section I and II must be written in separate answer sheets**

(6) **You can use only non-programmable calculator. No other gadget is allowed in the examination hall.**

(7) **Figures to the right indicate marks.**

(8) **In this question paper, all notations are standard.**

SECTION-I

Q-2(a) **Attempt any two of the following:** [08]

(i) Find the plane curve of fixed perimeter and maximum area.

(ii) Transform the initial value problem $y'' - 5y' + 6y = 0, y(0) = -5, y'(0) = -19$ into the corresponding integral equation.

(iii) Find the eigenvalues and the corresponding eigenfunctions of the integral equation $y(x) = \lambda \int_0^1 (2xt - 4x^2)y(t)dt$.

(b) **Attempt any four of the following:** [12]

(i) Show that the functional $\int_0^1 (2xy + tx' + t^2y')dt, x(0) = 0, x(1) = 1, y(0) = y(1) = \frac{1}{2}$ is stationary for $x(t) = t$ and $y(t) = \frac{1}{2}$, $t \in [0, 1]$.

(ii) Write the Bessel differential equation and reduce it to Strum-Liouville differential equation.

(iii) Find the eigenvalues and the corresponding eigenfunctions of the differential equation $y'' + \lambda y = 0, \lambda > 0$ with boundary conditions $y(0) = 0$ and $y'(\pi) = 0$.

(iv) Solve the integral equation $\int_0^x \frac{y(t)}{\sqrt{x-t}}dt = x(x + 1)$.

(v) Find the eigenvalues and corresponding eigenfunctions of the integral equation $y(x) = \lambda \int_0^1 e^{x-t}y(t)dt$.

(vi) Find the function $y(x)$ such that $I[y(x)] = \int_0^1 (y''^2 - 2xy)dx$ is extremum with conditions $y(0) = y'(0) = 0$ and $y(1) = y'(1) = 1$.

SECTION II

Q-3(a) **Attempt any three of the following** [18]

(i) Show that geodesics on a sphere of radius a are its great circles.

(ii) Solve the integro-differential equation $y' + 4y + 5\int_0^x y(t)dt = e^{-x}$, where $y(0) = 0$.

(iii) Solve the integral equation $y(x) = x + \lambda \int_0^1 (1 + x + t)y(t)dt$.

(iv) Solve the integral equation $y(x) = x^2 + \int_0^x \sinh(x - t)y(t)dt$.

(v) Find the eigenvalues and the corresponding eigenfunctions of the differential equation $y'' + \lambda y = 0$ with conditions $y(0) - y(1) = 0, y'(0) + y'(1) = 0$.

(b) **Attempt any two of the following** [12]

(i) If the functional $I[y(x)] = \int_{x_1}^{x_2} f(x, y, y')\,dx$ has the extremum value, prove that the integrand f satisfies the Euler's equation $\frac{\partial f}{\partial y} - \frac{d}{dx}(\frac{\partial f}{\partial y'}) = 0$.

(ii) Let $K(\cdot, \cdot) \in C([a, b] \times [a, b])$. Let $X = C([a, b])$ or $L^p([a, b])$ and $Y = C([a, b])$ or $L^p([a, b])$, where $1 \le p \le \infty$. Define $F : X \to Y$ as follows:

$$F(x)(s) = \int_a^b K(s, t)x(t)dm(t) \quad (s \in [a, b]; x \in X).$$

Then show that F is compact linear map from X into Y.

(iii) Find the eigenvalues and the corresponding eigenfunctions of the differential equation $y'' + \lambda y = 0$ on the interval $[0, 7]$ with the boundary conditions $y'(0) = 0$ and $y'(7) = 0$.

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