CHAROTAR UNIVERSITY OF SCIENCE AND TECHNOLOGY

Third Semester of M.Sc. (Mathematics) Examination November 2018

MA813 Topology-II

Date: 28/11/2018, Wednesday Time: 10:00 a.m. to 10:45 a.m. Maximum Marks: 20

Multiple Choice Questions

Important Instructions:

1. Tick the correct answer and it should be written in question paper itself.
2. You can use only non-programmable calculator. No other gadget is allowed in the examination hall.
3. Each question carries one mark.
4. All notations are standard.

Q - I Choose the correct answer from the given options for the followings: [20]

1. For the subsets A and B of a topological space ______ is true.

(a) $\overline{A} \cap \overline{B} = \overline{A \cap B}$ **(c)** $\overline{A} \cup \overline{B} = \overline{A \cup B}$

(b) $A^\circ \cup B^\circ = (A \cup B)^\circ$ **(d)** None of these

2. The basis element for the product topology on the set $X = \bigcup_{i=1}^{\infty} X_i$ are of the form ______.

(a) for $i = 1, 2, \ldots, n,\ B_i = U_i \in X_i$ and $B_i = X_i$

(b) for $i = 1, 2, \ldots, n,\ B_i = U_i \in X_i$ but $B_i = X_i$

(c) for all $i,\ U_i \in X_i$

(d) None of these

3. If $\mathbb{R}_\ell$ denotes the lower limit topology and $\mathbb{R}_u$ denotes the usual topology then ______.

(a) $\mathbb{R}_\ell$ is finer than $\mathbb{R}_u$ **(c)** $\mathbb{R}_\ell = \mathbb{R}_u$

(b) $\mathbb{R}_u$ is finer than $\mathbb{R}_\ell$ **(d)** None of these

4. The element of the form $\left\{\frac{a}{b}\right\},\ a, b \in \mathbb{Q},\ b \neq 0$ as a subspace of $\mathbb{R}_u$ is ______.

(a) open **(b)** not open **(c)** closed **(d)** None of these

5. If $I_{\mathbb{R}}$ denotes the identity map on $\mathbb{R}$, then ______.

(a) $I_{\mathbb{R}} : (\mathbb{R}, \tau_u) \to (\mathbb{R}, \tau_\ell)$ is continuous

(b) $I_{\mathbb{R}} : (\mathbb{R}, \tau_u) \to (\mathbb{R}, \tau_d)$ is continuous

(c) $I_{\mathbb{R}} : (\mathbb{R}, \tau_u) \to (\mathbb{R}, \tau_\ell)$ is not continuous

(d) $I_{\mathbb{R}} : (\mathbb{R}, \tau_d) \to (\mathbb{R}, \tau_u)$ is not continuous

6. If S^n denote the closed unit sphere in $\mathbb{R}^{n+1}$, then one point compactification of $\mathbb{R}^n$ is homeomorphic to ______.

(a) S^n **(c)** S^{n+1}

(b) $S^n - p,\ p = (0, 0, \ldots, 0, 1) \in \mathbb{R}^{n+1}$ **(d)** None of these

7. Every subset of $\mathbb{R}$ with co-finite topology is ______.

(a) finite **(b)** compact **(c)** not closed **(d)** None of these

8. The function $f : \mathbb{R} \to [0, 1]$ defined by $f(x) = x$, if $|x| \leq 1$ and $f(x) = \dfrac{1}{|x|}$, if $|x| \geq 1$, then ______.

(a) f is discontinuous

(b) f is one one

(c) f is not onto

(d) all of these

9. The set $(0, 1)$ has a compactification given by the map $f(x) =$ ______, $x \in \mathbb{R}$.

(a) $\frac{1}{x}$

(b) $\sin x$

(c) $\cos x$

(d) $(\cos 2\pi x, \sin 2\pi x)$

10. $\mathbb{R} \setminus \{0\}$ is homeomorphic to ______.

(a) $(0, 1)$

(b) $\{(x, y) \in \mathbb{R}^2 : y = x\}$

(c) $\{(x, y) \in \mathbb{R}^2 : xy = 1\}$

(d) None of these

11. $X = \{1, 2, \ldots, 2018\}$ with usual topology is ______.

(a) not a T_2 space

(b) not a compact space

(c) not a T_1 space

(d) a T_2 space

12. If a space is Hausdorff but not a regular, then it is ______ also.

(a) non-compact **(b)** compact **(c)** normal **(d)** None of these

13. Let $X = [0, 1]$, $A_n = \left[\frac{1}{n}, 1\right]$. Then $f : \mathbb{R} \to [0, 1]$ given by $f(0) = 1,\ f(x) = \frac{1}{x}, (0 < x \leq 1)$ is discontinuous ______.

(a) for all $x \in X$

(b) on A_n for all $n > 0$

(c) at $x = 0$

(d) None of these

14. Urysohn lemma gives a rich supply of continuous real valued functions from ______.

(a) 2^{nd} countable and Hausdorff spaces

(b) 2^{nd} countable spaces

(c) regular spaces

(d) 2^{nd} countable and regular spaces

15. Arbitrary product of connected spaces need not be connected with respect to ______ topology.

(a) product **(b)** box **(c)** usual **(d)** None of these

16. The sets $\{(x,y) \in \mathbb{R}^2 : x^2+y^2 = 1\}$ and $\{(x,y) \in \mathbb{R}^2 : x^2+y^2 = 4x-3\}$ are ______.

(a) not connected

(b) connected

(c) totally disconnected

(d) None of these

17. Let the space $X = [2016, 2017] \cup [2018, 2019]$ as a subspace of real line $\mathbb{R}$. Then ______.

(a) $[2016, 2017]$ and $[2018, 2019]$ are not closed sets

(b) $[2016, 2017]$ and $[2018, 2019]$ are not open sets

(c) $[2016, 2017]$ and $[2018, 2019]$ are clopen sets

(d) None of these

18. If $f(x) = \sin x,\ x \in \mathbb{R}$ then $f([0,1])$ is ______ in usual topology.

(a) not homeomorphic to $\mathbb{R}$

(b) homeomorphic to $\mathbb{R}$

(c) homeomorphic to $\mathbb{C}$

(d) None of these

19. Suppose $\{X_\alpha : \alpha \in \Lambda\}$ is a collection of locally compact spaces. Then $\prod\limits_{\alpha \in \Lambda} X_\alpha$ is locally compact if ______.

(a) each X_α is compact for finitely many but for all

(b) each X_α is compact for finitely many

(c) each X_α is compact for all but finitely many

(d) None of these

20. $[0,1]$ is ______ of $(0,1]$ by the map $f(x) = \sin \frac{1}{x}$

(a) Stone cěch compactification

(b) not a Stone cěch compactification

(c) homeomorphism

(d) None of these

Candidate's Seat No.________________

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Third Semester of M.Sc. (Mathematics) Examination November 2018
MA813 Topology-II

Date: 28/11/2018, Wednesday   Time:10:45 a.m. to 01:00 p.m.   Maximum Marks:50

Instructions:

1. **Section I and Section II must be written in separate answer books.**.
2. You can use only non-programmable calculator. No other gadget is allowed in the examination hall.
3. Figures to the right indicate marks.
4. All notations are standard.

SECTION I

Q - II (A) Attempt any TWO of the followings: [**8**]

(1) Is the statement "Every T_1 space need not be T_2." true or false? Justify.

(2) Prove that a second countable regular space is normal.

(3) Prove that a space is locally connected if and only if every component of an open set is open.

(B) Attempt any SIX of the followings: [**12**]

(1) State and prove the Pasting lemma.

(2) Prove that every metrizable space is a Hausdorff space.

(3) Is the statement "$(0, 1)$ and $S^1 - \{0\}$ are homeomorphic" true or false? Justify.

(4) Define first and second countable space.

(5) Prove that a compact Hausdorff space is regular.

(6) State Urysohn lemma.

(7) Prove that every normal space is completely regular.

(8) State Tietze extension theorem.

(9) Let C and D form a separation of X. If Y is a connected subspace of X, then prove that Y lies entirely within C or D.

SECTION II

Q - III (A) Attempt any THREE of the following. [**18**]

(1) **(a)** Let $\mathfrak{B}$ and $\mathfrak{B}'$ be bases for topologies τ and τ' on X respectively. Then prove that the following are equivalent:
(i) τ' is finer than τ.
(ii) For each $x \in X$ and each $B \in \mathfrak{B}'$ containing x, there exists $B' \in \mathfrak{B}'$ such that $x \in B' \subseteq B$.
(b) Let Y be a subspace of a space X. Then prove that $A \subseteq Y$is closed in Y(in a subspace topology) if and only if $A = C \cap Y$, for some closed subset C of X.

(2) **(a)** Let A, X and Y be topological spaces and $f_1 : A \to X, \quad f_2 : A \to Y$. Prove that the map $f : A \to X \times Y$ defined by $f(a) = (f_1(a), f_2(a))$, $a \in A$, is continuous if and only if f_1, f_2 are continuous.
(b) If a function $f : X \to Y$ is continuous, then prove that for each $x \in X$ and each neighborhood V of $f(x)$, there exists a neighborhood of U of X such that $f(U) \subseteq V$.

(3) Let X be a non-empty compact Hausdorff space such that every point of X is a limit point of X. Then prove that X is uncountable.

(4) Let X be a locally compact Hausdorff space which is not compact and Y be one point compactification of X. Then prove that Y is Hausdorff space, X is a subspace of Y, $Y - X$ is a single point and $\overline{X} = Y$.

(5) State and prove Tychonoff theorem.

(B) Attempt any TWO of the following. [**12**]

(1) If X, Y are compact then prove that $X \times Y$ is compact with respect to the product topology.

(2) Prove that $\mathbb{R}_\ell^2 = \mathbb{R}_\ell \times \mathbb{R}_\ell$ is not normal with respect to the product topology.

(3) Prove that the arbitrary product of connected space is connected with respect to the product topology but not with respect to the box topology.