St. Xavier's College – Autonomous Mumbai

Syllabus For 6th Semester Courses in Mathematics (June 2012 onwards)

Contents:

Theory Syllabus for Courses:

Course: S.Mat.6.01

Title: Real Analysis and Multivariable Calculus

Learning objectives: To understand Differentiability of vector fields, Parametric representation of a surface and Stokes' theorem.

Number of lectures: 45

Unit 1. Differential Calculus

(a) Limits and continuity of vector fields.
Basic results on limits and continuity of sum, difference, scalar multiples of vector fields.
Continuity and components of vector fields.
(b) Differentiability of scalar functions.
(i) Derivative of a scalar field with respect to a non-zero vector.
(ii) Direction derivatives and partial derivatives of scalar fields.
(iii) Mean value theorem for derivatives of scalar fields.
(iv) Differentiability of a scalar field at a point (in terms of linear transformation).
Total derivative ,differentiability at a point implies continuity, and existence of direction derivative at the point. The existence of continuous partial derivatives in a neighbourhood of a point implies differentiability at the point.
(v) Chain rule for scalar fields.
(vi) Higher order partial derivatives, mixed partial derivatives.
Sufficient condition for equality of mixed partial derivative.
Second order Taylor formula for scalar fields.

Unit 2. Differentiability of vector fields and its applications.

(i) Gradient of a scalar field. Geometric properties of gradient, level sets and tangent planes.
(ii) Differentiability of vector fields.
(iii) Definition of differentiability of a vector field at a point.
Differentiability of a vector field at a point implies continuity.
(iv) The chain rule for derivative of vector fields.

Unit 3. Parameterization of a surface.

(a) (i) Parametric representation of a surface.
(ii) The fundamental vector product, definition and it being normal to the surface.
(iii) Area of a parametrized surface.

Unit 4. Surface integral.

(a) (i) Surface integrals of scalar and vector fields (definition).
(ii) Independence of value of surface integral under change of parametric representation of the surface.

(iii) Stokes' theorem, (assuming general form of Green's theorem) Divergence theorem for a solid in 3-space bounded by an orientable closed surface for continuously differentiable vector fields.

List Of Recommended Reference Books

(1) Calculus. Vol. 2, T. Apostol, John Wiley.
(2) Calculus. J. Stewart. Brooke/Cole Publishing Co.
(3) Robert G. Bartle and Donald R. Sherbert. Introduction to Real Analysis, Second edition, John Wiley & Sons, INC.
(4) Richard G. Goldberg, Methods of Real Analysis, Oxford & IBH Publishing Co. Pvt. Ltd., New Delhi.
(5) Tom M. Apostol, Calculus Volume II, Second edition, John Wiley & Sons, New York.

Practicals:

1. Limits and continuity of vector fields, Partial derivative, Directional derivatives.
2. Differentiability of scalar fields.
3. Differentiability of vector fields.
4. Parametrisation of surfaces, area of parametrised surfaces.
5. Surface integrals.
6. Stokes' Theorem and Gauss' Divergence Theorem.
7. Miscellaneous Theoretical questions based on Units 1 and 2.
8. Miscellaneous Theoretical questions based on Units 3 and 4.

COURSE S.Mat.6.02

Title: ALGEBRA

Learning objectives:

Number of lectures: 45

Unit 1. Quotient Spaces **(12)**

Review of vector spaces over R:
(a) Quotient spaces:
(i) For a real vector space V and a subspace W, the cosets v + W and the quotient space V=W. First Isomorphism theorem of real vector spaces (Fundamental theorem of homomorphism of vector spaces.)
(ii) Dimension and basis of the quotient space V=W, when V is finite dimensional.
(b) (i) Orthogonal transformations and isometries of a real finite dimensional inner product space. Translations and reflections with respect to a hyperplane. Orthogonal matrices over R.
(ii) Equivalence of orthogonal transformations and isometries fixing origin on a finite dimensional inner product space. Characterization of isometries as composites of orthogonal transformations and isometries.
(iii) Orthogonal transformation of R^2. Any orthogonal transformation in R^2 is a reflection or a rotation.
(c) Characteristic polynomial of a square real matrix and a linear transformation of a finite dimensional real vector space to itself. Cayley Hamilton Theorem (Proof assuming the result A adj(A) = In for an square matrix over the polynomial ring R[t].)

Unit 2. Diagonalizability. **(10)**

(i) Diagonalizability of an real matrix and a linear transformation of a finite dimensional real vector space to itself.
Definition: Geometric multiplicity and Algebraic multiplicity of eigenvalues of an real matrix and of a linear transformation.
(ii) matrix A is diagonalisable if and only if R^n has a basis of eigen vectors of A if and only if the algebraic and geometric multiplicities of eigenvalues of A coincide.
(e) Triangularization.
(i) Triangularization of a real matrix having n real characteristic roots.
(f) Orthogonal diagonalization
(i) Orthogonal diagonalization of real symmetric matrices.
(ii) Application to real quadratic forms. Positive definite, semidefinite matrices. Classification in terms of principal minors. Classification of conics in R^2 and quadric surfaces in R^3.

Unit 3. Intrduction to Rings. **(14)**

(a) (i) Definition of a ring. (The definition should include the existence of a unity element.)
(ii) Properties and examples of rings, including Z, Q, R, C, Mn(R), Q[X], R[X], C[X], Z[i], Z[n]. (iii) Commutative ring. (iv) Units in a ring. The multiplicative group of units of a ring. (v) Characteristic of a ring. (vi) Ring homomorphisms. First Isomorphism theorem of rings. Second Isomorphism theorem of rings.
(vii) Ideals in a ring, sum and product of ideals.
(viii) Quotient rings. (b) Integral domains and fields. Definition and examples. (i) A finite integral domain is a field. (ii) Characteristic of an integral domain, and of a finite field. (c) (i) Construction of quotient field of an integral domain (Emphasis on Z, Q). (ii) A field contains a subfield isomorphic to Zp or Q. (d) Prime ideals and maximal ideals. Definition. Examples in Z. Characterization in terms of quotient rings.

Unit 4. Polynomial rings. (9)

Irreducible polynomials over an integral domain. Unique Factorization Theorem for polynomials over a field. (f) (i) Definition of a Euclidean domain (ED), Principal Ideal Domain (PID), Unique Factorization Domain (UFD). Examples of ED: Z, F[X], where F is a field, and Z[i]. (ii) An ED is a PID, a PID is a UFD. (iii) Prime (irreducible) elements in R[X], Q[X], Zp[X]. Prime and maximal ideals in R[X], Q[X]. (iv) Z[X] is a not a UFD (Statement only).

Recommended Books

1. I.N. Herstein. Topics in Algebra, Wiley Eastern Limited, Second edition.
2. N.S. Gopalakrishnan, University Algebra, Wiley Eastern Limited.
3. M. Artin, Algebra, Prentice Hall of India, New Delhi.
4. Tom M. Apostol, Calculus Volume 2, Second edition, John Wiley, New York, 1969.
5. W.B. Fraleigh, A first course in Abstract Algebra, Third edition, Narosa, New Delhi.
6. J. Gallian. Contemporary Abstract Algebra. Narosa, New Delhi.
Additional Reference Books
7) M. Artin. Algebra.
8) N.S. Gopalkrishnan. University Algebra.
9) P.B. Bhattacharya, S.K. Jain, S. Nagpaul. Abstract Algebra.

Suggested Practicals :
1. Rings, Integral domains and fields.
2. Ideals, prime ideals and maximal ideals.
3. Euclidean Domain, Principal Ideal Domain and Unique Factorization Domain.
4. Quotient spaces.
5. Orthogonal transformations,Isometries.
6. Diagonalization and Orthogonal diagonalization.
7. Miscellaneous Theoretical questions based on Unit 1,2.
8. Miscellaneous Theoretical questions based on Unit 3,4.

Course S.Mat.6.03

Title: Analysis

Learning objectives: Introduction to connectedness and compactness and Fourier Series.

Number of lectures: 45

Unit 1. Compactness (12 lectures)

(a) Definition of a compact set in a metric space (as a set for which every open cover has a finite subcover). Examples, properties such as
(i) continuous image of a compact set is compact.
(ii) compact subsets are closed.
(iii) a continuous function on a compact set is uniformly continuous.
(b) Characterization of compact sets in Rn: The equivalent statements for a subset of Rn to be compact:
(i) Heine-Borel property.
(ii) Closed and boundedness property.
(iii) Bolzano-Weierstrass property.
(iv) Sequentially compactness property.

Unit 2. connectedness.(10 lectures)

(c) (i) Connected metric spaces. Definition and examples.
(ii) Different characterizations of a connected space
(iii) Connected subsets of a metric space, connected subsets of R.
(iv) A continous image of a connected set is connected.
(d) (i) Path connectedness in Rn, definitions and examples.
(ii) A path connected subset of Rn is connected.
(iii) An example of a connected subset of Rn which is not path connected.

Unit 3. The function spaces (10 lectures)

(i) The function space C(X;R) of real valued continuous functions on a metric space X. The space C[a; b] with sup norm, Weierstrass approximation Theorem.(Statement only)
(ii) Fourier series of functions on C[-π , π], Bessel's inequality.

Unit 4.Sum of Fourier Series.(13 lectures.)

Dirichlet kernel, Fejer kernel, Cesaro summability of Fourier series of functions on C[-π , π], Pareseval's identity, convergence of the Fourier series in L_2 norm.

List Of Recommended Reference Books

1. S. Kumaresan. Topology of Metric spaces.
2. R.G. Goldberg Methods of Real Analysis, Oxford and IBH Publishing House, New Delhi.
3. W. Rudin. Principles of Mathematical Analysis. McGraw Hill, Auckland, 1976.
4. P.K. Jain, K. Ahmed. Metric spaces. Narosa, New Delhi, 1996.
5. G.F. Simmons. Introduction to Topology and Modern Analysis. McGraw Hill, New York,

6. T. Apostol. Mathematical Analysis, Second edition, Narosa, New Delhi, 1974.
7. E.T. Copson. Metric spaces. Universal Book Stall, New Delhi, 1996.
8. Sutherland. Topology.
9. D. Somasundaram, B. Choudhary. A _rst course in Mathematical Analysis. Narosa, New Delhi.
10. R. Bhatia. Fourier series. Texts and readings in Mathematics (TRIM series), HBA,

Suggested Practicals

1. Compactness in Rn (emphasis on R1, R2). Properties.
2. Connectedness.
3. Path connectedness.
4. Continuous image of compact and connected sets
5 Fourier series;
6 Parseval's identity.
7. Miscellaneous Theoretical Questions based on Unit 1and 2.
8. Miscellaneous Theoretical Questions based on Unit 3 and 4.

Course: S.MAT.6.04

Title: Complex Variables

Learning Objectives:-To learn about

i) Analytic functions & integration of such functions
ii) Conformal mapping , cross ratio, Bilinear transformation
iii) Laurent Series, Singularities and its types , Residues , Cauchy`s Residue theorem, Rouche`s theorem

Number of lectures: 45

Unit 1. Complex Numbers (10 Lectures)

Review of complex numbers ,the complex plane,Cartesian-polar-exponential form of a complex number ,Inequalities wrt absolute values, De Moivre`s theorem and its applications, Circular and Hyperbolic functions, Inverse circular and Hyperbolic functions, Separation of real and imaginary parts.

Unit 2. Functions of a Complex Variable(10 Lectures)
Limit ,Continuity ,dirivatives ,analytic functions ,Cauchy-Riemann equations (in cartesian and polar form), harmonic functions, orthogonal
curves. To find analytic function when its real/imaginary part or corresponding harmonic function is given. Conformal mapping , cross ratio, Bilinear transformation, fixed(invariant) points.

Unit 3. Complex Integration (10 Lectures)
Rectifiable curves ,integration along piecewise smooth paths , contours.
Cauchy`s theorem & its consequences , Cauchy`s integral formula for derivatives of analytic functions.

Unit 4. Laurent series,Types of singularities & Residues(15 Lectures)
Development of analytic functions as power series–Taylor & Laurent Series.
Entire functions ,Singularities and its types ,Residues , Cauchy`s Residue theorem and its applications-evaluation of standard integrals by Residue calculus method , the Argument principle, Rouche`s theorem & its applications- The Fundamental theorem of Algebra.

Reference Books:-

1) Theory & problems of Complex Variables
by Murray R.Spiegel , Schaum`s Outline series
McGraw-Hill Book Company,Singapore.
2) Functions of one complex variable
by John B.Conway, Narosa Publishing House ,New Delhi.
3) Complex variables and applications
by R.V.Churchill
4) Foundations of Complex Analysis
by S.Ponnusamy, Narosa Publishing House ,New Delhi.
5) john mathews, russel howell from Narosa

Practicals

1) Analytic functions, Cauchy-Riemann equations, Harmonic functions.
2) Conformal mappings, Bilinear transformations.
3) Integration along piecewise smooth paths, Cauchy`s theorem, Cauchy`s integral formula.
4) Taylor`s & Laurent Series
5) Singularities and its types ,Residues , Cauchy`s Residue theorem and its applications.
6) Rouche`s theorem & its applications
7) Miscellaneous Theoretical questions based on Unit 1 and 2.
8) Miscellaneous Theoretical questions based on Unit 3 and 4.

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Course:S.Mat.6.AC

Title: Computer programming and system analysis

Learning Objectives:-To learn about OOP through java programming, applets

Number of lectures: 50

Unit 1. Java Programming and applets (16 Lectures)

Introduction to Classes and Methods(continued)
Defining classes, creating- instance and class variables, creating objects of a class, accessing instance variables of a class, Creating methods, naming methods, accessing methods of class, constructor methods, overloading methods.

Arrays: Arrays (one and two dimensional) declaring arrays, creating array objects, accessing array elements.

Inheritance, interfaces and Packages: Super and sub classes, keywords- "extends", "super', 'final', finalizer methods and overriden methods, abstract classes, concept of interfaces and packages.

Java Applets Basics: Difference of applets and application, creating applets, life cycle of applet, passing parameters to applets.

Graphics, Fonts and Color
The graphics classes, painting the applet, font class, draw graphical figures (oval, rectangle, square, circle, lines,polygones) and text using different fonts.

Unit 2. Networking (09 Lectures)

Introduction
What is networking, need for networking, networking components- nodes, links (point to point and broadcast), networking topologies – bus, star, mesh, network services (connection oriented and connectionless).

Network Design
What is network design, requirement and tasks of a network, LAN MAN, WAN, VAN.

Network Architectures Layering principle, OSI Reference Model, TCP/ IP Reference Model. Comparison of OSI and TCP/P Reference Models.

Network Switching and Multiplexing
Bridges, interconnecting LANs with bridges spanning tree algorithm. What is multiplexing. Static multiplexing (FDM, TDM, WDM), dynamic multiplexing. What is switching, circuit switching, packet switching.

Routing and Addressing Router, router table, routing (direct and indirect), routing characteristics, shortest path routing Dijkstra`s algorithm. TCP/IP internetworking, IP addresses (class, classless), and sub netting and subnet mask, Domain names

Unit 3. C Programming. (16 lectures)
Loops and Controls
Control statements for decision making: branching (if statement, if-else statement, else-if statement, switch statement), looping (while loop, do while loop and for loop), breaking out of loops (break and continue statements).
Storage Classes
Automatic variables, external variables, register variables, static variables - scope and functions.
Functions and Arguments
Global and local variables, function definition, return statement, calling a function (by value, by reference), recursion, recursive functions.
Strings and Arrays
Arrays (one and two dimensional), declaring array variables, initialization of arrays, accessing
array elements, string functions (strcpy, strcat, strchr, strcmp, strlen, strstr, atoi, atof).Pointers
Fundamentals, pointer declarations, operators on pointers, passing pointers to functions, pointers and one dimensional array, pointers and two dimensional array.
Structures.Basics of structures, structures and functions.

Unit 4. Introduction to DBMS and RDBMS (9 Lectures)
Introduction to Database Concepts
Database systems vs file systems, view of data, data models, data abstraction, data independence,
three level architecture, database design, database languages - data definition language(DDL), data manipulation language(DML). E - R Model
Basic concepts, keys, E-R diagram, design of E-R diagram schema (simple example).
Relational structure
Tables (relations), rows (tuples), domains, attributes, candidate keys, primary key, entity integrity constraints, referential integrity constraints, query languages, normal forms 1,2,and 3 (statements only), translation of ER schemas to relational (database) schemas (logical design), physical design.

Recommended Books:-

(1)The complete reference java2: Patrick maughton, Hebert schind (TMH).
(Chapters 1 – 6, 8-9, 12, 21)
(2)Computer Networks – Andrew S. Tanenbaum (PHI) (Chapter 1: 1.1-1.4, chapter 2:2.5.4.2.5.5 Chapter 5:P 5.2.1-5.2.4,5.5.1-5.5.2,5.6.1-5.6.2, Chapter 7:7.1.1. 7.1.3).

(3)Programming in Ansi C - Ram Kumar and Rakesh Agarwal (Tata McGraw Hill)
(Chapters 2 - 8).
(4) Database System Concepts - Silberschatz, Korth, Sudarshan (McGraw-Hill Int. Edition) - 4th Edition (Chapter 1: 1.1 - 1.5, Chapter 2: 2.1 - 2.5, 2.8 - 2.9, Chapter 3: 3.1, Chapter 7: 7.1,7.2, 7.7)

Practicals:- Java programs that illustrate

1) the concept of java class
    (i) with instance variable and methods
    (ii) with instance variables and without methods
    (iii) without instance variable and with methods
  Create an object of this class that will invoke the instance variables and methods accordingly.
2) the concept of (one dimensional) arrays
3) the concept of (two dimensional) arrays
4) the concept of java class that includes inheritance
5) the concept of java class that includes overridden methods
6) the concept of java class that includes interfaces and packages
7) applets
8)Java programs on numerical methods.

C programs for:
1. Creating and printing frequency distribution.
2. (a) Sum of two matrices of order $m \times n$ and transpose of a matrix of order $m \times n$, where $m, n = 3$.
(b) Multiplication of two matrices of order m, where $m = 3$, finding square and cube of a square matrix using function.
3. Simple applications of recursive functions (like Factorial of a positive integer, Generating Fibonacci Sequence, Ackerman Function, univariate equation)
4. Sorting of Numbers (using bubble sort, selection sort), and strings.
5. Using arrays to represent a large integer (that cannot be stored in a single integer variable).
6. Counting number of specified characters (one or more) in a given character string.
7. Writing a function to illustrate pointer arithmetic.
8. Using structures to find and print the average marks of five subject along with the name of a student.
9. Program to find g.c.d. using Euclidean algorithm.
10. Numerical methods with C programs.
11. Program to decide whether given number is prime or not.
12. Finding roots of quadratic equation using C program.
13. Programs to find trace, determinant of a matrix.
14. Program to check given matrix is symmetric or not.