REVISED SYLLABUS OF B.Sc. Part –III (MATHEMATICS)

Implemented from June – 2010

Paper – V (ANALYSIS)

Section – I (REAL ANALYSIS)

UNIT – 1: **SETS AND FUNCTIONS** 7 lectures

1.1 Sets and Elements, Operations on sets

1.2 Functions

1.2.1 Definition of Cartesian product, Function, Extension and restriction of functions, onto function.

1.2.2 THEOREM: If $f : A \to B$ and if $X \subset B, Y \subset B$, then

$$f^{-1}(X \cup Y) = f^{-1}(X) \cup f^{-1}(Y).$$

1.2.3 THEOREM: If $f : A \to B$ and if $X \subset B, Y \subset B$, then

$$f^{-1}(X \cap Y) = f^{-1}(X) \cap f^{-1}(Y).$$

1.2.4 THEOREM: If $f : A \to B$ and if $X \subset A, Y \subset A$, then

$$f(X \cup Y) = f(X) \cup f(Y).$$

12.5 THEOREM: If $f : A \to B$ and if, $X \subset A, Y \subset A$, then

$$f(X \cap Y) \subset f(X) \cap f(Y).$$

1.2.6 Definition of composition of functions.

1.3 Real-valued functions

1.3.1 Definition : Real valued function. Sum, difference, product, and Quotient of real valued functions, $\max(f.g)$, $\min(f,g)$, $|f|$, Characteristic Function.

1.4 Equivalence, Countability

1.4.1 Definitions: one – to – one function, inverse function, 1–1 correspondence and equivalent sets, finite and infinite sets, countable and uncountable set.

1.4.2 Theorem: The countable union of countable sets is countable.

1.4.3 Corollary: The set of rational numbers is countable.

1.4.4 Theorem: If B is an infinite subset of the countable set A, then B is countable.

1.4.5 Corollary: The set of all rational numbers in $[0,1]$ is countable.

1.5 Real numbers

1.5.1 Theorem: The set $[0,1]=\{x:0\le x\le 1\}$ is uncountable.

1.5.2 Corollary: The set of all real numbers is uncountable.

1.6 Least upper bounds

1.6.1 Definition: Upper bound, lower bound of a set, least upper bound.

1.6.2 Least upper bound axiom,

1.6.3 Theorem: If A is any non-empty subset of R that is bounded below, then A has a greatest lower bound in R.

UNIT – 2: **SEQUENCES AND SERIES OF REAL NUMBERS** 15 lectures

2.1 Limit superior and limit inferior

2.1.1 Definition: Limit superior and limit inferior and Examples.

2.1.2 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then

$$\limsup_{n\to\infty} s_n = \lim_{n\to\infty} s_n .$$

2.1.3 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a convergent sequence of real numbers, then

$$\liminf_{n\to\infty} s_n = \lim_{n\to\infty} s_n .$$

2.1.4 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, then

$$\limsup_{n\to\infty} s_n \ge \liminf_{n\to\infty} s_n .$$

2.1.5 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, and if $\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = L$ and $L\in R$, then $\{s_n\}_{n=1}^{\infty}$ is convergent and $\lim_{n\to\infty} s_n = L$.

2.1.6 Theorem: If $\{s_n\}_{n=1}^{\infty}$ is a sequence of real numbers, and if $\limsup_{n\to\infty} s_n = \liminf_{n\to\infty} s_n = \infty$, then $\{s_n\}_{n=1}^{\infty}$ diverges to infinity.

2.1.7 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real numbers, and if $s_n \le t_n \quad (n\in I)$, then $\limsup_{n\to\infty} s_n \le \limsup_{n\to\infty} t_n$ and $\liminf_{n\to\infty} s_n \le \liminf_{n\to\infty} t_n$.

2.1.8 Theorem: If $\{s_n\}_{n=1}^{\infty}$ and $\{t_n\}_{n=1}^{\infty}$ are bounded sequences of real Numbers, then

$$\limsup_{n\to\infty}(s_n+t_n) \le \limsup_{n\to\infty} s_n + \limsup_{n\to\infty} t_n \;;$$

$$\liminf_{n\to\infty}(s_n+t_n) \ge \liminf_{n\to\infty} s_n + \liminf_{n\to\infty} t_n .$$

2.1.9 Theorem (Statement only): Let $\{s_n\}_{n=1}^{\infty}$ be bounded sequences of real Numbers.

a) If $\limsup_{n\to\infty} s_n = M$, then for any $\varepsilon > 0$, (a) $s_n < M + \varepsilon$ for all but a finite number of values of n; (b) $s_n > M - \varepsilon$ for infinitely many values of n.

b) If $\liminf_{n\to\infty} s_n = m$, then for any $\varepsilon > 0$, (c) $s_n > m - \varepsilon$ for all but a finite number of values of n; (d) $s_n < m + \varepsilon$ for infinitely many values of n.

2.1.10 Theorem: Any bounded sequence of real numbers has a convergent subsequence.

2.2 Cauchy sequences (Revision and statements of standard results without Proof)

2.3 Summability of sequences

2.3.1 Definition: (C,1) summablility and examples.

2.3.2 Theorem: If $\lim_{n\to\infty} s_n = L$, then $\lim_{n\to\infty} s_n = L$ (C,1).

2.4 Series whose terms form a non-increasing sequence

2.4.1 Theorem: If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=0}^{\infty} 2^n a_{2^n}$ converges, then $\sum_{n=1}^{\infty} a_n$ converges. Examples.

2.4.2 Theorem: If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=0}^{\infty} 2^n a_{2^n}$ diverges, then $\sum_{n=1}^{\infty} a_n$ diverges. Examples.

2.4.3 Theorem: The series $\sum_{n=1}^{\infty} \frac{1}{n^2}$ converges.

2.4.4 Theorem: If $\{a_n\}_{n=1}^{\infty}$ is a non increasing sequence of positive numbers and if $\sum_{n=1}^{\infty} a_n$ converges, then $\lim_{n\to\infty} na_n = 0$. Examples.

2.5 Summation by parts

2.5.1 Theorem: If $\{a_n\}_{n=1}^{\infty}$ and $\{b_n\}_{n=1}^{\infty}$ are two sequences of real numbers and let $s_n = a_1 + \cdots + a_n$. Then, for each positive integer n ∈ I,

$$\sum_{k=1}^{n} a_k b_k = s_n b_{n+1} - \sum_{k=1}^{n} s_k (b_{k+1} - b_n).$$

2.5.2 Abel′s lemma

2.5.3 Dirichlet′s test

2.5.4 Abel′s test

2.5.5 Examples

2.6 (C,1) Summability of series

2.6.1 Definition of (C,1) Summability of series.,

2.6.2 Theorem: If $\sum_{n=1}^{\infty} a_n$ is (C,1) summable and if $\lim_{n\to\infty} na_n = 0$, then $\sum_{n=1}^{\infty} a_n$ converges.

2.7 The Class l^2

2.7.1 Definition of the class l^2.

2.7.2 Theorem : The Schwarz inequality.

2.7.3 Theorem : Minkowski inequality.

2.7.4 Norm of an element in l^2.

2.7.5 Theorem: The norm for sequences in l^2 has the following properties:

N_1: $\|s\|_2 \geq 0$ $\quad (s \in l^2)$,

N_2: $\|s\|_2 = 0$ if and only if $s = \{0\}_{n=1}^{\infty}$,

N_3: $\|cs\|_2 = |c|.\|s\|_2$ $\quad (c \in R, s \in l^2)$,

N_4: $\|s + t\|_2 \leq \|s\|_2 + \|t\|_2$ $\quad (s, t \in l^2)$.

UNIT –3 : RIEMANN INTEGRATION **15 lectures**

3.1 Riemann integrability & integrals of bounded functions over bounded intervals:

3.1.1 **Definitions & simple examples:** subdivision & norm of subdivision, lower & upper sums, lower & upper integrals, oscillatory sum, Riemann integral.

3.2 Darboux's Theorem:

3.2.1 Lemma: Let f(x) be a function defined on [a, b] for which there is a k Є R such that |f(x)|≤ k. Let D_1 be a subdivision of [a, b] and D_2 be the subdivision of [a, b] consisting of all points of D_1 and at the most p more, with $|D_1| \leq \delta$. Then $S(D_1) - 2pk\delta \leq S(D_2) \leq S(D_1)$

3.2.2 Theorem: To every Є > 0, there corresponds δ > 0 such that

$S(D) < \int_a^b f(x)\,dx + \epsilon$, for every D with $|D| \leq \delta$.

3.2.3 Theorem: To every $\epsilon > 0$, there corresponds $\delta > 0$ such that $s(D) > \int_a^b f(x)dx - \epsilon$, for every D with $|D| \leq \delta$.

3.2.4 Theorem: For every bounded function f on [a, b], prove that the upper integral $\geq$ the lower integral.

3.3 Equivalent definition of integrability and integrals.

3.3.1 Theorem: If f is bounded and integrable over [a, b], then to $\varepsilon > 0$ there corresponds $\delta > 0$, such that for every subdivision $D = \{a = x_0, x_1, x_2, \ldots, x_n = b\}$ with $|D| \leq \delta$ and for every choice of $\xi_r \in [x_{r-1}, x_r]$, $\left|\sum_{r=1}^{n} f(\xi_r)(x_r - x_{r-1}) - \int_a^b f(x)dx\right| < \varepsilon$.

3.3.2 Theorem: If f is integrable over [a, b] and if there exists a number I such that to every $\varepsilon > 0$ there correspond $\delta > 0$ such that for every subdivision $D = \{a = x_0, x_1, x_2, \ldots, x_n = b\}$ with $|D| \leq \delta$ and for every choice of $\xi_r \in [x_{r-1}, x_r]$ with $|\sum_{r=1}^{n} f(\xi_r)(x_r - x_{r-1}) - I| < \epsilon$ then I is value of $\int_a^b f(x)dx$.

3.4 Conditions for integrability.

3.4.1 Theorem: The necessary and sufficient condition for the integrability of a bounded function f over [a, b] is that to every $\varepsilon > 0$, there corresponds $\delta > 0$ such that for every subdivision D of [a, b] with $|D| \leq \delta$, the oscillatory sum $w(D) < \varepsilon$.

3.4.2 Theorem: The necessary and sufficient condition for the integrability of a bounded function f over [a, b] is that to every $\varepsilon > 0$, there corresponds a subdivision D of [a, b] such that the corresponding oscillatory sum $w(D) < \varepsilon$.

3.5 Particular classes of bounded integrable functions:

3.5.1 Theorem: Every continuous function on [a, b] is Riemann

3.5.2 Theorem: Every monotonic function on [a, b] is Riemann integrable.

3.5.3 Theorem: Every bounded function on [a, b] which has only a finite number of points of discontinuities is Riemann integrable.

3.5.4 Theorem: If the function f bounded on [a, b] and the set of all points of discontinuities has a finite number of limit points then f is Riemann integrable over [a, b].

3.5.5 Examples on 3.5.

3.6 Properties of integrable functions:

3.6.1 Theorem: If a bounded function f is integrable on [a, b] then f is also integrable on [a, c] & [c, b], for a < c < b and conversely. In this case $\int_a^b f(x)dx = \int_a^c f(x)dx + \int_c^b f(x)dx$.

3.6.2 Lemma: The oscillation of a bounded function f on [a, b] is the least upper bound of the set $\{|f(\alpha) - f(\beta)| / \alpha, \beta \in [a,b]\}$.

3.6.3 Theorem: If f & g are both bounded and integrable functions on [a, b] then f $\pm$ g are also bounded & integrable over [a, b] and $\int_a^b [f(x) \pm g(x)]dx = \int_a^b f(x)dx \pm \int_a^b g(x)dx$.

3.6.4 Theorem: If f & g are both bounded and integrable functions on [a, b] then the product f.g is also bounded & integrable over [a, b].

3.6.5 Theorem: If f & g are both bounded and integrable functions on [a, b] and if there exists t > 0 with $|g(x)| \geq t$, $(a \leq x \leq b)$ then f/g is also bounded & integrable over [a, b].

3.6.6 Theorem: If f is both bounded and integrable function on [a, b] then | f | is also bounded & integrable over [a, b].

3.7 Inequalities for an integral:

3.7.1 Theorem: If f is bounded and integrable function on [a, b] and if M and m are the least upper and greatest lower bounds of f over [a, b] then $m(b-a) \leq \int_a^b f(x)dx \leq M(b-a)$, if $a \leq b$ and

$$m(a-b) \geq \int_a^b f(x)dx \geq M(a-b), \text{ if } b \leq a.$$

3.7.2 Theorem: If f is bounded and integrable over [a, b] with $|f(x)| \leq k$ then $\left|\int_a^b f(x)dx\right| \leq k.|b-a|$.

3.7.3 Theorem: If $\int_a^b |f(x)|dx$ exists then $\left|\int_a^b f(x)dx\right| \leq \int_a^b |f(x)|dx$.

3.8 Function defined by a definite integral:

3.8.1 Definition: the integral function of an integrable function f on [a, b].

3.8.2 Theorem: The integral function of an integrable function is continuous.

3.8.3 Theorem: The integral function Φ, of a continuous function f, is continuous and $\Phi' = f$.

3.9 Theorems of Integral Calculus (statements only):

3.9.1 Fundamental Theorem of Integral calculus.

3.9.2 First Mean Value Theorem of Integral calculus.

3.9.3 Second Mean Value Theorem of Integral calculus.

3.9.4 Integration by Change of variable.

3.9.5 Integration by Parts.

3.10 Examples on 3.9.

UNIT – 4 : <u>IMPROPER INTEGRALS</u> **8 lectures**

4.1 Definitions & simple examples: a point of infinite discontinuity of a function, a proper integral, an improper integral, convergence at the left end, convergence at the right end, convergence at both the ends, the case of finite number of infinite discontinuities, convergence at infinity convergence at minus infinity, convergence over (∞, – ∞), convergence over (∞, – ∞) together with a finite number of infinite discontinuities.

4. 2 Test for convergence at the left end: positive integrand.

4.2.1 The necessary & sufficient condition for the convergence of the improper integral $\int_a^b f(x)dx$ at a, when f(x)>0 for all x Є (a , b].

4.2.2 Comparison test for two improper integrals & examples.

4.2.3 Practical comparison test for two improper integrals & examples.

4.2.4 A useful comparison integral $\int_a^b \frac{dx}{(x-a)^n}$ & examples.

4.2.5 Examples on 4.2.

4.3 General test for convergence of the improper integral $\int_a^b f(x)dx$ at a ; f(x), not necessarily positive:

4.3.1 Definition of absolute convergence of the improper integral $\int_a^b f(x)dx$ at a.

4.3.2 Theorem: Every absolutely convergent integral is convergent.

4.3.3 Convergence at ∞ of the improper integral $\int_a^\infty f(x)dx$: positive integrand .

4.3.4 A useful comparison integral $\int_a^\infty \frac{dx}{x^n}$,a>0. The necessary & sufficient condition for its convergence.

4.3.5 Examples on 4.3.

4.4 Convergence at ∞ , the integrand being not necessarily positive.

4.4.1 General test for convergence.

4.4.2 Theorem: If an improper integral converges absolutely then it converges.

4.4.3 Test for absolute convergence of the integral of a product.

4.4.4 Examples on 4.4.

4.5 Tests for conditional convergence:

4.5.1 Abel's theorem for the convergence of the integral of a product.

4.5.2 Dirichlet's theorem for the convergence of the integral of a product.

4.5.3 Examples on 4.5.

SECTION – II

(METRIC SPACES)

UNIT –5: <u>LIMITS AND METRIC SPACES</u> **8 lectures**

5.1 Revision: Limits of a function on the real line.

5. 2 Metric space: Definition :of Metric space and examples inclusive of each of $R^1, R_d, R^n, l^\infty, l^2$.

5.3 Limits in metric spaces

5.3.1 Definition of $\lim_{x \to a} f(x) = L$.

5.3.2 If $\lim_{x \to a} f(x) = L$ and $\lim_{x \to a} g(x) = N$ then

(i) $\lim_{x \to a}[f(x) + g(x)] = L + N$;

(ii) $\lim_{x \to a}[f(x) - g(x)] = L - N$;

(iii) $\lim_{x \to a}[f(x)g(x)] = LN$ and

(iv) $\lim_{x \to a}[f(x)/g(x)] = L/N \quad (N \neq 0)$

5.4 Definition: Sequences and their convergence in metric space, Cauchy sequence in metric space.

5.4.1 Theorem : A sequence of points in any metric space cannot converge to two distinct limits

5.4.2 Theorem : Every convergent sequence in metric space is Cauchy.

5.4.3 Example to illustrate that every Cauchy sequence need not be convergent.

5.4.3 Theorem : Every Cauchy sequence of real numbers is bounded.

5.4.4 Theorem : If a Cauchy sequence has a convergent subsequence then the sequence itself is convergent.

5.4.5 Theorem : Every Cauchy sequence in R_d is convergent.

UNIT - 6: CONTINUOUS FUNCTIONS ON METRIC SPACES — 15 lectures

6.1 Functions continuous at a point on the real line.

6.1.1 Definition: Continuity of a function

6.1.2 Theorem (Statement only) : If real valued functions f and g are continuous at $a \in R^1$, then so are

$f+g,\quad f-g,\quad fg,\quad f/g,\quad f\circ g,\quad cf,\quad |f|$ where, $c \in R$ at a.

6.2 Reformulation:

6.2.1 Theorem : The real valued function f is continuous at $a \in R^1$ if and only if given $\varepsilon > 0$ there exists $\delta > 0$ such that

$$|f(x) - f(a)| < \varepsilon \qquad (|x-a| < \delta).$$

6.2.2 Definition: The open ball of radius r about a.

6.2.3 Theorem : The real valued function f is continuous at $a \in R^1$ if and only if the inverse image under f of any open ball $B[f(a);\varepsilon]$ about $f(a)$ contains an open ball $B[a;\delta]$ about a.

6.2.4 Theorem : A function f is continuous at a, if and only if

$$\lim_{n\to\infty} x_n = a \Rightarrow \lim_{n\to\infty} f(x_n) = f(a).$$

6.3 Functions continuous on a metric space

6.3.1 Definition: The open ball of radius r about a in a metric space.

6.3.2 Definition: Continuity of function defined on a metric space

6.3.3 Theorem : The function f is continuous at $a \in M_1$ if and only if any one of the following conditions hold

(i) Given $\varepsilon > 0$, there exists $\delta > 0$ such that

$$\rho_2[f(x), f(a)] < \varepsilon \qquad (\rho_1(x,a) < \delta).$$

(ii) The inverse image under f of any open ball $B[f(a);\varepsilon]$ about $f(a)$ contains an open ball $B[a;\delta]$ about a.

(iii) Whenever $\{x_n\}_{n=1}^{\infty}$ is a sequence of points in M_1 converging to a, then the sequence $\{f(x_n)\}_{n=1}^{\infty}$ of points in M_2 converging to $f(a)$.

6.3.4 Theorem : If f is continuous at $a \in M_1$ and g is continuous at $f(a) \in M_2$, then $g \circ f$ is continuous at a.

6.3.5 Theorem : Let M be a metric space, and let f and g be real valued functions which are continuous at $a \in M$, then so are $f+g, \quad f-g, \quad fg, \quad f/g, \quad |f|$ at a.

6.3.6 Definition of continuity of a function $f: M_1 \to M_2$.

6.3.7 Theorem : If f and g be continuous functions from a metric space M_1 into a metric space M_2, then so are $f+g, \quad f-g, \quad fg, \quad f/g, \quad |f|$ on M_1.

6.4 Open sets.

6.4.1 Definition: Open set

6.4.2 Any open ball in a metric space is an open set.

6.4.3 Theorem : In any metric space $<M, \rho>$, both M and $\varnothing$ are open sets.

6.4.4 Theorem : Arbitrary union of open sets is open.

6.4.5 Theorem : Every subset of R_d is open.

6.4.6 Theorem : Finite intersection open sets is open

6.4.7 Theorem : Every open subset G of R^1 can be written as $G = \bigcup I_n$ where $I_1, I_2, \ldots$ are a finite number or a countable number of open intervals which are mutually disjoint.

6.4.8 Theorem : A function is continuous if and only if inverse image of every open set is open.

6.5 Closed sets

6.5.1 Definition: Limit point, closure of a set.

6.5.2 Theorem : If E is any subset of the metric space M, then $E \subset \overline{E}$.

6.5.3 Definition: Closed set.

6.5.4 Theorem : Let E be a subset of the metric space M. Then the point $x \in M$ is a limit point of E if and only if every open ball $B[x;r]$ about x contains at least one point of E.

6.5.5 Theorem : Let E be a subset of the metric space M, then $\overline{E}$ is closed.

6.5.6 Theorem : In any metric space $<M, \rho>$, both M and $\varnothing$ are closed sets.

6.5.7 Theorem : Arbitrary intersection of closed sets is closed.

6.5.8 Theorem : Finite union of closed sets is closed.

6.5.9 Theorem :Let G be an open subset of the metric space M. Then $G' = M - G$ is closed. Conversely, if F is a closed subset of M, then $F' = M - F$ is open.

6.5.10 Theorem : Let $< M_1, \rho_1 >$ and $< M_2, \rho_2 >$ be metric spaces., and let $f : M_1 \to M_2$. Then f is continuous on M_1 if and only if $f^{-1}(F)$ is a closed subset of M_1 whenever F is a closed subset of M_2.

6.5.11 Theorem :Let f be a 1-1 function from a metric space M_1 onto a metric space M_2. Then if f has any one of the following properties, it has them all.

(i) Both f and f^{-1} are continuous (on M_1 and M_2, respectively).

(ii) The set $G_1 \subset M_1$ is open if and only if its image $f(G) \subset M_2$ is open.

(iii) The set $F \subset M_1$ is closed if and only if its image $f(F)$ is closed.

6.5.12 Definition : Homeomorphism, dense subset of a metric space.

6.5.13 Show that R^1 and R_d are not homeomorphic.

UNIT - 7: <u>CONNECTEDNESS, COMPLETENESS, AND COMPACTNESS</u>

15 lectures

7.1 More about open sets

7.1.1 Theorem : Let $< M, \rho >$ be a metric space and let A be a proper subset of M. Then the subset G_A of A is an open subset of $< A, \rho >$ if and only if there exists an open subset G_M of $< M, \rho >$ such that $G_A = A \cap G_M$.

7.2 Connected sets:

7.2.1 Theorem : Let $< M, \rho >$ be a metric space and let A be a subset of M. Then if A has either one of the following properties it has the other.

(a) It is impossible to find nonempty subsets A_1, A_2 of M such that

$$A = A_1 \cup A_2,\ \overline{A_1} \cap A_2 = \varnothing, A_1 \cap \overline{A_2} = \varnothing .$$

(b) When $< A, \rho >$ is itself regarded as metric space, then there is no set except A and $\varnothing$ which is both open and closed in $< A, \rho >$.

7.2.2 Definition: Connected set

7.2.3 Theorem : The subset A of R^1 is connected if and only if whenever $a \in A, b \in A$ with $a < b$, then $c \in A$ for any c such that $a < c < b$.

7.2.4 Theorem : A continuous function carries connected sets to connected sets.

7.2.5 Theorem : If f is a continuous real valued function on the closed bounded interval $[a,b]$, then f takes on every value between $f(a)$ and $f(b)$.

7.2.6 Theorem : A metric space is connected if and only if every continuous characteristic function on it is constant.

7.2.7 Theorem : If A_1 and A_2 are connected subsets of a metric space M, and if $A_1 \cap A_2 \neq \varnothing$, then $A_1 \cup A_2$ is also connected.

7.2.8 Theorem : The interval $[0,1]$ is not connected sunset of R_d.

7.3 Bounded and totally bounded sets

7.3.1 Definition: Bounded subset of metric space, totally bounded sets.

7.3.2 Theorem : Every totally bounded set is bounded.

7.3.3 Theorem : A subset A of R_d is totally bounded if and only if A contains only a finite number of points.

7.3.4 Definition: $\varepsilon -$dense set.

7.3.5 Theorem : The subset A of the metric space $< M, \rho >$ is totally bounded if and only if , for every $\varepsilon > 0$, A contains a finite subset $\{x_1, x_2, \ldots, x_n\}$ which is $\varepsilon -$dense in A.

7.3.6 Theorem : Let $< M, \rho >$ be a metric space. The subset A of M is totally bounded if and only if every sequence of points of A contains a Cauchy sequence.

7.4 Complete metric space.

7.4.1 Definition: Complete metric space.

7.4.2 Theorem : If $< M, \rho >$ be a complete metric space, and A is a closed subset of M , then $< A, \rho >$ is also complete.

7.4.3 Generalized nested interval theorem.

7.4.4 Definition: Contraction operator.

7.4.5 Theorem : Let $< M, \rho >$ be a complete metric space. If T is a contraction on M , then there is one and only one point x in M such that $Tx = x$.

7.4.6 R_d is complete and R^2 is complete.

7.5 Compact metric spaces

7.5.1 Definition: Compact metric space.

7.5.2 The metric space $<M,\rho>$ is compact if and only if every sequence of points in M has a subsequence converging to a point in M.

7.5.3 Theorem : A closed subset of a compact metric space is compact.

7.5.4 Theorem : Every compact subset of a metric space is closed.

7.5.5 Definition: Covering and open covering

7.5.6 Theorem : If M is a compact metric space, then M has the Heine-Borel property.

7.5.7 Theorem : If a metric space M has Heine-Borel property, the M is compact.

7.5.8 Definition: Finite intersection property.

7.5.9 Theorem : The metric space M is compact if and only if, whenever F is a family of closed subsets of M with finite intersection property, then $\bigcap_{F\in \mathcal{F}} F \neq \varnothing$.

7.5.10 Theorem : Finite subset of any metric space is compact.

UNIT- 8 : <u>SOME PROPERTIES OF CONTINUOUS FUNCTIONS ON METRIC SPACE</u> **7 lectures**

8.1 Continuous functions on compact metric space

8.1.1 Theorem : Let f be a continuous function of the compact metric space M_1 into a metric space M_2 . Then the range $f(M_2)$ of f is also compact.

8.1.2 Theorem : Let f be a continuous function of the compact metric space M_1 into a metric space M_2. Then the range $f(M_2)$ of f is a bounded subset of M_2.

8.1.3 Definition: Bounded function

8.1.4 Theorem : If the real valued function f is continuous on a closed bounded interval in R^1, then f must be bounded.

8.1.5 Theorem : If the real valued function f is continuous on the compact metric space M, then f attains a maximum value at some point of M. Also, f attains a minimum value at some point of M.

8.1.6 Theorem : If the real valued function f is continuous on a closed bounded interval $[a,b]$, then f attains a maximum and minimum value at some point of $[a,b]$.

8.1.7 Theorem : If f is a continuous real valued function on the compact connected metric space M, then f takes on every value between its minimum value and its maximum value.

8.2 Uniform continuity

8.2.1 Definition: Uniform continuity.

8.2.2 Let $< M_1, \rho_1 >$ be a compact metric space. If f is a continuous function from M_1 into a metric space $< M_2, \rho_2 >$, then f is uniformly continuous on M_1.

8.3.3 If the real valued function f is continuous on the closed bounded interval $[a,b]$, then f is uniformly continuous on $[a,b]$.

8.3.4 Let $< M_1, \rho_1 >$ be a metric space and let A be a dense subset of M_1. If f is a uniformly continuous function from $< A, \rho_1 >$ into a complete Metric space $< M_2, \rho_2 >$, then f can be extended to a uniformly continuous function F from M_1 into M_2.

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Paper No. – VI
(Algebra)
Section – I
(Modern Algebra)

Unit – 1 : **<u>GROUPS</u>** 15 lectures

1.1 Preliminaries: Divisibility in integers, the greatest common divisor (g. c. d.), function from a set A to a set B, a 1-1 function, an onto function, prime and composite numbers, congruence relation on the set of integers, Permutations, Cyclic Permutations, Transpositions, Disjoint Permutations, Even and odd permutations.

1.1.1 Theorem: (without proof): Euclid's algorithm: If m is a positive integer and n is any integer then there exists integers q and r such that $n = mq + r$, where $0 \leq r < m$.

1.1.2 Theorem: (without proof): For any two non-zero integers a & b, there exists a greatest common divisor d of a and b such that $d = ax + by$ for some integers x and y.

1.1.3 Theorem: (without proof): Two integers a & b are relatively prime if and only if there exist two integers x & y such that $ax + by = 1$.

1.2 Group and commutative group: Definitions & examples.

1.2.1 Theorems: In a group G

a) Identity element is unique.

b) Inverses of each $a \in G$ is unique.

c) $(a^{-1})^{-1} = a$, for all $a \in G$.

d) $(ab)^{-1} = b^{-1}a^{-1}$ for all $a, b \in G$.

e) $ab = ac \Rightarrow b = c$, $ba = ca \Rightarrow b = c$ for all $a, b, c \in G$.

1.2.2 Theorem : For elements a, b in a group G, the equations $ax = b$ and $ya = b$ have unique solutions for x and y in G.

1.3 Definition of Subgroup and examples

1.3.1 Theorem: A non empty subset H of a group G is subgroup of G iff

i) $a, b \in H \Rightarrow ab \in H$ ii) $a \in H \Rightarrow a^{-1} \in H$

1.3.2 Theorem: A non-empty subset H of a group G is subgroup of G iff

$a, b \in H \Rightarrow ab^{-1} \in H$

1.4 Definition of Centre of group G and Normalizer of a in G.

1.4.1 Theorem : Centre of group G is a subgroup of G

1.4.2 Normalizer is a subgroup of G.

1.5 Definition of left and right cosets and $Ha = \{x \in G \mid x \equiv a \bmod H\} = cl(a)$ for any $a \in G$.

1.6 Definition of order of group.

1.6.1 Lagrange's Theorem: If G is finite group and H is subgroup of G then o(H) divides o(G).

1. 6. 2Theorem: Ha = H if and only if a Є H.

1.6.3 **Theorem:** Ha = Hb if and only if ab^{-1} Є H .

1.6.4 **Theorem:** Ha is a subgroup of G if and only if a ЄH.

1.6.5 Theorem: HK is subgroup of G iff HK = KH.

1.7 Definition of Cyclic group and Order of element of a group.

1.7.1 Theorem: Order of a cyclic group is equal to the order of its generator.

1.7.2 Theorem: A subgroup of a cyclic group is cyclic.

1.7.3 Theorem: A cyclic group is abelian.

1.7.4 Theorem: If G is finite group, then order of any element of G divides order of group G.

1.7.5 Theorem: An infinite cyclic group has precisely two generators.

1.7.6 Definition of Eule's ϕ function.

1.7.7 Theorem: Number of generators of a finite cyclic group of order n is $\phi(n)$.

1.7.8 Euler's Theorem: Let a, n ($n \geq 1$) be any integers such that g.c.d. (a, n) = 1. Then $a^{\phi(n)} \equiv 1 \pmod{n}$.

1.7.9 Fermat's Theorem: For any integer a and prime p, $a^p \equiv a \pmod{p}$ and examples.

Unit – 2: <u>NORMAL SUBGROUPS, HOMOMORPHISMS, PERMUTATION GROUP</u> 10 Lectures

2.1 Normal Subgroups: Definition and Examples.

2.1.1 Theorem: A subgroup H of a group G is normal in G iff $g^{-1}Hg = H$ for all $g \in G$.

2.1.2 Theorem: A subgroup H of a group G is normal in G iff $g^{-1}hg \in H$ for all $h \in H$, $g \in G$.

2.1.3 Theorem: A subgroup H of a group G is normal in G iff product of two right cosets of H in G is again a right coset of H in G.

2.1.4 Definition of Quotient Group.

2.1.5 Theorem: If G is finite group and N is a normal subgroup of G then

$$o\left(\frac{G}{N}\right)=\frac{o(G)}{o(N)}$$

2.1.6 Theorem (without proof) : Every quotient group of a cyclic group is cyclic

2.2 Definitions of Homomorphism, Isomorphism, Epimorphism, Monomorphism, Endomorphism, and Automorphism.

2.2.1 Theorem: If $f:G\to G'$ is homomorphism then f(e) = e'

2.2.2 Theorem: If $f:G\to G'$ is homomorphism then $f(x^{-1}) = [f(x)]^{-1}$

2.2.3 Theorem: If $f:G\to G'$ is homomorphism then $f(x^n) = [f(x)]^n$, n an integer.

2.2.4 Definition of Kernel of homomorphism.

2.2.5 Theorem: If $f:G\to G'$ is homomorphism then Ker f is a normal subgroup of G.

2.2.6 Theorem: A homomorphism $f:G\to G'$ is one-one iff Ker f = $\{e\}$.

2.2.7 Fundamental Theorem of group homomorphism: If $f:G\to G'$ is an onto homomorphism with K = Ker f, then $\frac{G}{K}\cong G'$

2.2.8 Second Theorem of isomorphism: Let H and K be two subgroups of group G, where H is normal in G, then $\frac{HK}{H}\cong\frac{K}{H\cap K}$

2.2.9 Third Theorem of isomorphism: If H and K be two normal subgroups of group G, such that H $\subseteq$ K then $\frac{G}{K}\cong\frac{G/H}{K/H}$

2.2.10 Cayley's Theorem: Every group G is isomorphic to a permutation group.

Unit –3: <u>RINGS</u> **8 Lectures**

3.1 Definition of Ring, Commutative ring, Zero divisor, Integral Domain Division Ring, Field, Boolean ring.

3.1.1 Theorem: A field is an integral domain.

3.1.2 Theorem: A non-zero finite integral domain is field.

3.2 Definition of Subring

3.2.1Theorem: A non-empty subset S of ring R is a subring of R if and only if a, b $\in$ S $\Rightarrow$ ab, a – b $\in$ S

3.3 Characteristic of a ring.

3.3.1 Theorem: Let R be ring with unity. If 1 is additive order n then ch R = n. if 1 is of additive order infinity then ch R is 0.

3.3.2 Theorem: If D is an integral domain, then characteristic of D is either zero or prime number.

3.3.3 Definition of Nilpotent, Idempotent, product of rings.

3.4 Definition of Ideal

3.4.1 Definition of Sum of two ideals.

3.4.2 Theorem: If A and B of two ideals of R then A + B is an ideal of R containing both A and B.

3.5 Definition of Simple Ring.

3.5.1 Theorem: A division is a simple ring.

Unit –4: <u>HOMOMORPHISM AND IMBEDDING OF RING</u> **12 Lectures**

4.1 Definition of Quotient Rings, Homomorphism, Kernel of homomorphism.

4.1.1 Theorem: If $\theta : R \to R'$ be a homomorphism, then $\theta(0) = 0'$

4.1.2 Theorem: If $\theta : R \to R'$ be a homomorphism, then $\theta(-a) = -\theta(a)$

4.1.3 Theorem: If $f : R \to R'$ is homomorphism then Ker f is an ideal of R.

4.1.4 Theorem: If $f : R \to R'$ is homomorphism then Ker f = $\{0\}$ if and only if f is one –one.

4.1.5 Fundamental Theorem of ring homomorphism: If $f : R \to R'$ is an onto homomorphism with K = Ker f, then $\frac{R}{K} \cong R'$

4.1.6 First Theorem of isomorphism: Let B $\subseteq$ A be two ideal of ring R. then $\frac{R}{A} \cong \frac{R/B}{A/B}$

4.1.7 Second Theorem of isomorphism: Let A, B be two ideals of ring R then $\frac{A+B}{A} \cong \frac{B}{A \cap B}$

4.2 Definition of Imbedding ring.

4.2.1 Theorem: Any ring can be imbedded into a ring with unity.

4.3 Definition of Maximal Ideal and Prime Ideal.

4.3.1 Theorem: Let R be a commutative ring with unity. An ideal M of R is maximal ideal of R iff $\frac{R}{M}$ is a field.

4.3.2 Theorem: Let R be a commutative ring. An ideal P of R is prime iff $\frac{R}{P}$ is an integral domain.

Section – II
(Linear Algebra)

Unit –5: <u>VECTOR SPACES</u> **18 Lectures**

5.1 Definition of vector space and simple examples.

5.2 Theorem: In any vector space V(F) the following results hold

i) $0 \cdot x = 0$

ii) $\alpha \cdot 0 = 0$

iii) $(-\alpha)x = -(\alpha x) = \alpha(-x)$

iv) $(\alpha - \beta)x = \alpha x - \beta x$

5.3 Definition of subspace

5.4 Theorem: A necessary and sufficient condition for a non empty subset W of a vector space V(F) to be a subspace is that W is closed under addition and scalar multiplication.

5.5 Theorem: A non empty subset W of a vector space V(F) is a subspace of V if and only if $\alpha x + \beta y \in W \ for\ \alpha, \beta \in F, x, y \in W$.

5.6 Definition of sum of subspaces, direct sum, and quotient space, homomorphism of vector space and examples.

5.7 Theorem: Under a homomorphism $T : V \to U$

i) T(0) = 0

ii) T(-x) = -T(x)

5.8 Definition of Kernel and Range of homomorphism.

5.9 Theorem : Let $T : V \to U$ be a homomorphism, then Ker T is a subspace of V.

5.10 Theorem: Let $T : V \to U$ be a homomorphism, then Ker T = {0} if and only if T is one – one.

5.11 Theorem: Let $T : V \to U$ be a L.T. (linear transformation) then range of T is a subspace of U

5.12 Theorem: Let W be a subspace of V, then there exists an onto L.T. $\theta : V \to \frac{V}{W}$ such that Ker θ=W

5.13 Definition of Linear Span.

5.14 Theorem: L(S) is the smallest subspace of V containing S.

5.15 Theorem: If W is subspace of V then L(W) = W and conversely.

5.16 Definition of Finite dimensional vector space (F. D.V. S), Linear dependence and independence, basis of vector space and examples.

5.17 Theorem: If S = $\{v_1, v_2, v_3, \ldots, v_n\}$ is a basis of V then every element of V can be expressed uniquely as a linear combination of $v_1, v_2, v_3, \ldots, v_n$.

5.18 Theorem: Suppose S is a finite subset of a vector space V such that V = L(S) then there exists a subset of S which is a basis of V.

5.19 Definition of F.D.V.S.

5.20 Theorem: If V is a F.D.V.S. and $\{v_1, v_2, v_3, \ldots, v_r\}$ is a L.I. subset of V, then it can be extended to form a basis of V.

5.21 Theorem: If dim V = n and S = $\{v_1, v_2, v_3, \ldots, v_n\}$ spans V then S is a basis of V.

5.22 Theorem: If dim V = n and S = $\{v_1, v_2, v_3, \ldots, v_n\}$ is a L.I. subset of V then S is a basis of V.

Unit –6: <u>INNER PRODUCT SPACES</u> **10 Lectures**

6.1 Definition of Inner product space, norm of a vector and examples.

6.2 Theorem: Cauchy- Schwarz inequality. Let V be an inner product space. Then $|(u, v)| \leq \|u\| \|v\|$, for all u, v Є V.

6.3 Theorem: Triangle inequality. Let V be an inner product space. Then $\| u + v \| \leq \| u \| + \| v \|$, for all u, v Є V.

6.4 Theorem: Parallelogram law. Let V be an inner product space. Then $\| u + v \|^2 + \| u - v \|^2 = 2 (\| u \|^2 + \| v \|^2)$, for all u, v Є V.

6.5 Definition of Orthogonal vectors and orthonormal sets.

6.6 Theorem: Let S be a orthogonal set of non-zero vectors in an inner product space V. Then S is a linearly independent set.

6.7 Gram-Schmidt orthogonalisation process

6.7.1 Theorem : Let V be a non trivial inner product space of dimension n. Then V has an orthonormal basis.

6.7.2 Examples.

Unit –7: <u>LINEAR TRANSFORMATION</u> **10 Lectures**

7.1 Definition of L.T., Rank and Nullity and Examples.

7.2 Theorem : A L.T. $T: V \to V$ is one – one iff T is onto.

7.3 Theorem : Let V and W be two vector spaces over F. Let $\{v_1, v_2, v_3, \ldots, v_n\}$ be a basis of V and $w_1, w_2, w_3, \ldots, w_n$ be any vectors in W (not essentially distinct). Then there exists a unique L.T.

$T: V \to W \ s.t. T(V_i) = w_i \ \ i = 1,2,\ldots n$

7.4 Theorem : (Sylvester's Law) : Suppose V and W are finite dimensional vector spaces over a field F. Let T: V→W be a linear transformation. Then rank T + nullity T = dim V.

7.5 Theorem: If $T: V \to V$ be a L.T. Show that the following statements are equivalent.

i) Range T $\cap$ Ker T = {0}

ii) If T(T(v)) = 0 then T(v) = 0, v $\in$V.

7.6 Definition of Sum and Product of L.T., Linear operator, Linear functional and examples.

7.7 Theorem: Let T, T_1, T_2 be linear operators on V and let $I: V \to V$ be the identity mapping I(v) = v for all v then

i) I T = T I = T

ii) $T(T_1 + T_2) = TT_1 + TT_2$, $(T_1 + T_2)T = T_1T + T_2T$

iii) $\alpha(T_1T_2) = (\alpha T_1)T_2 = T_1(\alpha T_2) \quad \alpha \in F$

iv) $T_1(T_1T_2) = (T_1T_2)T_1$

7.8 Definition of Invertible L.T. and examples.

7.9 Theorem: A L.T. $T: V \to W$ is a nonsingular iff T carries each L.T. subset of V onto a L.I. subset of W.

7.10 Theorem: Let $T: V \to W$ be a L.T. where V and W are F.D.V.S. with same dimension. Then the following are equivalent.

i) T is invertible.

ii) T is nonsingular.

iii) T is onto.

7.11 Theorem: Let $T: V \to W$ and $S: W \to U$ be two L.T. Then

i) If S and T are one – one onto then ST is one-one onto and $(ST)^{-1} = T^{-1}S^{-1}$.

ii) If ST is one – one then T is one-one.

iii) If ST is onto then S is onto.

7.12 Definition of Matrix of L.T. and examples.

7.12.**Theorem:** $Hom(U,V) \cong M_{m\times n}(F)$.

7.13 Definition of Dual space.

7.13.1 Theorem: Let V be n dimensional vector space over a field F. Then the dimension the dual space of V over F is n.

Unit –8: <u>EIGEN VALUES AND EIGEN VECTORS</u> **7 Lectures**

8.1 Definition of Eigen values, Eigen vectors, Eigen space of order n.

8.1.1 examples.

8.2 Let A be a square matrix of order n. If λ is an eigen value of A then the set all eigen vectors of A corresponding to λ together with zero vector, forms a subspace of n dimensional unitary space.

8.3 Theorem: Let A be a square matrix of order n having k distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_k$. Let v_i be an eigen vector corresponding to the eigen value λ_i, i = 1, 2, …,k. Then the set $\{v_1, v_2, \ldots v_k\}$ is linearly independent.

8.4 Theorem: Let A be a square matrix of order n having n distinct eigen values $\lambda_1, \lambda_2, \ldots, \lambda_k$. Let v_i be an eigen vector corresponding to the eigen value λ_i, i = 1, 2, …,n. Then the set $\{v_1, v_2, \ldots v_n\}$ is basis for the domain space of A. The matrix of the linear transformation A with respect to the basis $\{v_1, v_2, v_3, . . . v_n\}$ is

$$\begin{bmatrix} \lambda_1 & 0 & 0 & \ldots & 0 \\ 0 & \lambda_2 & 0 & \ldots & 0 \\ 0 & 0 & \lambda_3 & \ldots & 0 \\ \ldots & \ldots & \ldots & \ldots & \\ 0 & 0 & 0 & \ldots & \lambda_n \end{bmatrix}$$

8.5 Examples on application of 8.4.

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Paper – VII
Complex Analysis & Integral Transform

Section – I (Complex Analysis)

Unit – 1 : <u>COMPLEX NUMBERS AND THEIR GEOMETRY</u> 12 lectures

1.1 Modulus of a complex number and properties of modulus.

1.2 Argument of a complex number and properties of argument.

1.3 Polar form of a complex number.

1.4 Complex variable z and function of a complex variable f(z) = u + iv.

1.5 Interpretation of arg $\left(\frac{z-\alpha}{z-\beta}\right)$**.**

1.6 Equation in complex form of (a) line, (b) circle.

1.7 Cross – ratio (z_1, z_2, z_3, z_4).

1.7.1 Theorem : If points z_1, z_2, z_3, z_4 are concyclic then (z_1, z_2, z_3, z_4) is real.

1.7.2 Theorem : If points z_1, z_2, z_3, z_4 are collinear then (z_1, z_2, z_3, z_4) is real.

1.7.3 Theorem : If (z_1, z_2, z_3, z_4) is real then z_1, z_2, z_3, z_4 are concyclic or collinear.

1.8 Joint family of straight lines and circles.

1.9 Family of circles

(a) $\left|\frac{z-\alpha}{z-\beta}\right|$ **= constant.**

(b) arg $\left(\frac{z-\alpha}{z-\beta}\right)$ **= constant.**

1.10 (a) Reflection point w. r. t. a line.

(b) Inverse point w. r. t. a circle.

1.11 Steriographic projection.

Unit – 2 : <u>ANALYTIC FUNCTIONS</u> 13 lectures

2.1 Limits and continuity of a function of a complex variable.

2.2 Differentiable function of a complex variable.

2.3 Rules of differentiation of a complex variable.

2.4 Necessary and sufficient condition for f(z) = u + iv to be analytic.

2.5 Polar form of Cauchy – Riemann Partial differential equations.

2.6 Examples on 2.4

2.7 Harmonic function, conjugate harmonic function.

2.8 Construction of analytic function f(z) = u + iv, when any one of u, v is known by –

(a) determination of conjugate function.

(b) Milne-Thomson Method.

2.9 Examples on 2.7 and 2.8.

Unit – 3 : COMPLEX INTEGRATION 12 lectures

3.1 Jordan curve, Orientation of Jordan curve.

3.2 Simply connected and multiply connected domains.

3.3 Rectifiable curves and their properties.

3.4 Integral along an oriented curve.

3.5 Examples on 3.4

3.6 Properties : (a) $\int_C [f(z)+g(x)]\,dz = \int_C f(z)dz + \int_C g(z)\,dz.$

(b) C has two parts C_1, C_2 then

$$\int_C f(z)dz = \int_{C_1} f(z)dz + \int_{C_2} f(z)\,dz.$$

(c) $\int_C f(z)\,dz = -\int_{-C} f(z)\,dz.$

3.7 Examples on evaluation of $\int_C f(z)dz$ **by reducing to real integrals , using result – " If C is defined by z = f(z) is continuously differentiable and f(z) is continuous over C, then the curve C is rectifiable and** $\int_C f(z)dz = \int_a^b [u(t) + i\,v(t)]\,dt$ **(without proof)".**

3.8 Cauchy′s (Integral) Theorem (Original form, using Green′s Theorem).

3.9 Extension of Cauchy′s Theorem to multiply connected regions.

3.10 Examples on 3.8, 3.9.

3.11 Cauchy′s Integral Formula for f(a).

3.12 Cauchy′s Integral Formula for f(a) for multiply connected regions.

3.13 Higher order derivative of analytic function $f^{(n)}(a) = \frac{n!}{2\pi i}\int_C \frac{f(z)dz}{(z-a)^{n+1}}.$

3.14 Examples on 3.11 to 3.13.

Unit – 4 : SINGULAR POINT AND CALCULUS OF RESIDUES 8 lectures

4.1 Zero′s, Poles, Types of Singularities.

4.2 Residues at an isolated singularity.

4.3 Examples on 4.1, 4.2.

4.4 Residue at infinity.

4.5 Computation of residues examples.

4.6 Cauchy′s residue theorem.

4.7 Examples on Cauchy′s residue theorem.

Section – II
(Integral Transform)

Unit – 5 : FOURIER TRANSFORM 20 lectures

A) Infinite Fourier Transform

5.1 Definitions : 1) The Fourier sine transform of f(x).

2) The Fourier cosine transform of f(x).

3) Fourier transform of f(x).

5.2 Examples.

5.3 Theorems :

1) Linearity property.

2) Change of scale property.

3) Infinite Fourier transform.

4) Fourier cosine transform.

5) Fourier sine transform.

6) Shifting property.

7) Modulating theorem.

8) Derivative theorem and its extension.

9) Convolution theorem.

5.4 Problems related to Integral equations.

5.5 Problems related to Fourier transform.

B) Finite Fourier Transform

5.6 Definitions : 1) The finite Fourier sine transform of f(x).

2) The finite Fourier cosine transform of f(x).

5.7 Theorems :

1) Fourier Integral.

2) Different forms of Fourier Integral formula.

3) Parseval′s Identity for Fourier series.

4) Parseval′s Identity for Fourier transform.

5.8 Problems related to Fourier Integral.

Unit – 6 : HANKEL TRANSFORM 15 lectures

A) Infinite Hankel Transform

6.1 Definition : The Infinite Hankel Transform.

6.2 Some important rules in Bessel functions.

6.3 Theorems :

1) Linearity property.

2) Change of scale property.

3) Hankel transform of derivative of a function.

4) Infinite Hankel transform of $\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} - \frac{n^2}{x^2} f$ **.**

5) Parseval′s theorem.

6.4 Problems.

B) Finite Hankel Transform

6.5 Definition : Finite Hankel Transform.

6.6 Theorems :

6.6.1 Finite Hankel transform of $\frac{\partial f}{\partial x}$ **.**

6.6.2 Hankel transform of $\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x}$ **.**

6.6.3 Hankel transform of $\frac{\partial^2 f}{\partial x^2} + \frac{1}{x}\frac{\partial f}{\partial x} - \frac{n^2}{x^2} f$ **.**

6.7 Problems.

Unit – 7 : APPLICATIONS OF FOURIER TRANSFORM TO BOUNDARY VALUE PROBLEM 5 lectures

7.1 Application of Infinite Fourier transform to boundary value problem.

7.2 Application of Finite Fourier transform to boundary value problem.

Unit – 8 : APPLICATIONS OF LAPLACE TRANSFORM 5 lectures

8.1 Applications of Partial differential equation.

8.2 Applications of Integral equation.

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Discrete Mathematics - Paper VIII (A)(optional)

Section I (Sets and Graphs)

Unit – 1 : <u>SETS AND FORMAL LANGUAGES</u> 12 lectures

1.1 Sets: Concept of a set, the empty set, the universal set, subset and equality relations between two sets, union, intersection, difference and symmetric difference of sets, Power set of a set, the Venn diagrams of sets.

1.2 Finite and infinite sets: The successor of a set, the natural numbers as the successors of the empty set Φ, definitions of finite, countably infinite and uncountably infinite sets.

1.2.1 **Theorem:** The set of all integers is countably infinite.

1.2.2 Theorem: The union of countably infinite number of countably infinite sets is countably infinite set.

1.2.3 Theorem: The set of real numbers between 0 & 1 is not a countably infinite set.

1.3 Principles of Mathematical inductions: The statements of 1^{st} and 2^{nd} principles of induction & examples.

1.4 Principle of inclusion & exclusion: Concept of cardinality of a finite set A i.e. |A|, cardinalities of union, intersection, difference & symmetric difference of finite sets.

1.4.1 T**heorem:** For finite sets A_1 and A_2, prove that

$$|A_1 \cup A_2| = |A_1| + |A_2| - |A_1 \cap A_2| .$$

1.4.2 **Theorem:** For finite sets A_1, A_2, and A_3, Prove that:

$$|A_1 \cup A_2 \cup A_3| = |A_1| + |A_2| + |A_3| - |A_1 \cap A_2| - |A_1 \cap A_3| - |A_2 \cap A_3| + + |A_1 \cap A_2 \cap A_3| .$$

1.4.3 Theorem: (Statement only): General principle of inclusion and exclusion for finite sets A_1, A_2, ..., Ar.

1.4.4 **Examples on 1.4.**

1.5 **Multi-sets:** Concept of a multi-set, union, intersection, difference and symmetric difference of multi-sets.

1.6 Propositions: Propositions, tautology, contradiction, atomic & compound propositions, conjunction, disjunction, negation, conditional, bi-conditional propositions, Equivalent propositions, examples on equivalent propositions using truth tables.

1.7 Ordered sets & Languages: An ordered pair & an ordered n – tuple of objects, sentence, language, union, intersection, difference and symmetric difference of languages.

1.8 **Phrase structure grammars:** Concept, definition & simple examples, Types of grammars & languages.

Unit – 2 : **Permutations, Combinations, Discrete Probabilities & Relations**

10 lectures

2.1 Experiments & outcomes: Definitions, the statements of the rules of product & sum for the outcomes of experiments.

2.2 Permutations: Definitions & examples.

2.3 Combinations: Definitions & examples.

2.4 Discrete probability: Definitions & examples.

2.5 Relations: a binary relation from a set A to a set B, a binary relation on a set A, tabular & graphical representation of a binary relation, union, intersection, difference and symmetric difference of relations, an n – array relation among the sets $A_1, A_2, \ldots, An$ i.e. a table on $A_1, A_2, \ldots, An$, Join & projection of tables on the sets $A_1, A_2, \ldots, An$.

2.6 Types of binary relations: The reflexive, symmetric, transitive, anti-symmetric relations on a set, the transitive extension & the transitive closure of a relation.

2.7 Equivalence relations & partitions: Definitions of equivalence relation and partition on a set, the product & sum of partitions, refinement of a partition.

2.7.1 **Theorem:** Every equivalence relation R on a set A corresponds to a partition P of A and conversely every partition corresponds to an equivalence relation.

2.8 Partial ordering relations: Definitions & simple examples of: partially ordered set, the Hasse diagram of a POSET, the Cartesian product of two POSET's, a maximal element, a minimal element, a lower bound, an upper bound, a greatest lower bound and a least upper bound of a POSET, a lattice as POSET.

2.9 Chains & anti-chains: Definitions and simple examples.

2.9.1 **Theorem:** If the length of the longest chains in a POSET $(P, \leq)$ is n then the elements of the set P can be partitioned into n disjoint anti-chains.

2.9.2 Theorem: If a POSET (P, $\leq$) consists of m.n + 1 elements then either there is an anti-chain containing (m + 1) elements or there is a chain of length n + 1 in the POSET (P, $\leq$).

2.10 Functions: a function, an injection, a surjection, a bijection, the statement of the pigeonhole principle and application to simple problems.

Unit – 3 : GRAPHS 12 lectures

3.1 Basic Terminology: Definitions & simple examples of: Directed and undirected graphs, isomorphism of two graphs, a sub – graph and the complement of a graph, the spanning sub-graph, directed & undirected complete graphs, multi – graphs & weighted graphs, a finite-state model of simple situations, the linear graphs, paths & circuits, connected graphs.

3.1.1 **Theorem:** In a graph with n vertices if there is a path from vertex v_1 to v_2 then there is a path of no more than n – 1 edges from v_1 to v_2.

3.2 Shortest path in a weighted graph : Definitions.

3.2.1 Algorithm: (E.W.Dijkstra's algorithm): To find shortest path in a weighted graph & examples.

3.3 Eulerian paths & circuits: Eulerian path & circuit, the degree of a vertex in an undirected graph, the statement of the hand-shaking lemma, the incoming & the outgoing degree of a vertex in a directed graph, the statement of the Hand-shaking dilemma.

3.3.1 Theorem: If an undirected graph possesses an Eulerian path then it is connected and has either no vertex of odd degree or exactly two vertices of odd degree.

3.3.2 Theorem: If an undirected connected graph has exactly two vertices of odd degree then it possesses an Eulerian path.

3.3.3 Theorem: (statement only): An undirected graph possesses an Eulerian circuit if and only if it is connected and its vertices are all of even degree.

3.4 Hamiltonian paths & circuits: Definitions.

3.4.1 Theorem: If a linear undirected graph G has n vertices and if the sum of the degrees for each pair of vertices in G is greater than or equal to n-1 then G is connected.

3.4.2 Theorem: (Statement only): If a linear undirected graph G has n vertices and if the sum of the degrees for each pair of vertices in G is greater than or equal to n –1 then there exists a Hamiltonian path in G.

3.4.3 Theorem: (Statement only): There is always a Hamiltonian path in a directed complete graph.

3.5 The traveling salesperson problem (TSP): Definitions.

3.5.1 Algorithm: The nearest neighbor method to solve the given TSP which gives reasonably good answer to the TSP & examples.

3.6 Planar graphs: a planar graph, finite & infinite regions.

3.6.1 Theorem: (Euler's formula): For any connected planar graph, prove: $v - e + r = 2$, where v, e, and r are the number of vertices, edges, and regions of the graph respectively.

3.6.2 Theorem: Prove: $e \leq 3v - 6$, in any connected linear planar graph that has no loops and has two or more edges.

3.6.3 Theorem: Prove that: $e \leq 2v - 4$, in a planar graph in which every region would be bounded by four or more edges. 4) Examples: The complete graphs K_5 & $K_{3,3}$ are not planar.

Unit – 4 : <u>TREES</u> 11 lectures

4.1 Trees: Definitions & simple examples.

4.1.1 **Theorem:** (Properties of trees):

i)There is a unique path between every two vertices in a tree.

ii) The number of vertices is one more than the number of edges in a tree.

iii) A tree with two or more vertices has at least two leaves.

4.1.2 Theorem: (Characterizations of trees): A graph in which there is a unique path between every pair of vertices is a tree.

4.1.3 A connected graph in which the number of vertices is one more than the number of edges is a tree.

4.1.4 A graph without circuit in which the number of vertices is one more than the number of edges is a tree.

4.2 Rooted trees: a directed tree, a rooted tree, an ordered rooted tree, isomorphism of ordered rooted trees, binary trees, regular trees, the path length of a vertex & the height of a tree, prefix codes, a binary tree for the weights w_1, w_2,…, w_n & the weight of it, an optimal tree, pre-fix & post-fix notations for an algebraic expression.

4.3 **Applications:**

4.3.1 Representation of the algebraic expressions as ordered binary trees, use them to write expressions in pre – fix & post – fix notations.

4.3.2 The relationship between i, the number of branch nodes and t, the number of leaves of a regular binary trees.

4.3.3 The bounds of the number of leaves of an m-array tree of height h.

4.3.4 The D. A. Huffman procedure for construction of an optimal tree for a given set of weights.

4.4 Spanning trees & cut – sets: a spanning tree of a connected graph, a cut-set of a connected graph, the fundamental circuit & cut–set of a connected graph relative to the spanning tree of the graph.

4.5 Properties of circuits & cut–sets

4.4.1 Theorem: A circuit and the complement of any spanning tree must have at least one edge in common.

4.4.2 Theorem: A cut–set and any spanning tree must have at least one edge in common.

4.4.3 Theorem: Every circuit has an even number of edges in common with every cut–set.

4.5 Minimum spanning trees: Definitions.

4.5.1 Algorithm: To determine a minimum spanning tree, MST, of a connected weighted graph & examples.

4.6 Transport networks: Definitions.

4.6.1 Theorem: The value of any flow in a given transport network is less than or equal to the capacity of any cut in the network.

4.6.2 Algorithm: (Labeling procedure): To find a maximum flow in a transport network & examples.

Section – II (Algorithms and Boolean Algebra)

Unit – 5 : **Finite state machines & algorithms** 11 lectures

5.1 Definitions & simple examples: Information processing machine with & without memory, finite state machine, tabular & graphical description of finite state machine.

5.2 Examples of finite state machines: Modeling of Physical systems.

5.3 Finite state machines as language recognizer: Use of FSM as language recognizer, Definition of finite state language.

5.4 Examples: Finite state languages & languages that are not finite state languages.

5.5 Finite state languages & type-3 languages: Definition of a nondeterministic finite state machine.

5.6 Examples:

5.6.1 Construction of a deterministic finite state machine from the given nondeterministic finite state machine that accepts exactly the same language.

5.6.2 Construction of a nondeterministic finite state machine that accepts the type-3 language specified by the grammar.

5.6.3 Construction of a grammar that specifies the language accepted by the finite state machine.

5.7 Analysis of algorithms: an algorithm for a problem, the time complexity of the algorithm, & the time complexity of the problem.

5.7.1 **Algorithms & complexities:** Discussion of complexities of :

i) An algorithm for finding the largest of n numbers.

ii) An algorithm for finding the smallest of n numbers.

iii) The BUBBLE-SORT algorithm for sorting n numbers.

5.8 Tractable & Intractable problems: Definitions: efficient algorithms, inefficient algorithms, tractable & intractable problems.

Unit – 6 : **Numeric and generating functions** 10 lectures

6.1 Manipulation of Numeric functions: Definitions & simple examples : Numeric function, the sum, product & the convolution of two numeric functions, a scaled version & the accumulated sum of a numeric function, the forward & backward difference of a numeric function, the numeric functions $S^i\mathbf{a}$ & $S^{-i}\mathbf{a}$ for any positive integer i, where **a** is any numeric function.

6.2 Generating functions : Definition.

6.2.1 Theorem: Let A (z) & B (z) be the generating functions of the numeric functions **a** and **b** respectively. Then i) the generating function of **a** + **b** is A (z) + B (z).

ii) the generating function of the convolution **a*b** is A (z) B (z).

iii) the generating function of the scaled version α**a** is αA (z).

iv) the generating function of S^i**a** is $z^i A(z)$.

v) the generating function of S^{-i} **a** is z^{-i} $[A(z) - a_0 - a_1 z - a_2 z^2 - \ldots$ $. - a_{i-1} z^{i-1}]$.

vi) the generating function of the forward difference of **a** is

$[A(z) - a_0 - zA(z)]/z$.

vii) the generating function of the backward difference of **a** is

$A(z) - zA(z)$.

6.2.2 Examples: Determination of numeric functions from the given generating functions.

6.3 Combinatorial problems: Definition.

6.3.1 The proof of the binomial theorem:

$(1+z)^n = C(n, 0) + C(n, 1)z + C(n, 2)z^2 + C(n, 3)z^3 + \ldots + C(n, n)z^n$.

By using i) the concept of generating function.

ii) the combinatorial arguments.

6.3.2 The proof of the relation: $C(n, r) = C(n-1, r) + C(n-1, r-1)$.

By using i) the algebraic manipulation,

ii) the combinatorial arguments

and iii) the generating functions.

6.4 **Solutions of combinatorial problems by combinatorial arguments.**

Unit – 7 : **Recurrence relations & recursive algorithm** 11 lectures

7.1 Recurrence relations: Definitions & simple examples: recurrence relations or difference equations, solution of a recurrence relation, linear recurrence relations with constant coefficients.

7.2 Homogeneous solutions: Definition.

7.2.1 Algorithm: To find a homogeneous solution of a linear difference equation with constant coefficients & examples.

7.3 Particular solutions: Definition.

7.3.1 Method of inspection: To find the particular solution of a linear difference equation with constant coefficients & examples.

7.4 Total solutions: Definition.

7.4.1 Theorem: (statement only): The necessary & sufficient conditions for the existence of unique total solution to a k^{th} order linear difference equation.

7.4.2 **Examples:** Finding total solutions for the difference equations.

7.5 Solution by the method of generating functions: Method & examples.

7.6 Sorting algorithms: Determination of complexities of the following sorting algorithms for sorting n given numbers by using recurrence relations : i) The BUBBLESORT algorithm.

and ii) The Bose-Nelson algorithm.

Unit – 8 : **Boolean algebra** 13 lectures

8.1 Algebraic systems: Definitions & simple examples: a binary, a ternary, an m-array operation on a set A, an algebraic system.

8. 2 Lattices & algebraic systems: Definitions & examples.

8. 2.1 Theorem: For any a and b in a lattice (A, ≤), prove that : i) $a \leq a \vee b$, ii) $b \leq a \vee b$, iii) $a \wedge b \leq a$ and iv) $a \wedge b \leq b$.

8.2.2 **Theorem:** For any a, b, c, d in a lattice (A, ≤), if $a \leq b$ and $c \leq d$ then prove that: i) $a \vee c \leq b \vee d$ and ii) $a \wedge c \leq b \wedge d$.

8.3 Principle of duality: The statement of the principle of duality for lattices.

8.4 Basic properties of algebraic systems defined by the lattices:

Let (A, V, Λ) be the algebraic system defined by the lattice (A, ≤).

1) **Commutative laws:** For any two elements a & b in A,

i) $a \vee b = b \vee a$ & ii) $a \wedge b = b \wedge a$.

2) **Associative laws:** For any three elements a, b, & c in A,

i) $(a \vee b) \vee c = a \vee (b \vee c)$ & ii) $(a \wedge b) \wedge c = a \wedge (b \wedge c)$.

3) **Idempotent laws:** For any element a in A, i) $a \vee a = a$ & ii) $a \wedge a = a$.

4) **Absorption laws:** For any two elements a & b in A,

i) $a \vee (a \wedge b) = a$ & ii) $a \wedge (a \vee b) = a$.

8.5 Distributive & complemented lattices: a distributive lattice, a universal lower & upper bounds of a lattice, a complement of an element, a complemented lattice.

8.5.1 Theorem: Let (A,≤) be a lattice with universal upper & lower bounds 1 & 0 respectively. Let a be any element of A. Then: $a \vee 1 = 1$, $a \wedge 1 = a$, $a \vee 0 = a$, and $a \wedge 0 = 0$.

8.5.2 **Examples of lattices:** i) a lattice which is distributive but not complemented.

ii) a lattice which is complemented but not distributive.

iii) a lattice which is neither complemented nor distributive.

iv) a lattice which is both complemented and distributive.

8.5.3 **Theorem:** In a distributive lattice, if an element has a complement then this complement is unique.

8.6 Boolean lattices & Boolean algebras: Definitions.

8.6.1 Theorem: (De-Morgan's laws): For any a & b in a Boolean algebra,

i) $\overline{a V b} = \bar{a} \Lambda \bar{b}$ & ii) $\overline{a \Lambda b} = \bar{a}$ V $\bar{b}$.

8.7 Uniqueness of finite Boolean algebras: Definitions & simple examples: a cover of an element, an atom in a Boolean algebra.

8.7.1 **Theorem:** In a finite Boolean algebra A,

i) For any nonzero element b there exists at least one atom a, such that $a \leq b$.

ii) If b & c are any two elements of A & if b Λ $\bar{c}$ = 0 then $b \leq c$.

iii) If b is a nonzero element and $a_1, a_2, \ldots, a_k$ be all the atoms of A such that $a_i \leq b$ then $b = a_1$ V a_2 V . . . V a_k.

iv) If b is a nonzero element and $a_1, a_2, \ldots a_k$ be all the atoms of A such that $a_i \leq b$ then $b = a_1 V a_2 V \ldots V a_k$ is the unique way to represent b as a join of atoms.

8.7.2 Theorem: (statement only): A finite Boolean algebra A has exactly 2^n elements for some integer $n > 0$. Moreover, there is a unique Boolean algebra of 2^n elements for every $n > 0$.

8.8 Boolean functions & Boolean expressions: a Boolean expression over a Boolean algebra, the value of a Boolean expression, equivalent Boolean expressions, a Boolean function, the two-valued Boolean algebra, a minterm expression, a disjunctive normal form of an expression, a maxterm expression, a conjunctive normal form of an expression.

8.8.1 **Algorithms:** Let A be a two-valued Boolean algebra.

i) To obtain a Boolean expression in disjunctive normal form for a function from A^n to A.

ii) To obtain a Boolean expression in conjunctive normal form for a function from A^n to A.

8.8.2 **Examples:** CNF & DNF (two-valued Boolean algebras).

8.8.3 **Two-valued Boolean algebras:**

i) Boolean algebra of propositions,

ii) Boolean algebra of logic gates &

iii) Boolean algebra of switching circuits.

REFERENCE BOOKS

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Special Theory of Relativity - Paper VIII (B)(Optional)

Section - I

Unit – 1 : <u>CLASSICAL THEORY OF RELATIVITY</u> 10 lectures

1.1 Review of Newtonian Mechanics.

1.1.1 Inertial system.

1.1.2 Event.

1.2 Galilean Transformations.

1.2.1 Newtonian Relativity.

1.2.2 Conservation laws in Newtonian Mechanics.

1.2.3 Ether.

1.2.4 Maxwell′s electromagnetic theory.

1.3 The Michelson – Morley experiment.

1.3.1 Fitzgerald and Lorentz Contraction hypothesis.

Unit – 2 : <u>LORENTZ TRANSFORMATIONS</u> 13 lectures

2.1 Einstein′s Special Relativity Theory.

2.1.1 Einstein′s principle of relativity.

2.1.2 Principle of constancy of light speed.

2.2 Lorentz Transformations.

2.3 Consequences of Lorentz Transformation

2.3.1 Lorentz – Fitzgearld length contraction

2.3.2 Time dilation

2.3.3 Clock paradox or twin paradox.

2.3.4 Simultaneity

2.4 Geometrical Interpretation of LT.

2.5 Group property of Lorentz Transformations and examples

Unit – 3 : <u>RELATIVISTIC KINEMATICS</u> 7 lectures

3.1 Introduction.

3.2 Transformation of particle velocity.

3.3 Relativistic addition law for velocities.

3.4 Transformation of the Lorentz contraction factor $\left(1 - {v^2}/{c^2}\right)^{\frac{1}{2}}$ **.**

3.5 The Transformations for the acceleration of a particle.

Unit – 4 : RELATIVISTIC MECHANICS 15 lectures

4.1 Introduction (Mass and Momentum).

4. 2 The mass of a moving particle m = $\frac{m_0}{\sqrt{\left(1 - {u^2}/{c^2}\right)}}$

4.2.1 Relativistic expression for Force.

4.2.2 Transverse and Longitudinal mass of the particle.

4.3 Mass energy equivalence E = mc^2.

4.4 Transformation equations for mass.

4.5 Transformation equations for momentum and energy.

4.5.1 Deduction to prove that p^2 – $\frac{E^2}{c^2}$ **is Lorentz invariant.**

4.6 Minkowski Space (Four Dimensional Continuum).

4.6.1 Time-like, Space- like, Light – like (null) intervals.

4.6.2 Events occurring at the same point and the same time.

4.6.3 Theorem : There exists an inertial system S′ in which the two events occur at one and the same point if the interval between two events is timelike.

4.6.4 Corollary : Two events which are separated by a timelike interval cannot occur simultaneously in any inertial system.

4.6.5 Theorem : There exists an inertial system S′ in which the two events occur at one and the same time if the interval between two events is spacelike.

4.6.6 World points and World lines.

4.6.7 Lorentz transformations in index form.

4.7 Past, Present, Future – Null Cone.

4.7.1 Proper time.

Section - II

Unit – 5 : Tensors 10 lectures

5.1 Introduction.

5. 2 Space Vn.

5.1.2 Einstein summation convention.

5.1.3 Definition : Dummy suffix , Free suffix (Real suffix)

5.1.4 Definition : Kronecker delta.

5. 3 Transformation of co-ordinates.

5.3.1 Scalar (Invariant) , Vector.

5.3.2 Definition : Contravariant vector, Covariant vector.

5.3.3 Rank (order) of tensor.

5.3.4 Tensors of higher order : (a) Contravariant tensor of rank r, (b) Covariant tensor of order r, (c) Mixed tensor of order (r + s).

5.3.5 Number of components of a tensor..

5.4 Definition : Symmetric tensor, Skew – symmetric tensor .

5.4.1 Number of distinct components of symmetric tensor and skew - symmetric tensor..

5.4.2 Results : (I) Symmetric property remains unchanged by tensor law of transformation.

(II) Anti – symmetric property remains unchanged by tensor law of transformation.

Unit – 6 : TENSOR ALGEBRA 10 lectures

6.1 Addition of tensors

6.1.1 Theorem : The sum (or difference) of two tensors is a tensor of the same rank and similar character.

6.2 Contraction

6.2.1 Property :Contraction reduces the rank of a tensor by two.

6.3 Product (Multiplication)of tensors

6.3.1 Outer multiplication (Definition)

6.3.2 Inner multiplication (Definition)

6.3.3 Theorem1 : The outer product (open product) of two tensors is a tensor.

6.4 Quotient law of tensors (Definition)

6.4.1 Theorem : A set of quantities, whose inner product with an arbitrary vector is a tensor, is itself a tensor.

6.5 Definition of Reciprocal Symmetric tensor (Conjugate Tensor)

6.6 Definition of Relative Tensor

6.7 Definitions : Riemannian Metric, Fundamental Tensor, Associate Tensors, Raising and Lowering of suffixes

Unit – 7 : TENSOR CALCULUS 10 lectures

7.1 Definition : Christoffel Symbols of 1st kind and 2nd kind

7.1.1 Theorem : To Prove that i) $\Gamma ij,k + \Gamma jk,i = \frac{\partial g_{ik}}{\partial x^j}$

ii) $\Gamma^i_{ij} = \frac{\partial}{\partial x^j} \log \sqrt{(-g)}$

iii) $\Gamma^i_{ij} = \frac{\partial}{\partial x^j} \log \sqrt{g}$

iv) $\frac{\partial g^{ij}}{\partial x^k} = -g^{ij}\Gamma^j_{lk} - g^{lj}\Gamma^j_{lk}$

7.1.2 Transformation law for Christoffel symbols : Theorem : Prove that Christoffel symbols are not tensors.

7.2 Definition : Covarant derivative of a covariant vector and cotravariant vector

7.2.1 Theorem : Covarant derivative of a covariant vector is a tensor of rank 2.

7.2.2 Theorem :Covarant derivative of a cotravariant vector is a tensor.

7.2.3 Covariant differential of tensors

7.2.4 Theorem : Covariant differentiations of tensor is a tensor.

Unit – 8 : RELATIVITY AND ELECTROMAGNETISM 15 lectures

8.1 Introduction.

8.1.1 Maxwell′s equations of electromagnetic theory in vaccum.

8.1.2 Propagation of electric and magnetic field strengths.

8.1.3 Scalar and Vector potential.

8.1.4 Four potential.

8.1.5 Transformations of the electromagnetic four potential vector.

8.2 Transformations of the charge density and current density.

8.2.1 Current four vector.

8.3 Gauge transformations.

8.4 Four dimensional formulation of the theory.

8.4.1 The electromagnetic field tensor.

8.4.2 Maxwell's equations in tensor form.

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Differential Geometry – Paper VIII (C)(Optional)

Section I (Curves and Surfaces)

Unit – 1 : <u>SPACE CURVES</u> 12 lectures

1.1 Parametric equations for space curves.

1.2 Vector representation of a curve.

1.3 Function of class r.

1.4 Path.

1.4.1 Equivalent paths.

1.4.2 Change of parameters.

1.5 Arc Length.

1.5.1 Arc Length of curve between two points.

1.5.2 Cartesian form.

1.5.3 Examples.

1.6 Tangent lines.

1.6.1 Equation of tangent line to a curve at a point.

1.6.2 Examples.

1.7 The osculating plane.

1.7.1 Definition 1, Definition 2.

1.7.2 Find the equation of osculating plane using definition 1 and definition 2.

1.8 The tangent plane at any point of the surface f(x,y,z) = 0.

1.8.1 Normal plane.

1.8.2 Normal plane is perpendicular to the osculating plane.

1.8.3 The osculating plane at a point of space curve given by the intersection of surfaces f(r) = 0; ψ(r) = 0.

1.8.4 Examples.

1.9 The principal Normal and Binomial (Definitions).

1.9.1 The direction of principal normal and binomial.

1.9.2 The unit vectors t, n, b.

1.9.3 Equations principal normal and binomial.

Unit – 2 : <u>CURVETURE</u> 13 lectures

2.1 Definition (Curveture and Torsion).

2.2 Screw Curveture (Definition).

2.3 Serret Frenet Formula.

2.4 Serret Frenet Formula (in cartesion form).

2.5 Examples.

2.6 Find curvatures and torsion of a curves.

2.6.1 When the equation of curve is r = f(u).

2.6.2 When the equation of curve is r = f(S).

2.7 Examples (Curveture and Torsion).

2.8 Helices (Definition and properties).

2.9 Involute and Evolute (Definitions).

2.9.1 Involute of space curve.

2.9.2 Find curvature κ_1 and torsion τ_1 of the involute.

2.9.3 Evolute of space curve.

2.9.4 find curvature and torsion of the evolute.

Unit – 3 : FUNDAMENTAL FORMS 10 lectures

3.1 First fundamental form or metric.

3.2 Geometrical interpretation of metric.

3.3 Properties I and II.

3.4 Second fundamental form.

3.5 Geometrical Interpretation.

3.6 Examples.

Unit – 4 : DERIVATIVE OF N 10 lectures

4.1 Derivative of N (The surface normal); Weingarton Equations.

4.2 Examples.

4.3 Angle between two parametric curves.

4.3.1 Necessary and Sufficient condition for the parametric curves for the surface to be orthogonal.

4.4 Orthogonal Trajectories (Definition).

Section – II (Curvature and Geodesics

Unit – 5 : LOCAL NON-INTRINSIC PROPERTIES OF A SURFACE CURVE ON A SURFACE 12 lectures

5.1 Curvature of Normal section.

5.2 Formula for curvature of normal section in terms of fundamental magnitudes.

5.3 Definitions of normal curvature and show that these definitions are equivalent.

5.4 Meusnier′s theorem.

5.5 Another Meusneir′s theorem.

5.6 Examples.

5.7 Principal curvature (Definition).

5.8 The equation giving principal curvatures.

5.9 Differential equation of principal directions.

5.10 Mean curvature or Mean Normal curvature. First curvature, Gaussian curvature, Minimal surface.

Unit – 6 : LINES OF CURVATURE 10 lectures

6.1 Definition 1 , Definition 2.

6.1.1 Differential equation of lines of curvature.

6.1.2 An important property of lines of curvature.

6.1.3 The differential equation of lines of curvature through a point on the surface z = f(x, y).

6.1.4 Lines of curvature as parametric curves (Theorem).

6.2 General surface of revolution.

6.2.1 Parametric curves and surface of revolution.

6.2.2 Lines of curvature on a surface of revolution.

6.2.3 Principal curvatures on surface of revolution.

6.2.4 Surface of revolution having constant negative Gaussian curvature.

6.3 Examples.

6.4 The fundamental equations of surfaces theory.

6.5 Gaussian formulae (Theorem).

6.6 Examples.

6.7 Gauss characteristics equation.

6.8 Mainardi – Codazzi equations.

6.9 Examples.

Unit – 7 : GEODESICS AND MAPPING OF SURFACES – I 13 lectures

7.1 Geodesics.

7.2 Differential equation of Geodesics.

7.3 Necessary and Sufficient condition that the curve v = c be a geodesic.

7.4 Canonical geodesic equation.

7.5 Examples.

7.6 Normal property of Geodesics.

7.7 Differential equation of Geodesic via normal property.

7.8 Geodesics on a surface of Revolution.

7.9 Examples.

7.10 Clairaut′s theorem.

7.11 Examples.

7.12 Geodesic curvature.

7.13 Formulae for k_9.

7.14 Liouville′s formula for k_9.

7.15 Examples.

Unit – 8 : GEODESICS AND MAPPING OF SURFACES – II 10 lectures

8.1 Gauss – Bonnet theorem.

8.2 Examples.

8.3 Torsion of a Geodesic.

8.4 Examples.

8.5 Bonnet′s theorem in relation to geodesic.

8.6 Geodesic Parallels.

8.7 Geodesic polars.

8.8 Mapping of surface.

8.9 Isometric lines and isometric correspondence.

8.10 Examples.

8.11 Minding theorem.

8.12 Conformal mapping

8.13 Examples.

REFERENCE BOOKS

1. **Differential Geometry,** Miattal and Agarwal, Krishna Prakashan Media [P] Ltd. 27th edition (1999), 11, Shivaji Road, Meerut – 1 (U.P.)

2. **Introduction to Differential Geometry,** J. A. Thorpe, Springer Verlag.

3. **Lecture notes on elementary Topology and Geometry,** I. M. Singer and J. A. Thorpe, Springer Verlag 1967.

4. **Elementary Differential Geometry,** B. O. Neill, Academic Press, 1966.

5. **Lectures on Differential Geometry of Curves and Surfaces,** S. Sternberg, Printice – hall 1976.

6. **Differential and Riemannian Geometry,** D. Laugwitz, Academic Press, 1965.

7. **Elements of Differential Geometry,** R. S. Millman, and G. D. Parker, Springer Verlag.

8. **An Introduction to Differential and Riemannian Geometry,** T. J. Willmor, Oxford University Press 1965.

Mathematical Modelling -Paper VIII (D)(Optional)

Section I

Unit – 1 : MATHEMATICAL MODELLING : NEED, TECHNIQUES, CLASSIFICATION AND SIMPLE ILLUSTRATION 10 lectures

1.1 Simple situations requiring Mathematical Modelling.

1.2 The technique of Mathematical Modelling.

1.3 Classification of Mathematical Models.

1.4 Some characteristics of Mathematical Models.

1.5 Mathematical Modelling through Geometry.

1.6 Mathematical Modelling through Algebra.

1.7 Mathematical Modelling through Trignometry.

1.8 Mathematical Modelling through Calculus.

1.9 Limitations of Mathematical Modelling .

Unit – 2 : MATHEMATICAL MODELLING THROUGH ORDINARY DIFFERENTIAL EQUATIONS OF FIRST ORDER 12 lectures

2.1 Mathematical Modelling through differential equations.

2.2 Linear Growth and Decay Models.

2.3 Non-linear Growth and Decay Models.

2.4 Compartment Models.

2.5 Mathematical Modelling in Dynamics through Ordinary differential equations of First Order.

Unit – 3 : MATHEMATICAL MODELLING THROUGH SYSTEMS OF ORDINARY DIFFERENTIAL EQUATIONS OF FIRST ORDER

12 lectures

3.1 Mathematical Modelling in Population Dynamics.

3.2 Mathematical Modelling of Epidemics through System of Ordinary differential equations of First order.

3.3 Compartment Models through Systems of Ordinary Differential Equations.

3.4 Mathematical Modelling in Economics through System of Ordinary differential equations of First order.

3.5 Mathematical Models in Medicine, Arms Race Battles and Internationals Trades in terms of System of Ordinary Differential Equations.

Unit – 4 : MATHEMATICAL MODELLING THROUGH ORDINARY DIFFERENTIAL EQUATIONS OF SECOND ORDER 11 lectures

4.1 Mathematical Modelling of Planetary Motions.

4.2 Mathematical Modelling of Circular Motion and Motion of Satellites.

4.3 Mathematical Modelling through Linear Differentials Equations of Second Order.

4.4 Miscellaneous Mathematical Models through Ordinary Differential Equations of the Second Order.

Section II

Unit – 5 : MATHEMATICAL MODELLING THROUGH DIFFERENCE EQUATIONS 12 lectures

5.1 The need for Mathematical Modelling through Difference Equations : Some Simple Models.

5.2 Basic Theory of Linear Difference Equations with Constant Coefficients.

5.3 Mathematical Modelling through Difference Equation in Economics .

5.4 Mathematical Modelling through Difference Equations in Population Dynamics.

Unit–6:MATHEMATICAL MODELLIG THROUGH GRAPHS 12 lectures

6.1 Situation that can be modelled through Graphs.

6.2 Mathematical Models in terms of Directed Graphs.

6.3 Mathematical Models in terms of Signed Graphs.

6.4 Mathematical Models in terms of Weighted Diagraphs.

Unit – 7 : **LAPLACE TRANSFORMS AND THEIR APPLICATIONS TO DIFFERENTIAL EQUATIONS** 11 lectures

7.1 Introduction.

7.2 Properties of Laplace Transform.

7.2.1 Transforms of Derivative.

7.2.2 Transforms of Integrals.

7.3 Unit Step Functions.

7.4 Unit Impulse Functions

7.5 Application of Laplace transforms.

7.5.1 Vibrating Motion.

7.5.2 Vibration of Coupled Systems.

7.5.3 Electric Circuits.

Unit – 8 : <u>MATHEMATICAL MODELLING THROUGH DECAY – DIFFERENTIAL - DIFFERENCE EQUATIONS</u> 10 lectures

8.1 Single Species Population Models.

8.2 Prey-Predator Model.

8.3 Multispecies Model.

8.4 A Model for Growth of Population inhibited by Cumulative Effects of Pollution.

8.5 Prey- Predator Model in terms of Integro – Differential Equations.

8.6 Stability of the Prey – Predator Model.

8.7 Differential – Differences Equations Models in Relation to other Models.

REFERENCE BOOKS

1. **Mathematical Modelling,** J. N. Kapur, New Age International (P) Ltd., Publishers Reprint 2003.
2. **Differential Equations and Their Application ,** Zafar Ahsan ,Prentice Hall of India , Delhi
3. **Mathematical Modelling,** J.G. Andrews and R. R. Mclone (1976). Butterwerths London.
4. **Mathematical Modelling Techniques,** R. Aris (1978) , Pitman.
5. **Differential Equation Models,** Martin Braun, C. S. Coleman, D.A.Drew , Vol. 1.
6. **Political and Related Models,** Steven J. Drams, Kl. F Lucas, P. D. Straffin (Eds), Vol. 1.
7. **Discrete and System Models,** W. F. Lucas, F. S. Roberts, R. M. Thrall, Vol. 3.
8. **Life Science Models,** H. M. Roberts And M. Thompson, Vol. 4.
9. **“ Thinking with Models ” (Mathematical Models in Physical, Biological and Social Sciences),** T. Saaty and J.Alexander Pergamon Press, New York.

PAPER – VIII (E) (OPTIONAL)

Application of Mathematics in Finance and Insurance

Section – I
(Application of Mathematics in Finance)

Unit – 1 : FINANCIAL MANAGEMENT 10 lectures

1.1 An overview.

1.2 Nature and Scope of Financial Management.

1.3 Goals of Financial Management and main decisions of financial management.

1.4 Difference between risk, speculation and gambling.

Unit – 2 : TIME VALUE OF MONEY 20 lectures

2.1 Interest rate and discount rate.

2.2 Present value and future value,

2.3 discrete case as well as continuous compounding case.

2.4 Annuities and its kinds.

2.5 Meaning of return.

2.5.1 Return as Internal rate of Return (IRR).

2.5.2 Numerical Methods like Newton Raphson Method to calculate IRR.

2.5.3 Measurement of returns under uncertainty situations.

2.6 Meaning of risk.

2.6.1 Difference between risk and uncertanity.

2.6.2 Types of risks. Measurements of risk.

2.7 Calculation of security and Portfolio Risk and Return- Markowitz Model.

2.7.1 Sharpe's Single Index Model.

2.7.2 Systematic Risk and Unsystematic Risk.

Unit – 3 : TAYLOR SERIES AND BOND VALUATION 5 lectures

3.1 Calculation of Duration and Convexity of bonds.

Unit – 4 : FINANCIAL DERIVATIVES. 10 lectures

4.1 Futures. Forward.

4.2 Swaps and Options.

4.3 Call and Put Option.

4.3.1 Call and Put Parity Theorem.

4.4 Pricing of contingent claims through Arbitrage and Arbitrage Theorem.

Section – II
(Applications of Mathematics in Insurance)

Unit – 5 : <u>INSURANCE FUNDAMENTALS</u> 20 lectures

5.1 Insurance defined.

5.2 Meaning of loss. Chances of loss, peril, hazard and proximate cause in insurance.

5.3 Costs and benefits of insurance to the society and branches of insurance-life insurance and various types of general insurance.

5.4 Insurable loss exposures features of a loss that is ideal for insurance.

Unit – 6 : <u>LIFE INSURANCE MATHEMATICS</u>. 5 lectures

6.1 Construction of Mortality Tables.

6.2 Computation of Premium of Life Insurance for a fixed duration and for the whole life.

Unit – 7 : <u>DETERMINATION OF CLAIMS FOR GENERAL INSURANCE</u> 10 lectures

7.1 Determination of claims for general insurance using Poisson Distribution.

7.2 Determination of claims for general insurance using and Negative Binomial Distribution.

7.2.1 The Polya Case.

Unit – 8 : <u>DETERMINATION OF THE AMOUNT OF CLAIMS IN GENERAL INSURANCE</u> 10 lectures

8.1 Compound Aggregate claim model and its properties and claims of reinsurance.

8.2 Calculation of a compound claim density function.

8.3 F-recursive and approximate formulae for F.

REFERENCE BOOKS

1. **Corporate Finance -Theory and Practice,** Aswath Damodaran, John Wiley & Sons. Inc.
2. **Options, Futures, and Other Derivatives,** John C. Hull, Prentice - Hall of India Private Limited.
3. **An Introduction to Mathematical Finanace,** Sheldon M . Ross, Cambridge University Press.
4. **Introduction to Risk Management and Insurance,** Mark S. Dorfman,Prentice Hall, Englwood Cliffs, New Jersey

Computational Mathematics Laboratory - IV

(Operations Research Techniques)

Sr.No.	Topic	No. Of Practicals
	Linear Programming :	
1	Simplex Method : Maximization Case	1
2	Simplex Method : Minimization Case	1
3	Two-Phase Method	1
4	Big-M-Method	1
	Transportation Problems :	
5	North- West Corner Method	1
6	Least Cost Method	1
7	Vogel's Approximation Method	1
8	Optimization of T.P. by Modi Method	1
	Assignment Problems:	
9	Hungarian Method	1
10	Maximization Case in Assignment Problem	1
11	Unbalanced Assignment Problems	1
12	Travelling Salesman Problem	1
	Theory of Games:	
13	Games with saddle point	1
14	Games without saddle point : (Algebraic method)	1
15	Games without saddle point : a) Arithmetic Method b) Matrix Method	1
16	Games without saddle point : Graphical method	1

REFERENCE BOOKS

1. Operations Research [Theory and Applications], By J.K.Sharma
   Second edition, 2003, Macmillan India Ltd., New Delhi.
2. Operations Research : S. D. Sharma.

Computational Mathematics Laboratory – V

(Complex Variables and Applications of Differential Equations)

Sr.No.	Topic	No. Of Practicals
	(I) COMPLEX VARIABLES	
1	**Conformal Mapping (I)**	1
2	**Conformal Mapping (II)**	1
3	**Complex Line Integral**	1
4	**Power Series expansion of f(z)**	1
5	**Singularities and Residues of f(z)**	1
6	**Evaluation of the integrals of the form** $\int_0^{2\pi} f(\cos\theta, \sin\theta)\, d\theta$	1
7	**Evaluation of the integrals of the form** $\int_{-\infty}^{\infty} f(x)\, dx$ **where f(x) is a real function of the variable x.**	1
8	**Evaluation of the integrals of the form** $\int_{-\infty}^{\infty} f(x)\sin(mx)dx$ **and** $\int_{-\infty}^{\infty} f(x)\cos(mx)dx$ **, where m > 0 and f(x) is rational function of x.**	1

Sr.No.	Topic	No. Of Practicals
	(II) APPLICATIONS OF DIFFERENTIAL EQUATIONS	
9	**Biological Growth**	**1**
10	**Compound Interest**	**1**
11	**Problem in Epidemiology**	**1**
12	**Mixture Problem**	**1**
13	**Law of Mass Action**	**1**
14	**Motion of Rocket**	**1**
15	**A microeconomic Market Model**	**1**
16	**Arms Race**	**1**

REFERENCE BOOKS

1. **Shanti Narayan and P.K.Mittal , Theory Of Functions Of A Complex Variable, S. Chand & Co. Ltd., New Delhi, 2005.**
2. **Zafar Ashan, Differential Equations And Their Applications, Prentice Hall of India, New Delhi, 1999.**

Computational Mathematics Laboratory – VI
(Numerical Recipes)

Sr.No.	Topic	No. of Practical
1	***C++ Introduction:*** History, Identifiers, Keywords, constants, variables, C++ operations. Data typesin C++: Integer, float, character. Input/Output statements, Header files in C++, iostreanm.h, iomanip.h,math.h.	1
2	***Expressions in C++:*** (i) constant expression, (ii) integer expression, (iii) float expression, (iv) relational expression,(v) logical expression, (vi) Bitwise expression. Declarations in C++. Program Structure of C++ . Simple program to " WEL COME TO C++ ". ***Control Statements:*** (a) if, if – else, nested if. (b) for loop, while loop, do-while loop. (c) break, continue, goto, switch statements. *Euclid's algorithm to find gcd and then to find lcm of two numbers a, b * To list 1!, 2!, 3!, … , n! (n). * To print prime numbers from 2 to n.	1
3	***Arrays :*** (a) Sorting of an array. (b) Linear search. (c) Binary search. (d) Reversing string.	1
4	***Functions:*** User defined functions of four types with illustrative programs each.	1
5	***Microsoft Excel Knowledge*** *The student is expected to familiarize with Microsoft-Excel software for numerical Computations*. Opening Microsoft Excel, Overview of Excel Naming parts of the Excel Window, File New, File Open, File Close,File Save/Save As, AutoFill and Data Series, Cut, Copy, Paste, Insert, Menu Bar, Toolbar, Right clicking, Fill Handle, Inserting, deleting, and moving, Rows, Columns, Sheets, Mathematical symbols (Preset Functions) AutoSum, Copying a calculation using the fill handle, Formula Bar, Editing Formula Using preset functions, Order of operations, Print a worksheet. 1) Mean and S.D. of raw data, arrange given numbers in ascending or descending order. 2) Find the inverse of Matrix, transpose of matrices, determinant of square matrix, addition, multiplication of matrices.	3
6	***MATLAB Knowledge*** *The student is expected to familiarize with MATLAB software for numerical computation.* Basic of MATLAB, Tutorial Lessons. 1) Using MATLAB functions find the	3

	inverse of Matrix, transpose of matrices, determinant of square matrix, addition, multiplication of matrices, eigen values, eigen vectors. 2) Using MATLAB creating graphs of simple functions	
Numerical Methods - Find the solutions of following methods either using C++ programming OR M-Excel OR MATLAB software		
7	Interpolation : (a) Lagrange′s interpolation formula. (b) Newton Gregory forward interpolation formula. (c) Newton Gregory backward interpolation formula.	2
8	Numerical Methods for solution of A system of Linear Equations: (Unique solution case only) (a) Gauss – Elimination Method. (b) Gauss – Jorden Method.	2
9	Numerical Methods for solution of Ordinary Differential Equations: (a) Euler Method (b) Euler′s Modified Method (c) Runge Kutta Second and Fourth order Method.	2

REFERENCE BOOKS

1. Programming with C++, D. Ravichandran Second Edition, Tata Mac- Graw-Hill publishing Co. Ltd., New Delhi (2006).
2. Working with Excel 97 A Hands on Tutorial, Tata Mac- Graw-Hill Series, publishing Co. Ltd., New Delhi (2006).
3. Getting Started with MATLAB 7, Rudra Pratap, OXFORD UNIVERSITY PRESS.(2009)
4. Numerical Technique Lab MATLAB Based Experiments, K. K. Mishra, I. K. International Publishing House Pvt. Ltd., New Delhi.(2007)
5. MATLAB An Introduction with Applications, Amos Gilat, S. P. Printers, Delhi.(2004)

Computational Mathematics Laboratory – VII

(Project Work, Seminar, Study Tour, Viva- Voce)

A. PROJECT : **[25 Marks]**

Each student of B.Sc. III (Mathematics) is expected to read, collect, understand culture of Mathematics, its historic development. He is expected to get acquainted with Mathematical concepts, innovations, relevance of Mathematics. Report of the project work should be submitted through the respective Department of Mathematics.

Topics for Project work :

1. Contribution of the great Mathematicians such as Rene Descart, Leibnitz, Issac Newton, Euler, Langrange,Gauss, Riemann, Fourier, Bhaskaracharya, Srinivas Ramanujan etc.
2. On the following topics or on other equivalent topics :

   (i) Theorem on Pythagorus and pythagorian triplets.

   (ii) On the determination of value of π.

   (iii) Remarkable curves.

   (iv) Orthogonal Latin Spheres.

   (v) Different kinds of numbers.

   (vi) Law of quadratic reciprocity of congruence due to Gauss.

   (vii) Invention of Zero.

   (viii) Vedic Mathematics.

   (ix) Location of objects in the celestial sphere.

   (x) Kaprekar or like numbers.

   (xi) Playing with PASCAL′S TRIANGLE and FIBONACCI NUMBERS.

   (xii) PERT and CPM.

   (xiii) Magic squares.

   (xiv) Software such as Mupad, Matlab, Mathematica, Xplore, etc.

   (xv) Pigeon hole principle.

Evaluation of the project report will be done by the external examiners at the time of annual examination.

B. SEMINAR **[10 Marks]**

Topics for seminar should be selected as follows (or Equivalent) :

(i) Archimedian Solids.

(ii) Pascal′s Triangle and prime numbers.

(iii) Perfect numbers.

(iv) Sieripinski′s Carpet.

(v) Cantor set, Cantor Conjucture.

(vi) Euler′s Conjucture.

(vii) Some famous paradoxes in Mathematics.

(viii) Diagonalization of Matrices.

(ix) Riemann Surfaces.

**** Internal evaluation by the members of Mathematics teachers of the Department of Mathematics of the respective college.**

Synopsis of the seminar should be attached with project report.

C. STUDY TOUR **[5 Marks]**

List of suggested places : Banglore, Pune, Kolhapur, Mumbai, Ahamdabad, Hydrabad, etc.

D. VIVA-VOCE (on the project report). **[10 Marks]**

REFERENCE BOOKS

1. The World of Mathematics, James R. Newman & Schuster, New York.

2. Men Of Mathematics, E.T.Bell.

3. Ancient Indian Mathematics, C. N. Srinivasayengar.

4. Vedic Mathematic , Ramanand Bharati.

5. Fascinating World of Mathematical Science Vol. I, II, J. N. Kapur.

JOURNALS

1. Mathematical Education.

2. Mathematics Today.

3. Bona Mathematical.

4. Ramanujan Mathematics News Letter.

5. Resonance.

6. Mathematical Science Trust Society (MSTS), New friends′ colony, New Delhi 4000 065.

DETAILS OF SYLLABI
B.Sc. PART - III MATHEMATICS

This Syllabus of Mathematics carries **600**marks.

The distribution of marks as follows :

(1) Mathematics Paper – V " **ANALYSIS** " …. Marks – 100.

Section – I : **…………………. Marks - 50**

Section – II : **…………………. .Marks - 50**

(2) Mathematics Paper – VI " **ABSTRACT ALGEBRA**" …. Marks – 100.

Section – I : **…………………. Marks - 50**

Section – II : **…………………. .Marks - 50**

(3) Mathematics Paper – VII " **COMPLEX ANALYSIS AND INTEGRAL TRANSFORM**" …. Marks – 100.

Section – I : **…………………. Marks - 50**

Section – II : **…………………. .Marks - 50**

(4) Mathematics Paper – VIII (A) (Optional) " **DISCRETE MATHEMATICS**" …. Marks – 100.

Section – I : **…………………. Marks - 50**

Section – II : **…………………. .Marks - 50**

(5) Mathematics Paper – VIII (B) (Optional)

" **SPECIAL THEORY OF RELATIVITY** " …. Marks – 100.

Section – I : **…………………. Marks - 50**

Section – II : **…………………. .Marks - 50**

(6) Mathematics Paper – VIII (C) (Optional)

" **DIFFERENTIAL GEOMETRY** " …. Marks – 100.

Section – I : **…………………. Marks - 50**

Section – II : **…………………. .Marks – 50**

(7) Mathematics Paper – VIII (D) (Optional)

" **MATHEMATICAL MODELING** '' …. Marks – 100.

Section – I : **………………… Marks - 50**

Section – II : **………………… .Marks - 50**

(8) Mathematics Paper – VIII (E) (Optional)

" **APPLICATIONS OF MATHEMATICS IN FINANCE AND INSURANCE** ''

…. Marks – 100.

Section – I : **………………… Marks - 50**

Section – II : **………………… .Marks - 50**

(9) **COMPUTATIONAL MATHEMATICS LABORATRY - IV**

This carries <u>50</u> marks.

Examination : 40 Marks
Journal : 10 Marks

(10) **COMPUTATIONAL MATHEMATICS LABORATRY - V**

This carries <u>50</u> marks.

Examination : 40 Marks
Journal : 10 Marks

(11) **COMPUTATIONAL MATHEMATICS LABORATRY - VI**

This carries <u>50</u> marks.

Examination : 40 Marks
Journal : 10 Marks

(12) **COMPUTATIONAL MATHEMATICS LABORATRY - VII**

This carries <u>50</u> marks.

Project : 25 Marks (External Examiner)
Seminar : 10 Marks (College Department)
Study Tour : 05 Marks (External Examiner)
Viva Voce : 10 Marks (External Examiner)

Note : Each student of a class will select separate topic for project work and a separate topic for seminar . He/ She should submit the reports of his / her project work . Study tour report and Synopsis of the seminar to the department and get the same certified.

(13) (i) Total teaching periods for Paper – V , VI, VII, VIII are **<u>12</u>**

(Twelve) per week.

3 (Three) periods per paper per week.

(ii) Total teaching periods for Computational Mathematics Laboratory –III, IV, V, VII for the whole class are **<u>20</u> (Twenty)** per week.

5 (Five) periods per Lab. per week.

(14). Equivalence of the papers may be as follows

New Syllabus	Old Syllabus
Compulsory Papers	
Mathematics Paper – V (Analysis)	Mathematics Paper – V (Analysis)
Mathematics Paper –VI (Algebra)	Mathematics Paper – VI (Algebra)
Mathematics Paper – VII (Complex Analysis & Integral Transform)	Mathematics Paper – VII (Complex Analysis & Integral Transform)
OPTIONAL PAPERS	
Mathematics Paper –VIII(A) (Discrete Mathematics)	Mathematics Paper – VIII(A) (Discrete Mathematics)
Mathematics Paper – VIII(B) (Special Theory Of Relativity)	Mathematics Paper – VIII(B) (Special Theory Of Relativity)
Mathematics Paper –VIII(C) (Differential Geometry)	Mathematics Paper –VIII(C) (Differential Geometry)
Mathematics Paper –VIII(D) (Mathematical Modelling)	Mathematics Paper –VIII(D) (Mathematical Modelling)
Mathematics Paper –VIII(E)	Mathematics Paper –VIII(E)

(Application of Mathematics in Finance and Insurance)	(Application of Mathematics in Finance and Insurance)

Computational Mathematics Laboratory

Computational Mathematics Laboratory --IV (Operations Research Techniques)	Computational Mathematics Laboratory –IV (Operations Research Techniques)
Computational Mathematics Laboratory --V (_Complex Variables and Applications of Differential Equations)	Computational Mathematics Laboratory –V (Complex Variables and Applications of Differential Equations)
Computational Mathematics Laboratory --VI (Numerical Recipes in C++)	Computational Mathematics Laboratory --VI (Numerical Recipes in C++)
Computational Mathematics Laboratory --VII (Project, Viva, Seminar, Tour Report)	Computational Mathematics Laboratory –VII (Project, Viva, Seminar, Tour Report)

Nature of Theory Question Paper

Section-I

Q1. Multiple choice questions -	(10 Marks)
Q2. Attempt Any Two out of Three	(20 Marks)
Q3. Short Answers (any four out of six)	(4 X 5 Marks)

Section-II

Q4. Multiple choice questions -	(10 Marks)
Q5. Attempt Any Two out of Three	(20 Marks)
Q6. Short Answers (any four out of six)	(4X 5 Marks)