

Corporate Finance Sample Exam Questions with Answers

February 17, 2010

1. Consider an entrepreneur who owns a project with return R in the success state and return zero in the failure state (these are the only states), and with success probability p which depends on the entrepreneur's effort. Suppose high effort implies $p = p^H$ and low effort implies $p = p^L$ and a private benefit B to the entrepreneur. You should assume risk neutrality, zero discount rate, and a perfectly competitive capital market. The investment cost of the project is i .

(a) If $p^L R < i < p^H R$, what is the maximum amount of funds that can be raised from the capital market? Explain.

(b) Suppose the entrepreneur has made the investment using private funds, and that there is α percent probability that high effort is chosen and $1 - \alpha$ probability that low effort is chosen. The entrepreneur is now considering a second project with cost j , and this project increases the success probability from p (which is either p^H or p^L depending on type) to $p + \tau$. The entrepreneur has no wealth and must finance the investment cost by issuing claims. Assume $\tau R > j$. Demonstrate that under certain conditions, investment in the new project does not occur in equilibrium for one type of entrepreneur. What are these conditions, and what type is unable to invest?

Distribution of marks: 25 in total (a) 10 marks (b) 15 marks. Answers: (a) Incentive compatibility constraint (for high effort) implies that $p^H(R - R_I) \geq p^L(R - R_I) + B$ or $R_I \leq R - \frac{B}{p^H - p^L}$. The maximum amount of outside financing is, therefore, $p^H \left(R - \frac{B}{p^H - p^L} \right)$; (b) Need to show that there exists no separating equilibrium where both types invest – then if a pooling equilibrium breaks down we are done. The first is relatively trivial (need to look at the IC constraints). A pooling equilibrium requires $(p^L + \alpha(p^H - p^L) + \tau)R_I = i$ (where R_I is the return promised to the outside investors). IC constraints are $p^L R \leq (p^L + \tau)(R - R_I)$ for the low effort type and $p^H R \leq (p^H + \tau)(R - R_I)$ for the high effort type, which reduce to $\frac{p^L + \tau}{(p^L + \alpha(p^H - p^L) + \tau)}i \leq \tau R$ for the low effort type and $\frac{p^H + \tau}{(p^L + \alpha(p^H - p^L) + \tau)}i \leq \tau R$. The first is always satisfied by the second may not be, so it is always the high effort type (if any) that exists the market. Exit happens when p^H is high compared to the average probability, and τ , i.e. when uncertainty about type is

high and when the new investment is a small part of the firm's total assets.

2. (a) What is pecking order theory of finance? Explain why the overriding principle behind pecking order theory is that the entrepreneur seeks to minimize the information sensitivity of claims issued to the capital market. Try to give an intuitive answer to this question, there is no need to produce algebra in answering this question.
- (b) Explain why free riding in takeovers can explain the empirical observations that the stock response of bidders to takeover announcements is close to zero, whereas the stock response to targets to the same announcements is significantly positive.
- (c) Explain Jensen and Meckling's agency cost of outside equity. You may like to use algebra or a diagram to illustrate your answer.

Distribution of marks: 25 in total (a) 7 marks (b) 10 marks (c) 8 marks. Answers: (a) Main idea is that the issuer makes a gain from selling claims to finance a positive NPV project, but may at the same time make a loss or a gain on financing depending on whether they sell undervalued or overvalued claims – leading to over or under investment. Reducing the information sensitivity of claims issued reduces the financing gain/loss so we are left with only the good projects being financed. (b) Standard Grossman-Hart analysis – if success-probability is β , synergy gains are 1, and bid-premium is P , then in equilibrium $\beta = P$: if tendering the target shareholder receives $v + P$, if not tendering the target shareholder receives $\beta(v+1) + (1-\beta)(v) = v + \beta$, and obviously indifference is required. (c) Again, standard approach: financial value B and the financial equivalent of the non-financial benefits F such that $B + F = 1$ (RHS normalized). If firm has only internal equity the utility max problem yields $\frac{\partial u}{\partial B} = \frac{\partial u}{\partial F} = \lambda$ (the Lagrange multiplier). If firm has α outside equity the utility max problem yields $\alpha \frac{\partial u}{\partial B} = \frac{\partial u}{\partial F} = \lambda$, which implies that the indifference curve has slope $-\alpha$ in the latter case and slope -1 in the first case. Since the 'budget' constraint $B + F = 1$ has slope -1 the former is optimal whereas the latter is not – and there is overproduction of non-financial benefits.

3. A firm has no assets but owns an investment project with cash flow $20 + \tilde{\epsilon}$ with probability $\frac{1}{2}$ and $5 - \tilde{\epsilon}$ with probability $\frac{1}{2}$. The project costs 5, and the random variable $\tilde{\epsilon} \sim \text{Uniform} [0, 5]$. Assume the binary uncertainty (the up/down risk) is systematic risk which is unknown to the manager, whereas the ϵ uncertainty is unsystematic, firm-specific, risk that is known to the manager, at the time of investment. The firm has no other assets or cash, but has a repayment obligation D which is due after the investment is done.

- (a) Conditional on ϵ , what is the optimal investment plan if the firm has zero borrowing (i.e. $D = 0$)?
- (b) Conditional on ϵ , what is the optimal investment plan if the firm has positive borrowing (i.e. $D > 0$)?
- (c) What is the value of debt ex ante (before ϵ is known), as a function of D ?
- (d) What is the debt capacity of the firm ex ante (i.e. the maximum amount the firm can borrow)?

Distribution of marks: 25 marks in total; (a) 5 marks; (b) 5 marks; (c) 10 marks; (d) 5 marks. Answers: (a) Invest iff $\frac{1}{2}(20 + \epsilon) + \frac{1}{2}(5 - \epsilon) - 5 = 12.5 - 5 \geq 0$ which is true. (b) Invest iff $\frac{1}{2}(20 + \epsilon - D) + \frac{1}{2}(\max(0, 5 - \epsilon - D)) \geq 5$, which is true for $D \leq 10 - \epsilon$. (c) For $0 \leq D \leq 5$: $V(D) = \int_0^{5-D} \frac{D}{5} d\epsilon + \int_{5-D}^5 (\frac{D}{2} + \frac{5-\epsilon}{2}) \frac{1}{5} d\epsilon = D - \frac{D^2}{20}$; for $5 < D \leq 10$: $V(D) = \int_0^{10-D} (\frac{D}{2} + \frac{5-\epsilon}{2}) \frac{1}{5} d\epsilon = \frac{3}{2}D - \frac{3}{20}D^2$. (d) maximizing the two expressions: for $0 \leq D \leq 5$ we find $D = 10$ so debt capacity is $D \geq 5$, and for $0 < D \leq 10$ we find $D = 5$ so debt capacity is for $D = 5$ where debt is worth 3.75.

4. Consider an entrepreneur who has no wealth but who owns a project with cost I . The project's payoff is determined by the entrepreneur's effort. Suppose the payoff is high (X^H) with probability $p = \min(e, 1)$ and low (X^L) with probability $1 - p$. The number e is the entrepreneur's effort. The cost of effort is $\frac{c}{2}e^2$, and assume this cost is borne by the entrepreneur only and needs no extra financing (i.e. it is a private non-pecuniary cost).

- a. [5 marks] Derive the first best level of effort. For what values of c and I do we find that investment is profitable and that the first best effort level is between zero and one? Assume, for the remaining questions, that these restrictions are satisfied.

- b. [5 marks] Suppose the entrepreneur can raise outside financing by issuing a claim with payoff R^H for high output and R^L for low output, where $0 \leq R^i \leq X^i$ ($i = H, L$) and $R^L \leq R^H$. If the entrepreneur can commit to a given effort level \hat{e} in the financing contract sold to the outside investors, what effort level \hat{e} will be implemented? What financing plan is chosen in equilibrium?
- c. [10 marks] Now assume that the financing contract cannot be written such as to commit the entrepreneur on effort. What financing plan is chosen in this case? Explain.
- d. [5 marks] Are there frictions in the financing process in the situation above? If so, which firms are more likely to experience frictions? Finally, how would we expect the firm to respond to financial frictions, and is there empirical evidence to support such behaviour? Explain.

a. $\max_e eX^H + (1 - e)X^L - I - \frac{c}{2}e^2$ yields first best $e^* = \frac{X^H - X^L}{c}$ so interior solution for $c > X^H - X^L$. Investment is profitable iff NPV evaluated at e^* is positive, i.e. $\frac{X^H - X^L}{c}X^H + \left(1 - \frac{X^H - X^L}{c}\right)X^L - \frac{c}{2}\left(\frac{X^H - X^L}{c}\right)^2 = X^L + \frac{(X^H - X^L)^2}{2c} \geq I$.

b. $\max_{\hat{e}} \hat{e}R^H + (1 - \hat{e})R^L + \hat{e}(X^H - R^H) + (1 - \hat{e})(X^L - R^H) - I - \frac{c}{2}\hat{e}^2$ subject to $\hat{e}R^H + (1 - \hat{e})R^L \geq I$ yields $\max_{\hat{e}} \hat{e}X^H + (1 - \hat{e})X^L - \frac{c}{2}\hat{e}^2$ which is equivalent to the problem in a so will yield the first best. Financing is irrelevant as long as the budget constraint is satisfied.

c. The programme is in this case $\max_e \bar{e}R^H + (1 - \bar{e})R^L + e(X^H - R^H) + (1 - e)(X^L - R^L) - I - \frac{c}{2}e^2$ subject to $\bar{e} = e$ (investors have rational beliefs) and $\bar{e}R^H + (1 - \bar{e})R^L \geq I$ (the budget constraint). The first order condition implies $e^{**} = \frac{(X^H - R^H) - (X^L - R^L)}{c} = e^* - \frac{R^H - R^L}{c}$. In this case the financing plan matters, so we need to consider the optimal choice of contract: $\max_{R^H, R^L} X^L + \left(e^* - \frac{R^H - R^L}{c}\right)(X^H - X^L) - \frac{c}{2}\left(e^* - \frac{R^H - R^L}{c}\right)^2$ subject to the budget constraint. This programme depends on $R^H - R^L$ only, and we find that the objective function is monotonic and decreasing in $R^H - R^L$ – hence the programme is equivalent to $\min_{R^H, R^L} R^H - R^L$ subject to $R^L + \left(e^* - \frac{R^H - R^L}{c}\right)(R^H - R^L) \geq I$. This programme is solved by putting $R^L = X^L$ and R^H minimal – i.e. a debt contract.

d. Frictions appear when debt is risky. The firm the least able to issue risk free (or near risk free) debt will experience the greatest frictions. One of the ways in which firms respond to the increased cost of outside financing is to suit its investment plan to the availability of free cash flow which underpins much of the literature on financial constraints. There is considerable evidence that firms we expect to be financially constrained ex ante, have higher cash flow sensitivity in their investment policy than firms we expect to be unconstrained. (Students may quote some specific papers here which were covered in the lecture.)

5.
 - a. [3 marks] Prove the Modigliani-Miller Irrelevance of Capital Structure Theorem. What assumptions does this theorem hinge on?
 - b. [3 marks] Explain the Static Trade Off Theory of Capital Structure. There is no need to produce algebra.
 - c. [3 marks] Explain the free rider problem in takeovers. What factors can alleviate this problem? There is no need to produce algebra.
 - d. [6 marks] A firm has cash flow 50 at time 1 and can make an investment at time 1 of I that produces a cash flow at time 2 of $\ln(1 + I)^{31}$. There is zero discounting. The manager has preferences for investment and does not respond to other forms of incentive structures. What is the efficient level of investment at time 1? Suppose the firm can borrow short term (a payment obligation D_1 due at time 1) and long term (a payment obligation D_2 due at time 2). What debt structure D_1 and D_2 will implement the first best? Does your answer depend on the possibility that another investment project might appear at time 2? Explain.
 - e. [10 marks] A firm has a nominal long term senior debt liability of D and no existing assets, but owns an investment opportunity that has payoff $(20 + \tilde{\epsilon})$ with probability $\frac{1}{2}$ and $5 - \tilde{\epsilon}$ with probability $\frac{1}{2}$, with $\tilde{\epsilon} \sim \text{Uniform}[0, 5]$, and with an investment cost of 6. Assume risk neutrality and zero discounting, and also assume the investment decision is made before the ‘fundamental’ project risk (the high/low state variable) is realized but after the ‘idiosyncratic’ project risk (the ϵ state variable) is realized. Work out

the investment region and the no-investment region, and also work out the ex ante values (i.e. the value before $\tilde{\epsilon}$ is known) of the debt contract for various values of D . Would the debt holders ever choose to unilaterally write off parts of the debt liability D ex ante? Explain.

a. *Standard proof – the simplest one based on the fact that in arbitrage free markets there exist positive state prices. Requires no frictions, taxes, deadweight costs in financing or investment, or asymmetric information.*

b. *Again standard – debt is tax-efficient but leads to an increase in the present value of future deadweight bankruptcy costs. The optimal capital structure is chosen such that the marginal tax benefit equals the marginal increase in the deadweight costs.*

c. *If non-tendering shareholders can capture the full economic value of their shares, there is an incentive to hold out for the full value if offered something less. If everybody else tender their shares it is optimal not to tender since the takeover will take place for sure – this creates a disincentive to tender.*

d. *Efficient investment is determined by $\max_I \ln(1 + I)^{31} - I$ which generates $I^* = 30$. It is optimal to put $D_1 = 20$ such that the manager invests only the free cash flow $50 - 30 = 20$. Long term debt D_2 does not matter, but if the firm expects another investment project to appear then D_2 should be minimal such as to prevent debt overhang. The firm can always retain the option to borrow short term at time 1.*

e. *NPV is $\frac{1}{2}(20 + 5) - 6 = 6.5$ so investment is efficient regardless. NPV to equity holders is $\frac{1}{2}(20 + 5) - 6 - D = 6.5 - D$ for $5 - \epsilon \geq D \geq 0$ so is always positive in this region, and $\frac{1}{2}(20 + \epsilon - D) - 6 = 4 + \frac{\epsilon}{2} - \frac{D}{2}$ for $20 + \epsilon \geq D > 5 - \epsilon$ so is positive for $D \leq 8 + \epsilon$. Investment region is, therefore, for $0 \leq D \leq 8 + \epsilon$ and there is no investment otherwise. The debt payoff is D for $5 - \epsilon \geq D \geq 0$ and $\frac{1}{2}D + \frac{1}{2}(5 - \epsilon) = 2.5 + \frac{D - \epsilon}{2}$ for $8 + \epsilon \geq D > 5 - \epsilon$, and 0 for $D > 8 + \epsilon$.*

In the region $0 \leq D \leq 5$, the debt payoff is D for $0 \leq \epsilon \leq 5 - D$ and $2.5 + \frac{D - \epsilon}{2}$ for $D - 5 < \epsilon \leq 5$, so the debt value is

$$\int_0^{5-D} D \frac{1}{5} d\epsilon + \int_{5-D}^5 \left(2.5 + \frac{D - \epsilon}{2} \right) \frac{1}{5} d\epsilon = D - \frac{D^2}{20}$$

In the region $5 < D \leq 8$ the debt value is (similarly)

$$\int_0^5 \left(2.5 + \frac{D - \epsilon}{2}\right) \frac{1}{5} d\epsilon = 1.25 + \frac{D}{2}$$

and in the region $8 < D \leq 13$ the debt value is (similarly)

$$\int_{D-8}^5 \left(2.5 + \frac{D - \epsilon}{2}\right) \frac{1}{5} d\epsilon = \frac{169}{20} - \frac{D^2}{20}$$

and in the region $13 < D$ the debt value is zero. We notice that the debt values increase in D until $D > 8$. Therefore, in this region the debt holders would benefit from a unilateral write off until $D = 8$.

6. A firm owned by an entrepreneur has already made an investment that has high payoff (X^H) with probability θ and low payoff (X^L) with probability $1 - \theta$, and is considering a new project that costs I and will improve the probability of high payoff with an amount Δ (assume $\theta + \Delta < 1$). We assume that the project has positive NPV: $\Delta(X^H - X^L) > I$. The entrepreneur has own wealth W which we assume is less than I , so it is necessary the new project has some financing from outside investors. To finance the shortfall, the entrepreneur issues a contract with payoff R^H if the firm has high payoff and R^L if the firm has low payoff, where $0 \leq R^i \leq X^i$ ($i = H, L$) and $R^L \leq R^H$. Assume the entrepreneur may be of a ‘good type’ with $\theta = \theta^G$ or a ‘bad type’ with $\theta = \theta^B$, where $\theta^B < \theta^G$, and assume the entrepreneur knows his own type but that the market has a probability distribution over types with $\Pr(\theta = \theta^G) = \alpha$ and $\Pr(\theta = \theta^B) = 1 - \alpha$.
- [3 marks] Assume the market has perfect information about the entrepreneur’s type. What is the equilibrium describing investment and financing? Does financing matter?
 - [3 marks] Now assume the market has imperfect information (i.e. the market know only the probability distribution described above). What is the equilibrium concept used to describe investment and financing in this case? What restrictions, if any, dictate the investors’ beliefs?
 - [7 marks] Derive a pooling equilibrium. State clearly any assumptions made.
 - [7 marks] Derive a separating equilibrium where one type chooses not to finance the project. State clearly any assumptions made.

e. [5 marks] Is the entrepreneur's wealth instrumental in arriving at a pooling or a separating equilibrium? Explain.

a. Entrepreneur's wealth (type θ) is $W + X^L + \theta(X^H - X^L) + (\Delta(X^H - X^L) - I)\mathbf{I}_{Investment} - (\theta - \hat{\theta})(R^H - R^L)\mathbf{I}_{Investment}$ where $\mathbf{I}_{Investment}$ is a dummy variable indicating investment and $\hat{\theta}$ is the market's belief about type. The budget constraint is $R^L + (\hat{\theta} + \Delta)(R^H - R^L) \geq I - W$. If perfect information $\theta - \hat{\theta} = 0$, therefore both types invest (since NPV is positive) and adjust their financing to fit the budget constraint.

b. Perfect Bayesian Equilibrium. Agents' actions are incentive compatible with respect to agents' beliefs, and beliefs are Bayesian rational along actions that are part of the equilibrium path. This implies that in a pooling equilibrium $\hat{\theta}$ is equal to the prior beliefs of the market, and in a separating equilibrium $\hat{\theta} = \theta$ for all actions that are part of the equilibrium. Off equilibrium actions lead to arbitrary beliefs as they are on the null-sets where Bayes law does not apply.

c. Pooling equilibrium where both types invest can be supported by θ^B as off-equilibrium beliefs (can show that there exists no pooling equilibrium with both types not investing as the NPV is strictly positive and the financing loss must be non-negative for at least one type regardless). Entrepreneurs' wealth increases if $\Delta(X^H - X^L) - I \geq (\theta - \hat{\theta})(R^H - R^L)$. For the bad type this is always satisfied as in this case the RHS is zero or negative, for the good type this is satisfied if $R^H - R^L$ is sufficiently small.

d. Here we assume θ^B as off-equilibrium beliefs. The bad type invests and the good type does not (can show that there is no equilibrium the other way – however there exists a separating equilibrium with both types investing if the good type can issue risk free debt – some students may choose this one). The wealth of the bad type is $W + X^L + \theta^B(X^H - X^L) + \Delta(X^H - X^L) - I$ which is the same as the entrepreneur gets if investing with an off-equilibrium financing plan, and strictly greater than no-investment yields. The wealth of the good type is $W + X^L + \theta^G(X^H - X^L)$. Deviating into investment yields $W + X^L + \theta^G(X^H - X^L) + \Delta(X^H - X^L) - I - (\theta^B - \theta^G)(R^H - R^L)$ regardless of financing plan. Therefore, this is an equilibrium provided $(\theta^G - \theta^B)(R^H - R^L) \geq \Delta(X^H - X^L) - I$.

e. The critical conditions are $\Delta(X^H - X^L) - I \geq (\theta^G - \hat{\theta})(R^H - R^L)$ for a pooling equilibrium and $\Delta(X^H - X^L) - I \leq (\theta^G - \theta^B)(R^H - R^L)$ for a separating equilibrium. There are several approaches here but the intuition is that the greater the wealth, the less risky can you make the external financing contract, and the more likely is it that you can avoid a separating equilibrium and arrive at a pooling equilibrium.

7. Consider a firm with earnings flow y following a Brownian motion with drift α and volatility σ . The expected value of all future earnings conditional on current earnings flow y_t is

$$E_t \left(\int_t^\infty e^{-rs} y_s ds \middle| y_t \right) = \frac{y_t}{r - \alpha}$$

You should assume the drift rate α is a risk neutral drift rate, and that the instantaneous discount rate r remains constant throughout.

- What is the value of a perpetual risk free bond contract with constant coupon flow c ?
- What is the value of a perpetual risky bond contract with constant coupon flow c when the firm's equity holders can choose timing of default optimally to their advantage? You should assume the bond holders lose α percent of the firm's assets in default, and that the firm's assets are restructured as a firm with zero debt.
- What is the credit spread s for this bond contract at the current earnings level of y_t ?
- What is the recovery rate in default for this bond contract, relative to the value of a perpetual bond contract with yield $i = r + s$ (where s is the credit spread of the bond at par value)?

Distribution of marks: 25 marks in total; (a) 5 marks (b) 10 marks (c) 5 marks (d) 5 marks. Answers: (a) $\frac{c}{r}$; (b) Debt contract $B = Ay^\lambda + \frac{c}{r}$ for λ the negative root of $\frac{1}{2}\sigma^2\lambda(\lambda - 1) + \alpha - r = 0$. The value at default is $(1 - \alpha)\frac{y^}{r - \alpha}$, so $A = \left((1 - \alpha)\frac{y^*}{r - \alpha} - \frac{c}{r} \right) y^{-\lambda y^*}$. The equity value is $\frac{y}{r - \mu} - \left(\frac{y^*}{r - \alpha} \right) \left(\frac{y}{y^*} \right)^\lambda - \frac{c}{r} \left(1 - \left(\frac{y}{y^*} \right)^\lambda \right)$, and maximizing with respect to y^* yields $y^* = \frac{c}{r} (r - \mu) \frac{\lambda}{\lambda - 1}$; (c) Yield is $i = \frac{c}{B}$, and comparing to a risk free bond $r = \frac{c}{c/r}$ we find the credit spread as $s = i - r = \frac{c}{\left((1 - \alpha)\frac{y^*}{r - \alpha} \right) \left(\frac{y}{y^*} \right)^\lambda + \frac{c}{r} \left(1 - \left(\frac{y}{y^*} \right)^\lambda \right)} - r$; (d) the recovery rate is the actual recovered value in default $(1 - \alpha)\frac{y^*}{r - \alpha}$ over the 'notional' value $\frac{c}{i}$, i.e. $\frac{i}{c}(1 - \alpha)\frac{y^*}{r - \alpha} = \frac{i}{r}(1 - \alpha)\frac{\lambda}{\lambda - 1}$.*