IIT-JEE 2012

EXAMINATION

(Held on 08-04-2012)

PAPER-1

(ANSWERS & SOLUTIONS)

ALLEN CAREER INSTITUTE
KOTA (RAJASTHAN)
Path to Success

Corporate Office

ALLEN CAREER INSTITUTE

"SANKALP", CP-6, INDRA VIHAR, KOTA-324005

PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003

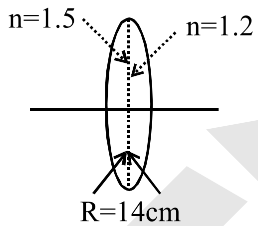
E-mail: info@allen.ac.in Website: www.allen.ac.in

PART - I : PHYSICS

SECTION–I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE is correct.**

1. A biconvex lens is formed with two thin plano-convex lenses as shown in the figure, Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this biconvex lens, for an object distance of 40 cm, the image distance will be :-



(A) –280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

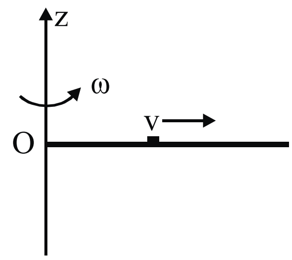
Ans. (B)

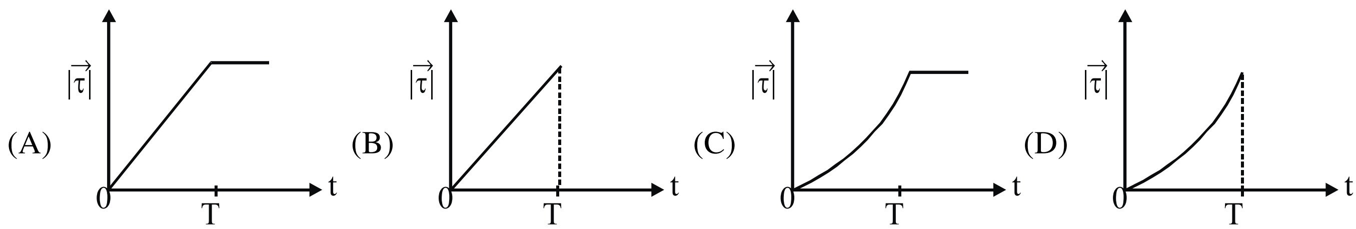
Sol. Equivalent focal length of system is $\frac{1}{f}=\frac{1}{f_1}+\frac{1}{f_2}$

$$\frac{1}{f}=(1.5-1)\left(\frac{1}{14}\right)+(1.2-1)\left(\frac{1}{14}\right)=\frac{1}{20}$$

from lens equation $\frac{1}{v}-\frac{1}{u}=\frac{1}{f}$ we get $\frac{1}{v}-\frac{1}{(-40)}=\frac{1}{20}$; v = + 40 cm

2. A thin uniform rod, pivoted at OP, is rotating in the horizontal plane with constant angular speed ω. as shown in the figure. At time t = 0, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. If reaches the end of the rod at t= T and stops. The angular speed of the system remains ω throughout. The magnitude of the torque $(|\tau|)$ on the system about O, as a function of time is best represented by which plot?





Ans. (B)

Sol. Net torque on a system is rate of change of angular momentum. Let insect be at a radial distance r from axis at an instant.

Angular momentum of system as function of r is given by $|\vec{L}| = \left(\frac{ML^2}{3} + mr^2\right)\omega$

As ω is of constant magnitude, angular momentum varies due to variation in r.

$$|\tau| = \left|\frac{dL}{dt}\right| = \omega\frac{dI}{dt} = \omega\frac{d}{dt}\left(\frac{ML^2}{3} + mr^2\right)$$

$$= \omega\left(0 + m2r\frac{dr}{dt}\right) = 2m\omega(r)\left(\frac{dr}{dt}\right)$$

$$= 2m\omega(vt)(v) = (2m\omega v^2 t) \Rightarrow |\tau| \propto t$$

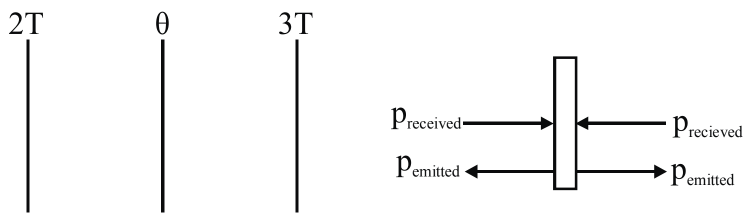
Thus graph is linear passing through origin.

3. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures 2T and 3T respectively. The temperature of the middle (i.e. second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{1/4} T$ (B) $\left(\frac{97}{4}\right)^{1/4} T$ (C) $\left(\frac{97}{2}\right)^{1/4} T$ (D) $(97)^{1/4} T$

Ans. (C)

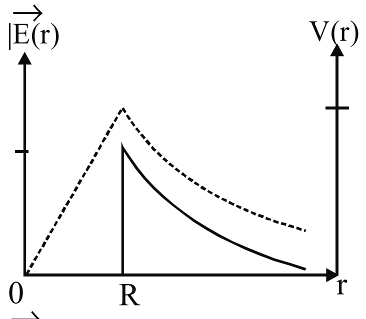
Sol.

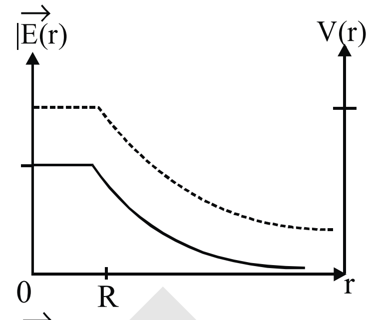


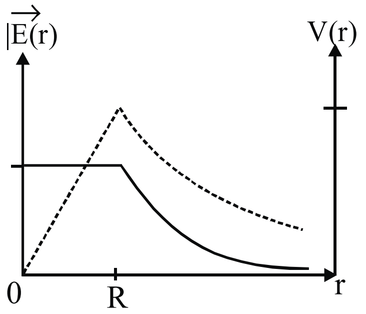
For steady state, $P_{net\ received} = P_{net\ emitted}$

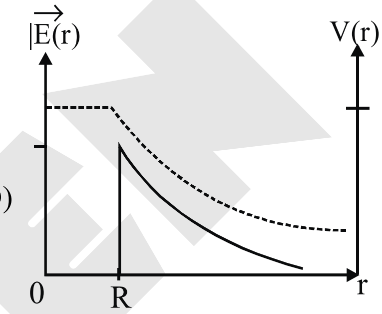
$$\sigma A(\theta^4 - (2T)^4) = \sigma A[(3T)^4 - \theta^4] \Rightarrow \theta^4 - 16T^4 = 81T^4 - \theta^4 \Rightarrow \theta = \left(\frac{97}{2}\right)^{1/4} T$$

4. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential V(r) with the distance r from the centre, is best represented by which graph?

(A)


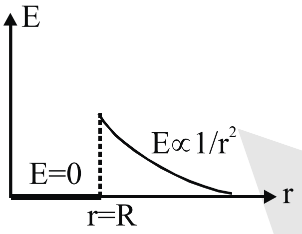
(B)


(C)


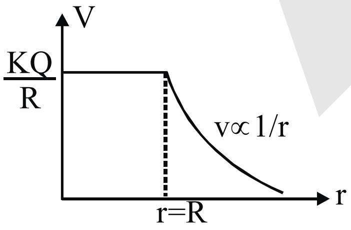
(D)


Ans. (D)

Sol. Electric field inside a uniformly charged spherical shell :



$E_{inside} = 0;$ $r < R$ $E_{outside} = \frac{kQ}{r^2};\ r \geq R$



Electric potential : $V_{inside} = \frac{KQ}{R};\ r \leq R$ $V_{outside} = \frac{KQ}{r};\ r \geq R$

5. In the determination of Young's modulus $\left(Y = \frac{4MLg}{\pi\ell d^2}\right)$ by using Searle's method, a wire of length L = 2m and diameter d = 0.5 mm is used. For a load M = 2.5 kg, an extension l = 0.25 mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) due to the errors in the measurements of d and l are the same.

(B) due to the error in the measurement of d is twice that due to the error in the measurement of l.

(C) due to the error in the measurement of l is twice that due to the error in the measurement of d.

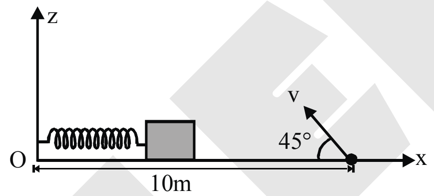
(D) due to the error in the measurement of d is four times that due to the error in the measurement of l.

Ans. (A)

Sol. $y = \dfrac{4MLg}{\pi \ell d^2} \Rightarrow \dfrac{\delta y}{y} = \dfrac{\delta(\ell)}{\ell} + \dfrac{2\delta(d)}{d}$

$\dfrac{\delta \ell}{\ell} = \dfrac{[0.5/100]}{0.25} = \dfrac{1}{50}$ and $\dfrac{2\delta(d)}{d} = \dfrac{2[0.5/100]}{0.5} = \dfrac{1}{50}$

6. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. They system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at t = 0. It then executes simple harmonic motion with angular frequency $\omega = \dfrac{\pi}{3}$ rad/s. Simultaneously at t = 0, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at t = 1 s, the value of v is (take g = 10 m/s^2)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Ans. (A)

Time of flight = $\dfrac{2v\sin 45^\circ}{g} = 1 \quad \Rightarrow v = \dfrac{g}{\sqrt{2}} = \sqrt{50}$ m/s

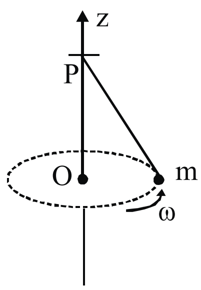
7. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

Ans. (D)

Sol. Fringe width is given by $(\beta) = \dfrac{D\lambda}{d}$ and order of wavelength in electromagnetic spectrum is given by $\lambda_R > \lambda_G > \lambda_B$

8. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

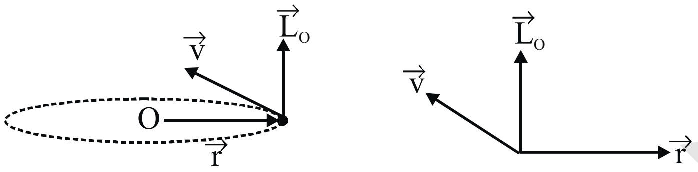
(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

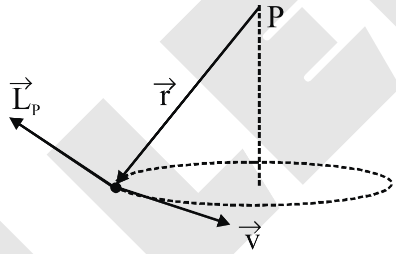
(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

Ans. (C)

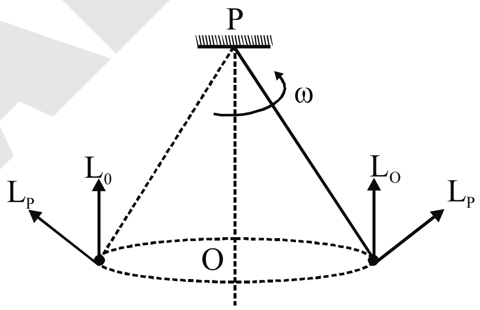
Sol.



Angular momentum of particle about O is vertical as shown in figure.



Angular momentum $\vec{L}_P$ about point P is as shown in figure at an instant



$\vec{L}_O$ is constant & $\vec{L}_P$ is changing in direction.

9. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}(helium)}{v_{rms}(\arg on)}\right)$ is

(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

Ans. (D)

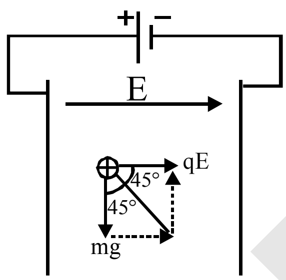
Sol. $v_{rms} = \sqrt{\frac{3RT}{M}} \Rightarrow \frac{(v_{rms})_{He}}{(v_{rms})_{Ar}} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{\frac{40}{4}} = \sqrt{10} = 3.16$

10. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

Ans. (C)

Sol.



As proton moves at 45° in vertical plane it implies magnitude of electric force and gravitational force is same.

for 45° direction |mg| = |qE|

$$\frac{mg}{q} = E = \frac{V}{d}$$

$$V = \frac{mgd}{q} = \frac{(1.6 \times 10^{-27} kg)(1 \times 10^{-2} m)}{(1.6 \times 10^{-19} C)} = 1 \times 10^{-9} \text{ volt}$$

Section-II : Multiple Correct Answer(s) Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

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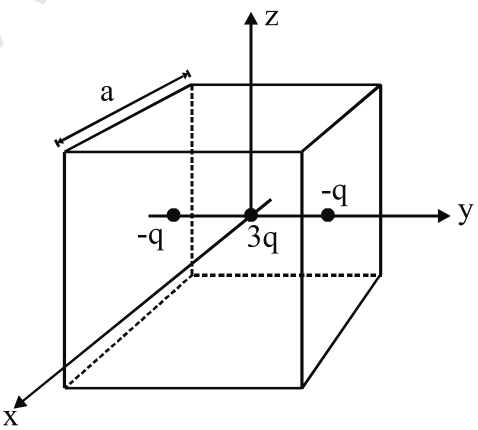
11. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, –q at (0, –a/4, 0), +3q at (0,0,0) and –q at (0, +a/4, 0). Choose the correct option(s).



(A) The net electric flux crossing the plane x = +a/2 is equal to the net electric flux crossing the plane x = – a/2

(B) The net electric flux crossing the plane y = +a/2 is more than the net electric flux crossing the plane y = –a/2.

(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(D) The net electric flux crossing the plane z = +a/2 is equal to the net electric flux crossing the plane x = +a/2.

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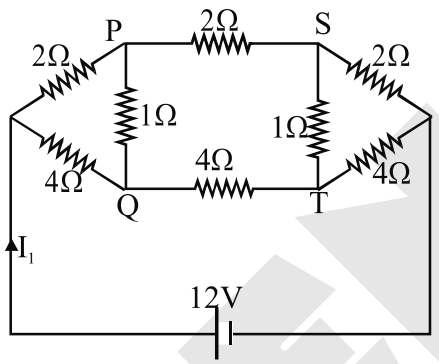
Ans. **(A,C,D)**

Sol. For (A): Position of charges is symmetrical with respect to planes at x = + a/2 and x = – a/2

For (C) : Net charge enclosed by cube is q.

For (D) : Charges are symmetrically placed iwth given planes.

12. For the resistance network shown in the figure, choose the correct option(s).

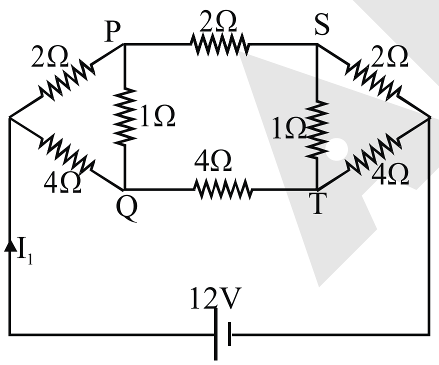


(A) the current through PQ is zero (B) $I_1 = 3A$

(C) The potential at S is less than that at Q (D) $I_2 = 2A$

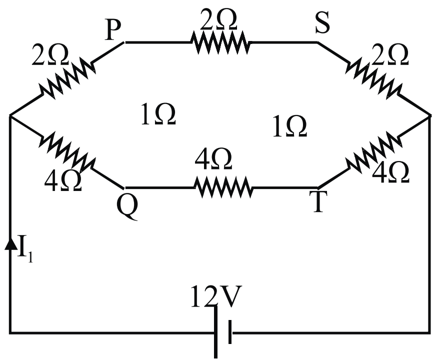
Ans. **(A,B,C,D)**

Sol.



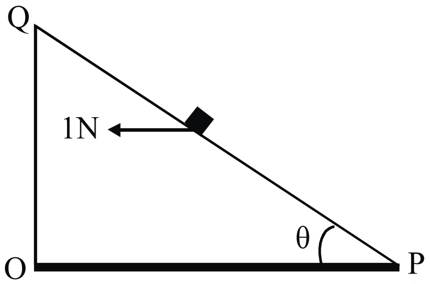
Since resitances in two arms upper and lower are in same ratio.

Current through PQ is zero.



$I_2 = \frac{12}{6} = 2A$, $I_1 = \frac{12}{6} + \frac{12}{12} = 3A$ and potential of S is less than that at Q.

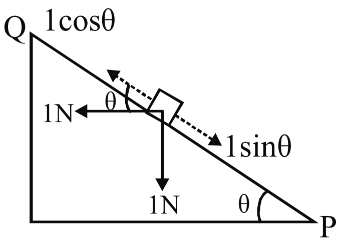
13. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its center of mass as shown in the figure. The block remains stationary if (take g = 10 m/s^2)



(A) θ = 45°
(B) θ > 45° and a frictional force acts on the block towards P
(C) θ > 45° and a frictional force acts on the block towards Q
(D) θ < 45° and a frictional force acts on the block towards Q

Ans. (A,C)

Sol.

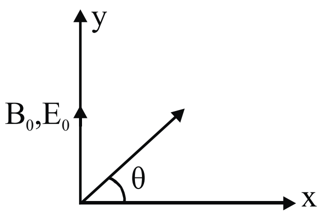


(1) If $\sin\theta = \cos\theta \Rightarrow \theta = 45° \Rightarrow$ no friction will act and the block will remain at rest.
(2) If $\sin\theta > \cos\theta \Rightarrow \theta > 45° \Rightarrow$ friction will act towards Q
(3) If $\sin\theta < \cos\theta \Rightarrow \theta < 45° \Rightarrow$ friction will act towards P

14. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time t =0, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option (s) is (are) correct for time t > 0?
(A) If θ = 0°, the charge moves in a circular path in the x-z plane.
(B) If θ = 0°, the charge undergoes helical motion with constant pitch along the y-axis.
(C) If θ = 10°, the charge undergoes helical motion with its pitch increasing with time, along the y-axis
(D) If θ = 90°, the charge undergoes linear but accelerated motion along the y-axis

Ans. (C,D)

Sol.



vcosθ is perpendicular to magnetic field and it plays role of tangential velocity (responsible for circular motion).
vsinθ is axial velocity parallel to magnetic field responsible for axial motion.
If θ = 0°, the path of the charge particle will be helical with increasing pitch.
If θ = 10°, the path of the charge particle will be helical with increasing pitch.
If θ = 90°, the path of the charge particle will be linear and accelerating.

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15. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe.

(A) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(B) a low -pressure pulse starts travelling up the pipe, if the other end of the pipe is open

(C) a low pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

(D) a high-pressure pulse starts travelling up the pipe, if the other end of the pipe is closed

Ans. (B,D)

Section-III : Integer Answer Type

This section contains **5 questions**. The answer to each question is **a single digit Integer**, ranging from **0 to 9** (both inclusive)

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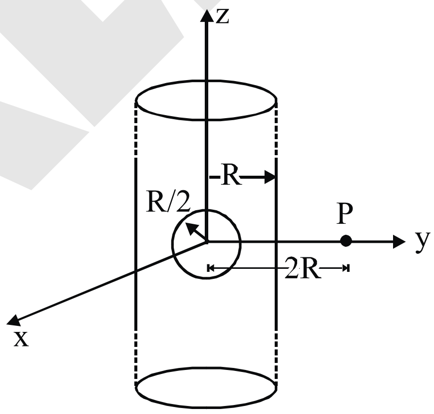
16. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius R/2 with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance 2R from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



Ans. 6

Sol. Electric field due to cylinder = $\frac{2K\lambda}{2R}$

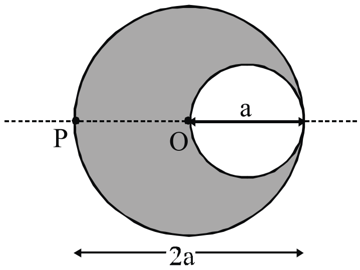
Electric field due to sphere = $\frac{KQ}{(2R)^2}$ [where $K = \frac{1}{4\pi \in_0}$]

From principle of superposition.

$$E = \frac{2K(\lambda)}{2R} - \frac{K(Q)}{(2R)^2} = \frac{K\left(\rho\pi R^2\right)}{R} - \frac{K\left(\rho \times \frac{4\pi}{3}\left(\frac{R^3}{8}\right)\right)}{4R^2}$$

$$= K\left(\rho\pi R\right) - \frac{K(\rho)(\pi R)}{24} = \frac{23\rho\pi R}{4\pi\varepsilon_0 \times 24} = \frac{23\rho R}{16 \times 6\varepsilon_0} \Rightarrow k = 6$$

17. A cylindrical cavity of diameter a exists inside a cylinder of diameter 2a as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



Ans. 5

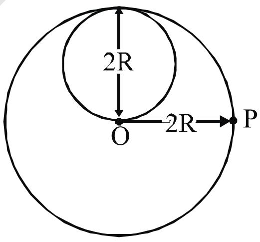
Sol. Magnetic field on the surface of cylinder is given by $\frac{\mu_0 Ja}{2}$

Magnetic field at radial distance r outside is given by $\frac{\mu_0 ja^2}{2r}$

From principle of superposition we get

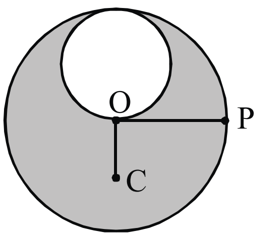
$$\vec{B} = \frac{\mu_0 J(a)}{2} - \frac{\mu_0 J}{2\pi\left(\frac{3a}{2}\right)} \times \frac{\pi a^2}{4} = \frac{5\mu_0 aJ}{12} \Rightarrow N = 5$$

18. A lamina is made by removing a small disc of diameter 2R from a bigger disc of uniform mass density and radius 2R, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_0 and I_P respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



Ans. 3

Sol.



Let mass of compelte disc of radius 2R is 4M & mass of removed part is M.

$$I_O = \frac{4M(2R)^2}{2} - \left[\frac{MR^2}{2} + MR^2\right] = \frac{13}{2}MR^2$$

$$I_P = \left[\frac{4M(2R)^2}{2} + 4M(2R)^2\right] - \left[\frac{M(R)^2}{2} + M(5R)^2\right] = \frac{37}{2}MR^2$$

$$\frac{I_P}{I_o} = \frac{37}{13} \approx 3$$

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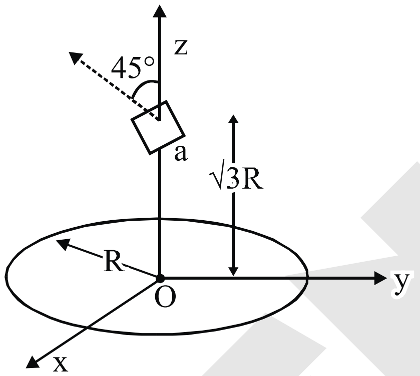
19. A circular wire loop of radius R is placed in the x-y plane centred at the origin O. A square loop of side a (a<<R) having two turns is placed with its center at z = $\sqrt{3}$ R along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is



Ans. 7

Sol. Magnetic field at a distance x along axis of a circular coil is given by $B(x) = \frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}$

Flux linked with N coils of square loop inclined at θ = 45° with vertical is given by

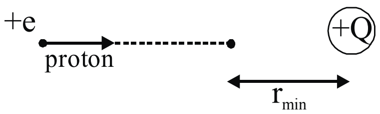
$$\phi(x) = \left[\frac{\mu_0 i R^2}{2(R^2 + x^2)^{3/2}}\right] N(a^2)(\cos 45°)$$

As $P = Mi$ so M = $\frac{\mu_0 R^2 (2)}{2\left[R^2 + (\sqrt{3}R)^2\right]^{3/2}} \left(\frac{a^2}{\sqrt{2}}\right) = \frac{\mu_0 a^2}{2^{7/2} R}$ thus p = 7

20. A proton is fired from very far away towards a nucleus with charge Q = 120 e, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is (take the proton mass, m_P = (5/3) × 10^{-27} kg; h/e = 4.2 × 10^{-15} J.s/C; $\frac{1}{4\pi\varepsilon_0}$ = 9 × 10^9 m/F; 1 fm = 10^{-15} m)

Ans. 7

Sol.



From work energy theorem we get $W_{electric} = \Delta KE$; $\frac{(Qq)}{4\pi \in_0 r_{min}} = \frac{1}{2} m v^2$

and de-Broglie wavelength is given by

$$\lambda = \frac{h}{p} = \frac{h}{\sqrt{2mK}} = \frac{h}{\sqrt{2m\left(\frac{Qq}{4\pi \in_0 r_{min}}\right)}} = \frac{h}{\sqrt{\frac{2m(120e)(e)}{4\pi \in_0 r_{min}}}}$$

on substituting numerical values we get λ = 7× 10^{-15} m

PART-II : CHEMISTRY

SECTION – I : Single correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. As per IUPAC nomenclature, the name of the complex $[Co(H_2O)_4(NH_3)_2]Cl_3$ is :

(A) Tetraaquadiaminecobalt(III) chloride

(B) Tetraaquadiamminecobalt(III) chloride

(C) Diaminetetraaquacobalt(III) chloride

(D) Diamminetetraaquacobalt(III) chloride

Ans. **(D)**

Sol. $[Co(H_2O)_4(NH_3)_2]Cl_3$

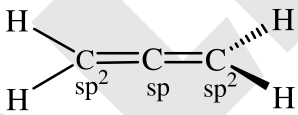
Diamminetetraaquacobalt(III) chloride

22. In allene (C_3H_4), the type(s) of hybridisation of the carbon atoms is (are)

(A) sp and sp^3 (B) sp and sp^2 (C) only sp^2 (D) sp^2 and sp^3

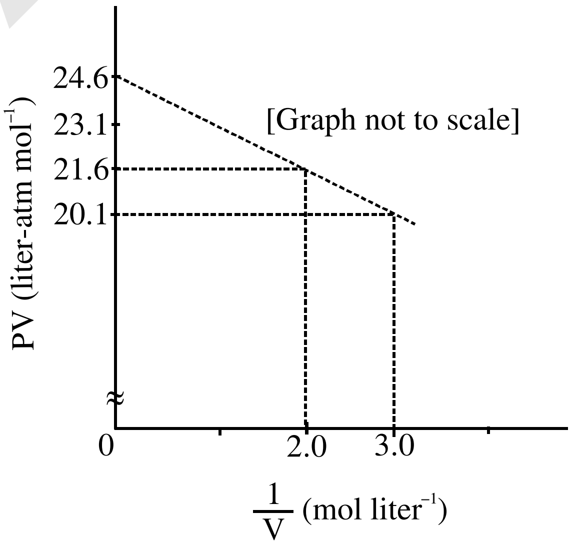
Ans. **(B)**

Sol. Allene structure



The type of hybridisation of carbon atoms are sp & sp^2

23. For one mole of a van der Waals gas when b = 0 and T=300K, the PV vs. 1/V plot is shown below. The value of the van der Waals constant a (atm. liter^2 mol^{-2}) is



(A) 1.0 (B) 4.5 (C) 1.5 (D) 3.0

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Ans.(C)

Sol. $\left(P+\dfrac{a}{V^2}\right)(V-0)=RT$ as $(n = 1)$

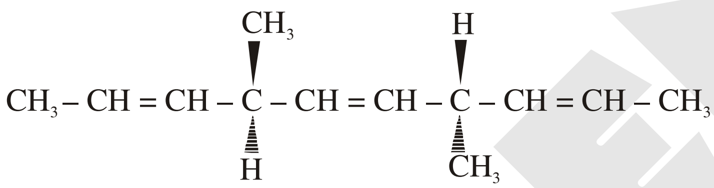
$PV = RT - \dfrac{a}{V}$

On comparing slope from graph

$-a = \dfrac{20.1-21.6}{3-2}$

$-a = \dfrac{-1.5}{1} = a = 1.5$

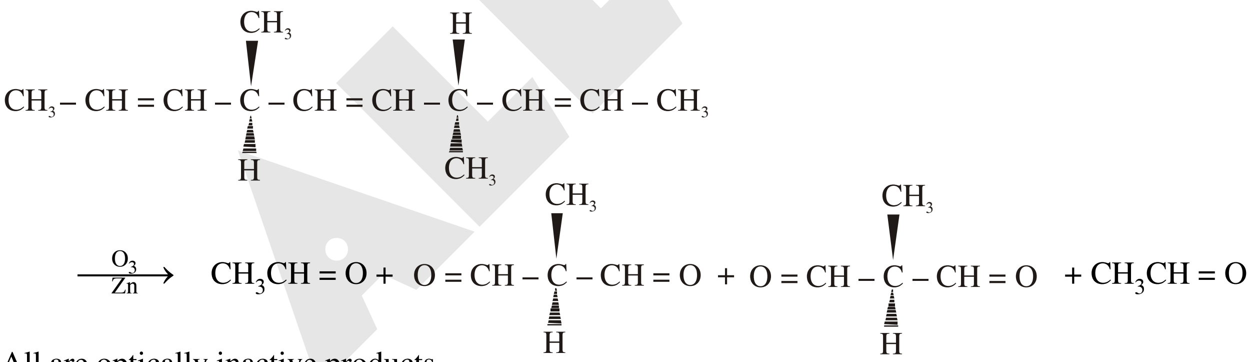
24. The number of optically active products obtained from the complete ozonolysis of the given compound is :



(A) 0 (B) 1 (C) 2 (D) 4

Ans. (A)

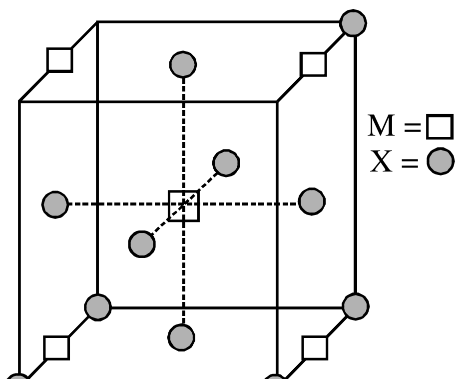
Sol.



All are optically inactive products

No. of optically active product = 0

25. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is :



(A) MX (B) MX_2 (C) M_2X (D) M_5X_{14}

Ans.(B)

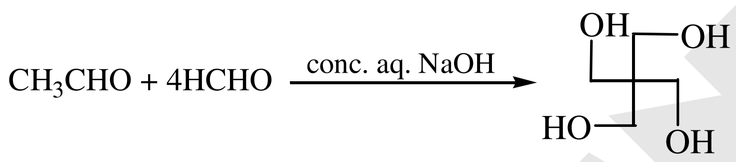
Sol. 'X' is forming fcc.

$\Rightarrow$ Effective "X" atoms per unit cell = 4

Effective "M" atoms per unit cell = $\frac{1}{4}\times 4+1=2$

Formula = MX_2.

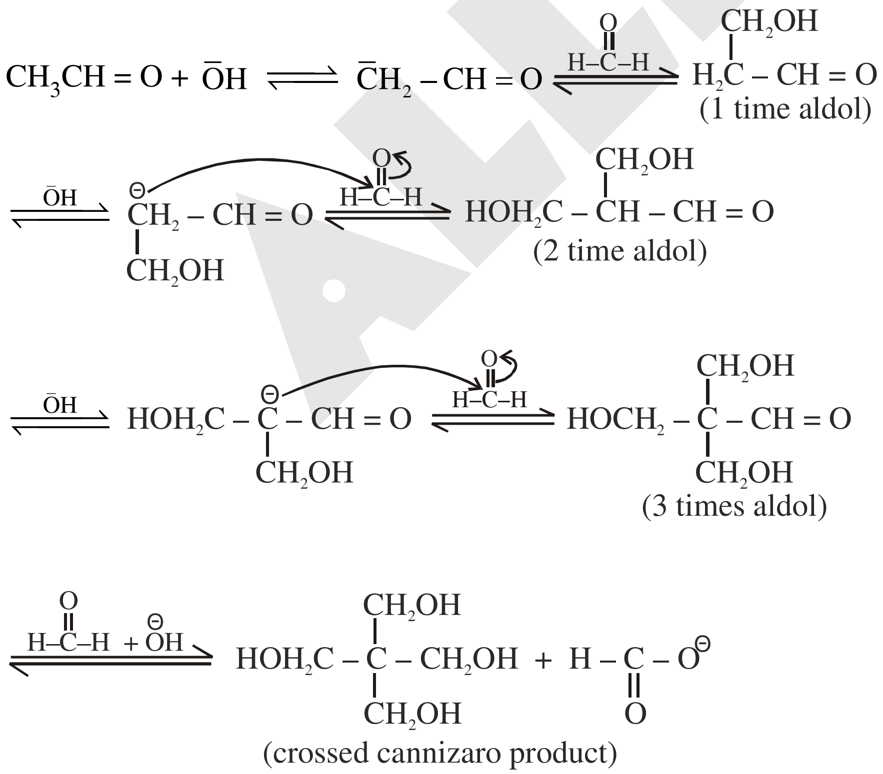
26. The number of aldol reaction(s) that occurs in the given transformation is



(A) 1 (B) 2 (C) 3 (D) 4

Ans. (C)

Sol.



the carboxyl functional group ($-\underset{\underset{O}{\|}}{C}-OH$) is presen in.

27. The colour of light absorbed by an aqueous solution of $CuSO_4$ is -

(A) orange-red (B) blue-green (C) yellow (D) violet

Ans. (A)

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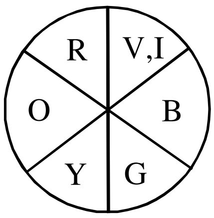
Sol.

$$\underrightarrow{\text{V I B G Y O R}}$$
$$\lambda \uparrow \nu \downarrow E \downarrow$$

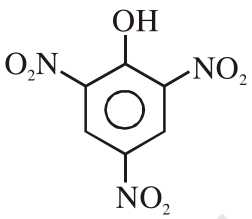


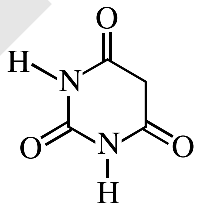
The colour of aqueous solution of $CuSO_4$ is blue so colour of light absorbed by an aqueous solution of $CuSO_4$ is orange-red.

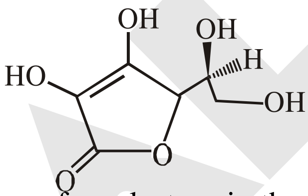
28. The carboxyl functional group (–COOH) is present in -

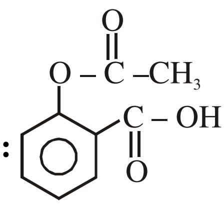
(A) picric acid (B) barbituric acid (C) ascorbic acid (D) aspirin

Ans. (D)

Sol. **Picric acid :** 

Barbituric acid : 

Ascorbic acid : 

Aspirin :  (Acetyl salicylic acid)

29. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]

(A) $\frac{h^2}{4\pi^2ma_0^2}$ (B) $\frac{h^2}{16\pi^2ma_0^2}$ (C) $\frac{h^2}{32\pi^2ma_0^2}$ (D) $\frac{h^2}{32\pi^2ma_0^2}$

Ans. (C)

Sol. $v = \frac{nh}{2\pi mr}$

K.E. $= \frac{1}{2}m.\frac{n^2h^2}{4\pi^2m^2r^2}$

for second orbit

K.E. $= \frac{4h^2}{8\pi^2m^2r_2^2}$

since $r_1 = a_0$

$r_2 = 4a_0$

K.E. $= \frac{h^2}{32\pi^2ma_0^2}$

30. Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen-

(A) HNO_3, NO, NH_4Cl, N_2 (B) HNO_3, NO, N_2, NH_4Cl

(C) HNO_3, NH_4Cl, NO, N_2 (D) NO, HNO_3, NH_4Cl, N_2

Ans. (B)

Sol. HNO_3 — O.S $1 + X + 3(-2) = 0$; $1 + X = 6$; $X = +5$

NO — O.S $X + (-2) = 0$; $X = +2$

N_2 — O.S. = zero

$NH_4^+Cl^-$ — O.S. $X + 4 = 1$; $X = -3$

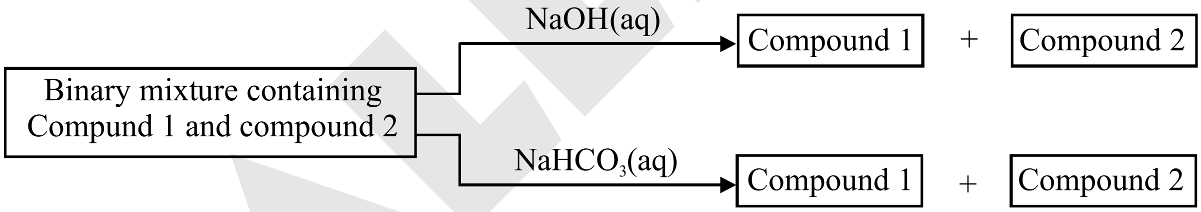
so decreasing order is HNO_3 , NO, N_2 NH_4Cl

SECTION-II : Multiple Correct Answser(s) Type

This section contains **5 multiple choice question**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

---

31. Identify the binary mixtures (s) that cna be separated into the individual compounds, by differential extraction, as shown in the given scheme -



(A) C_6H_5OH and C_6H_5COOH (B) C_6H_5COOH and $C_6H_5CH_2OH$

(C) $C_6H_5CH_2OH$ and C_6H_5OH (D) $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

Ans (B, D)

Sol. Acidic structure order of some organic compounds

$C_6H_5COOH, C_6H_5CH_2CO_2H > H_2CO_3 > C_6H_5OH > H_2O > C_6H_5CH_2OH$

(A) C_6H_5OH , C_6H_5COOH $\longrightarrow$ separated by $NaHCO_3$ only but not by NaOH

(B) C_6H_5COOH , $C_6H_5CH_2OH$ $\longrightarrow$ separated by $NaHCO_3$ & NaOH both.

(C) $C_6H_5CH_2OH$, C_6H_5OH $\longrightarrow$ separated by NaOH only but not by $NaHCO_3$

(D) $C_6H_5CH_2OH$, $C_6H_5CH_2COOH$ $\longrightarrow$ separated by $NaHCO_3$ & NaOH both.

32. Choose the correct reason(s) for the stability of the **lyophobic** colloidal particle.

(A) Preferential adsorption of ions on their surface from the solution

(B) Preferential adsorption of solvent on their surface from the solution

(C) Attraction between different particles having opposite charges on their surface

(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles

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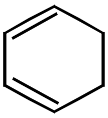
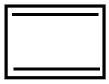
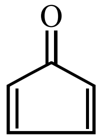
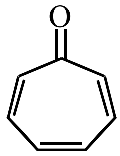
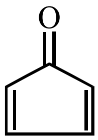
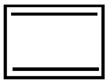
Ans. (A,D)

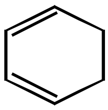
33. Which of the following molecules, in pure from , is (are) **unstable** at room temperature ?

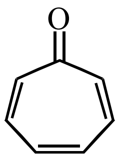
(A)  (B)  (C)  (D) 

Ans (B, C)

Sol.  

Both are antiaromatic. So are unstable.

 Non aromatic stable at room temperature.

 Aromatic, stable at room temperature.

34. Which of the following hydrogen halides react(s) with $AgNO_3$(aq) to give a precipitate that dissolves in $Na_2S_2O_3$(aq):

(A) HCl (B) HF

(C) HBr (D) HI

Ans. (A, C, D)

Sol. $AgNO_3 + HCl \longrightarrow AgCl\downarrow$ (white ppt.)

$AgCl\downarrow + Na_2S_2O_3 \longrightarrow [Ag(S_2O_3)_2]^{3-}$
soluble complex

$AgNO_3 + HBr \longrightarrow AgBr\downarrow$ pale yellow ppt.

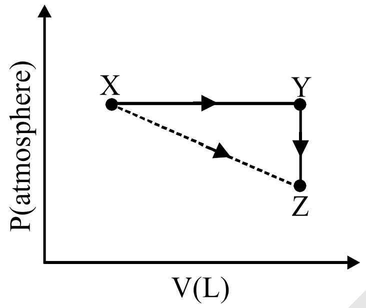
$AgBr\downarrow + Na_2S_2O_3 \longrightarrow [Ag(S_2O_3)_2]^{3-}$
soluble complex

$AgNO_3 + HI \longrightarrow AgI\downarrow$ yellow ppt.

$AgI\downarrow + Na_2S_2O_3 \longrightarrow [Ag(S_2O_3)_2]^{3-}$
soluble complex

$AgNO_3 + HF \longrightarrow$ No precipitation

35. For an ideal gas, consider only P-V work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice(s) is (are) correct ? [take ΔS as change in entropy and w as work done]



(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$ (B) $W_{x\to z} = W_{x\to y} + W_{y\to z}$

(C) $W_{x\to y\to z} = W_{x\to y}$ (D) $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

Ans.(A,C)

Sol. For option A : As entropy is state function.

$\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$

For option C : since $W_{y\to z} = 0$ as (V constant)

$W_{x\to y\to z} = W_{x\to y} + W_{y\to z}$

$\Rightarrow \quad W_{x\to y\to z} = W_{x\to y}$

SECTION–III : Integer Answer Type

This Section contains **5 questions**. The answer to each question is a **single digit integer,** ranging from 0 to 9 (both inclusive)

---

36. The substitutes $\mathbf{R_1}$ and $\mathbf{R_2}$ for nine peptides are listed in the table given below. How many of these peptides are positively changed at pH = 7.0 ?

$$H_3\overset{\oplus}{N}-\underset{\underset{H}{|}}{CH}-CO-NH-\underset{\underset{R_1}{|}}{CH}-CO-NH-\underset{\underset{R_1}{|}}{CH}-CO-NH-\underset{\underset{H}{|}}{CH}-CO\overset{\ominus}{O}$$

Peptide	$\mathbf{R_1}$	$\mathbf{R_2}$
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$	CH_3

Ans. 4

Sol.

(I)	H	H	Neutral peptide	0
(II)	H	CH_3	Neutral peptide	0
(III)	CH_2–COOH	H	Acidic peptide	(–)ve charge
(IV)	CH_2–$CONH_2$	$(CH_2)_4NH_2$	Basic peptide	(+)ve charge
(V)	CH_2–$CONH_2$	CH_2CONH_2	Neutral peptide	0
(VI)	$(CH_2)_4$–NH_2	$(CH_2)_4$–NH_2	Basic peptide	(+)ve charge
(VII)	CH_2–COOH	CH_2CONH_2	Acidic peptide	(–)ve charge
(VIII)	CH_2OH	$(CH_2)NH_2$	Basic peptide	(+)ve charge
(IX)	$(CH_2)_4NH_2$	CH_3	Basic peptide	(+)ve charge

Hence IV, VI, VIII & IX

$-CH_2OH$, amides $-CH_2\overset{\overset{\displaystyle O}{\|}}{C}-NH_2$ are neutral groups.

37. The periodic table consists of 18 groups. An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element **X** as shown below. To which group , element **X** belongs in the periodic table ?

$${}^{63}_{29}Cu + {}^{1}_{1}H \rightarrow 6\,{}^{1}_{0}n + \alpha + 2\,{}^{1}_{1}H + \mathbf{X}$$

Ans. 8

Sol. $${}^{63}_{29}Cu + {}_{1}H^{1} \rightarrow 6\,{}^{1}_{0}n + {}^{4}_{2}He + 2\,{}^{1}_{1}H + {}_{26}X^{52}$$

$\Rightarrow$ X has atomic number = 26 = atomic number of iron

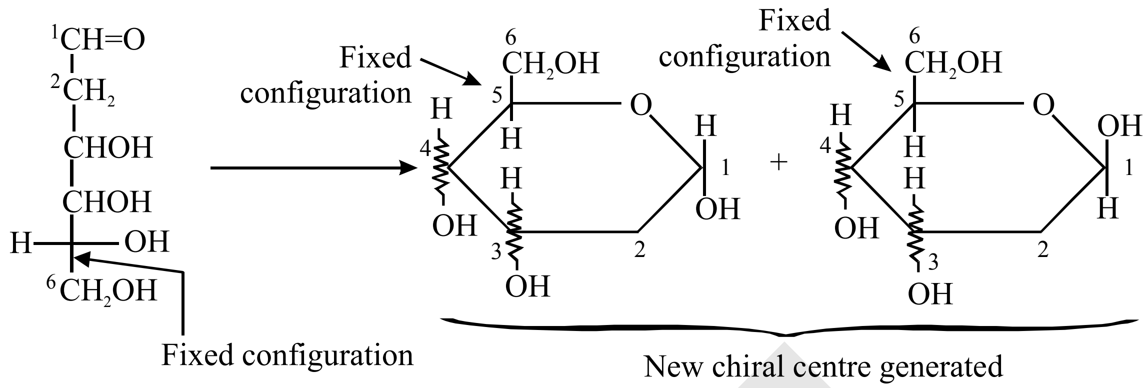
$\Rightarrow$ group number = 8

38. When the following aldohexose exists in its d-configuration , the total number of stereoisomers in its pyranose form is -

CHO
|
CH_2
|
CHOH
|
CHOH
|
CHOH
|
CH_2OH

Ans. 8

Sol.



D-configuration (Four stereoisomers w.r.t. to C_3 & C_4 chiral centres)

New chiral centre generated at C1 carbon in pyranose form hence total 8 pyranose form w.r.t. to C_3 & C_4 variable configuration centres

39. 29.2 % (w/w) HCl stock solution has a density of 1.25 g mL^{-1}. The molecular weight of HCl is 36.5g mol^{-1}. The volume (mL) of stock solution required to prepare a 200 mL solution of 0.4 M HCl is.

Ans. 8

Sol. Molarity of given solution $= \dfrac{\left(\dfrac{w}{w}\%\right)\times \text{density}\times 10}{\text{mol. wt.}} = \dfrac{29.2\times 1.25\times 10}{36.5} = 10$

during dilution, $M_1V_1 = M_2V_2$

$10 \times V_1 = 0.4 \times 200$ $\boxed{V_1 = 8\,\text{mL}}$

40. An organic compound undergoes first-order decomposition . The time taken for its decomposition to 1/8 and 1/10 of its initial concentration are $t_{1/8}$ and $t_{1/10}$ respectively. What is the value of $\dfrac{[t_{1/8}]}{t_{1/10}}\times 10$? (take $\log_{10}2 = 0.3$)

Ans. 9

Sol. $\because$ $t = \dfrac{2.303}{k}\log\dfrac{a}{(a-x)}$

$$t_{1/8} = \frac{2.303}{k}\log\frac{a}{a/8}$$

$$t_{1/8} = \frac{2.303}{k}\log 8$$

$$t_{1/10} = \frac{2.303}{k}\log\frac{a}{a/10}$$

$$t_{1/10} = \frac{2.303}{k}\log 10$$

$$\frac{t_{1/8}}{t_{1/10}} = \frac{\log 8}{\log 10} = 3\log 2 = 3\times 0.3 = 0.9$$

$\therefore$ $\dfrac{t_{1/8}}{t_{1/10}}\times 10 = 0.9\times 10 = 9$

PART - III : MATHEMATICS

SECTION–I : Single Correct Answer Type

This section contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

41. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is-

(A) $20(x^2 + y^2) - 36x + 45y = 0$
(B) $20(x^2 + y^2) + 36x - 45y = 0$
(C) $36(x^2 + y^2) - 20x + 45y = 0$
(D) $36(x^2 + y^2) + 20x - 45y = 0$

Ans. (A)

Sol. Let mid point be (h, k),

then chord of contact :

$hx + ky = h^2 + k^2$ ....(i)

Let any point on the line $4x - 5y = 20$ be $\left(x_1, \frac{4x_1 - 20}{5}\right)$

$\therefore$ Chord of contact :

$5x_1x + (4x_1 - 20)y = 45$ ....(ii)

(i) and (ii) are same

$$\therefore \quad \frac{5x_1}{h} = \frac{4x_1 - 20}{k} = \frac{45}{h^2 + k^2}$$

$$\Rightarrow \quad x_1 = \frac{9h}{h^2 + k^2} \text{ and } x_1 = \frac{45k + 20(h^2 + k^2)}{4(h^2 + k^2)}$$

$$\Rightarrow \quad \frac{9h}{h^2 + k^2} = \frac{45k + 20(h^2 + k^2)}{4(h^2 + k^2)}$$

$\Rightarrow$ $20(h^2 + k^2) - 36h + 45k = 0$

$\therefore$ Locus is $20(x^2 + y^2) - 36x + 45y = 0$

42. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is -

(A) 75 (B) 150 (C) 210 (D) 243

Ans. (B)

Sol. Balls can be distributed as 1, 1, 3 or 1, 2, 2 to each person.

When 1, 1, 3 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!\,1!\,3!} \cdot \frac{1}{2!} . 3! = 60$$

When 1, 2, 2 balls are distributed to each person, then total number of ways :

$$= \frac{5!}{1!\,2!\,2!} \cdot \frac{1}{2!} . 3! = 90$$

$\therefore$ total = 60 + 90 = 150

43. Let $f(x)=\begin{cases} x^2\left|\cos\frac{\pi}{x}\right| & , \quad x\neq 0 \\ 0 & , \quad x=0 \end{cases}, x\in \mathrm{IR}$, then f is -

(A) differentiable both at x = 0 and at x = 2

(B) differentiable at x = 0 but not differentiable at x = 2

(C) not differentiable at x = 0 but differentiable at x = 2

(D) differentiable neither at x = 0 nor at x = 2

Ans. (B)

Sol. At x = 0

$$\text{R.H.D} = \lim_{h\to 0}\frac{(0+h)-(0)}{h} = \lim_{h\to 0}\frac{h^2\left|\cos\frac{\pi}{h}\right|-0}{h}$$

$$=\lim_{h\to 0} h\left|\cos\frac{\pi}{h}\right| = 0\times\cos(\infty)$$

= 0 × finite = 0

$$\text{LHD} : =\lim_{h\to 0}\frac{f(0-h)-f(0)}{-h}$$

$$=\lim_{h\to 0}\frac{h^2\cos\left(\frac{\pi}{-h}\right)-0}{-h} = \lim_{h\to 0} -h\cos\left(\frac{\pi}{h}\right)$$

= 0

∵ LHD = RHD at x = 0

⇒ $f(x)$ is differentiable at x = 0

At x = 2

$$\text{RHD} = \lim_{h\to 0}\frac{f(2+h)-f(2)}{h}$$

$$=\lim_{h\to 0}\frac{(2+h)^2.\cos\left(\frac{\pi}{2+h}\right)-0}{h}$$

$$=4\lim_{h\to 0}\frac{\cos\left(\frac{\pi}{2+h}\right)}{h}$$

$$=-4\lim_{h\to 0}\frac{\sin\left(\frac{\pi}{2+h}\right).\left(-\frac{\pi}{(2+h)^2}\right)}{1} = \pi$$

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LHD : $\lim_{h\to 0}\frac{f(2-h)-f(2)}{-h}$

$$=\lim_{h\to 0}\frac{(2-h)^2\left(-\cos\left(\frac{\pi}{2-h}\right)\right)}{-h}$$

$$=\lim_{h\to 0}\frac{4\left(\sin\frac{\pi}{(2-h)}\right)\left(\frac{\pi}{(2-h)^2}\right)}{-1} = -\pi.$$

LHD $\neq$ RHD at x = 2

$\therefore$ Not differentiable at x = 2.

44. The function $f : [0,3] \to [1,29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is :

(A) one-one and onto
(B) onto but not one-one
(C) one-one but not onto
(D) neither one-one nor onto

Ans. (B)

Sol. $f(x) = 2x^3 - 15x^2 + 36x + 1$

$\Rightarrow f'(x) = 6(x^2 - 5x + 6)$

$= 6(x - 2)(x - 3)$

$\because f(x)$ is non monotonic in $x \in [0,3]$

$\Rightarrow f(x)$ is not one-one

$f(x)$ is increasing in $x \in [0,2)$ and decreasing in $x \in (2,3]$

$f(0) = 1,\ f(2) = 29\ \&\ f(3) = 28$

$\therefore$ Range of $f(x)$ is [1,29]

$\Rightarrow f(x)$ is onto.

45. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then -

(A) a = 1, b = 4
(B) a = 1, b = –4
(C) a = 2, b = –3
(D) a = 2, b = 3

Ans. (B)

Sol. $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$

$$\Rightarrow \lim_{x\to\infty}\left(\frac{(1-a)x^2+x(1-a-b)+1-b}{x+1}\right)=4$$

$$\Rightarrow \lim_{x\to\infty} \frac{\left((1-a).x+1-a-b+\left(\frac{1-b}{x}\right)\right)}{1+\frac{1}{x}} = 4$$

for limit to exist finitely

$1 - a = 0$ and $1 - a - b = 4$

$\Rightarrow a = 1$ and $b = -4$.

46. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a **cannot** take the value -

(A) -1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Ans. (D)

Sol. $z^2 + z + 1 - a = 0$

$\because$ z is imaginary $\Rightarrow D < 0$

$1 - 4(1 - a) < 0$

$4a < 3$

$a < \frac{3}{4}$.

Aliter : $a = z^2 + z + 1$

$\because \quad a = \bar{a}$ (given a is real)

$\therefore \quad z^2 + z = \bar{z}^2 + \bar{z}$

$\Rightarrow \quad z^2 - \bar{z}^2 = \bar{z} - z$

$\Rightarrow \quad z + \bar{z} = -1$ ($\because$ Im(z) is non zero)

$\Rightarrow \quad \text{Re}(z) = -\frac{1}{2}$

$\therefore$ z can be taken as $-\frac{1}{2} + iy$

where $y \in R$

$\therefore \quad a = \left(-\frac{1}{2}+iy\right)^2 + \left(\frac{-1}{2}+iy\right)+1$

$\Rightarrow \quad a = \frac{1}{4} - \frac{1}{2} + 1 - iy + iy - y^2$

$\Rightarrow \quad a = \frac{3}{4} - y^2 \Rightarrow a < \frac{3}{4}$

$\therefore \quad a \neq \frac{3}{4}$

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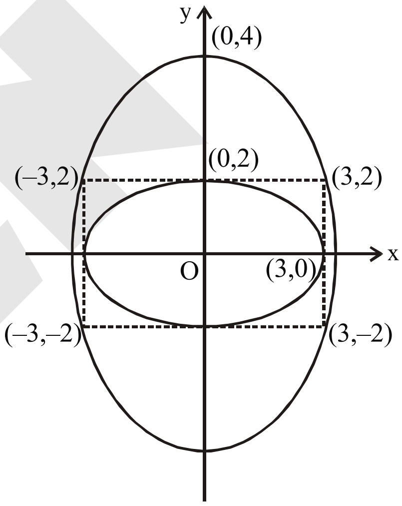
47. The ellipse $E_1 : \frac{x^2}{9}+\frac{x^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0,4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is -

(A) $\frac{\sqrt{2}}{2}$ (B) $\frac{\sqrt{3}}{2}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Ans. **(C)**

Sol. Let equation of E_2 be



$\frac{x^2}{a^2}+\frac{y^2}{16}=1$ ($\because$ E_2 passes through (0, 4))

$\because$ E_2 passes through (3,2)

$\therefore$ $\frac{9}{a^2}+\frac{4}{16}=1$

$\Rightarrow$ $a^2 = 12$

$\therefore$ $e^2 = 1-\frac{a^2}{16}=1-\frac{3}{4} \Rightarrow e=\frac{1}{2}$

48. Let P =[a_{ij}] be a 3 × 3 matrix and let Q = [b_{ij}], where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is -

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

Ans. **(D)**

Sol. $|Q| = \begin{vmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{vmatrix}$

$\Rightarrow$ $|Q| = 2^2.2^3.2^4.\begin{vmatrix} a_{11} & 2a_{12} & 2^2 a_{13} \\ a_{21} & 2a_{22} & 2^2 a_{23} \\ a_{31} & 2a_{32} & 2^2 a_{33} \end{vmatrix}$

$= 2^2.2^3.2^4|P|.2^3$

$= 2^2.2^3.2^4.2.2^3 = 2^{13}$

49. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}}dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

Ans. (C)

Sol. Let $I = \int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}}dx$

$$= \int \frac{\sec x(\sec x + \tan x)\sec x}{(\sec x + \tan x)^{11/2}}dx$$

Put secx + tanx = t

$\Rightarrow$ (secx tanx + sec^{2}x) dx = dt

Also $\because$ sec^{2}x – tan^{2}x = 1

$\Rightarrow$ secx – tanx = $\frac{1}{t}$

$\therefore$ secx = $\frac{1}{2}\left(t + \frac{1}{t}\right)$

$$\therefore \quad I = \frac{1}{2}\int \frac{\left(1 + \frac{1}{t}\right)dt}{t^{11/2}}$$

$$\Rightarrow \quad I = \frac{1}{2}.\int \left(t^{-9/2} + t^{-13/2}\right)dt \Rightarrow I = \frac{1}{2}\left(-\frac{2t^{-7/2}}{7} - \frac{2t^{-11/2}}{11}\right) + K$$

$$\Rightarrow \quad I = -\frac{1}{t^{11/2}}\left(\frac{1}{11} + \frac{t^2}{7}\right) + K$$

$$\Rightarrow \quad I = -\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$$

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50. The point P is the intersection of the straight line joining the points Q(2,3,5) and R(1,–1,4) with the plane 5x – 4y – z = 1. If S is the foot of the perpendicular drawn from the point T(2,1,4) to QR, then the length of the line segment PS is -

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Ans. **(A)**

Sol. Line QR :

$$\frac{x-2}{1}=\frac{y-3}{4}=\frac{z-5}{1}=\lambda$$

Any point on line QR :

$(\lambda + 2, 4\lambda + 3, \lambda + 5)$

$\therefore$ Point of intersection with plane :

$5\lambda + 10 - 16\lambda - 12 - \lambda - 5 = 1$

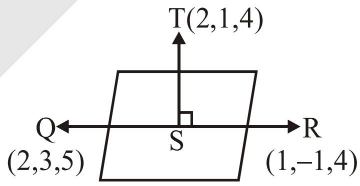
$\Rightarrow \lambda = -\frac{2}{3}$

$\therefore P\left(\frac{4}{3},\frac{1}{3},\frac{13}{3}\right)$

Also

$\because$ TQ = TR = $\sqrt{5}$

$\Rightarrow$ S is the mid-point of QR



$\Rightarrow S\left(\frac{3}{2}, 1, \frac{9}{2}\right)$

$\Rightarrow PS = \frac{1}{\sqrt{2}}$ units

SECTION–II : Multiple Correct Answer Type

This section contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE** are correct.

51. Let $\theta, \varphi \in [0,2\pi]$ be such that

$$2\cos\theta(1-\sin\varphi) = \sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi - 1,$$

$$\tan(2\pi-\theta) > 0 \text{ and } -1 < \sin\theta < -\frac{\sqrt{3}}{2}.$$

Then φ **cannot** satisfy-

(A) $0 < \varphi < \frac{\pi}{2}$ (B) $\frac{\pi}{2} < \varphi < \frac{4\pi}{3}$ (C) $\frac{4\pi}{3} < \varphi < \frac{3\pi}{2}$ (D) $\frac{3\pi}{2} < \varphi < 2\pi$

Ans. (A,C,D)

Sol. $\tan(2\pi - \theta) > 0$

$\Rightarrow$ $2\pi - \theta$ lies in I on III quadrant

$\Rightarrow$ $\because$ lies in II or IV Quadrant ....(i)

$\because$ $-1 < \sin\theta < -\frac{\sqrt{3}}{2} \Rightarrow \theta \in \left(\frac{4\pi}{3}, \frac{5\pi}{3}\right)$ ......(ii)

$\therefore$ By (i) and (ii) : $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right)$

Also, given $2\cos\theta(1-\sin\phi) = \sin^2\theta\left(\tan\frac{\theta}{2} + \cot\frac{\theta}{2}\right)\cos\phi - 1$

$\Rightarrow$ $2\cos\theta - 2\cos\theta\sin\phi = \dfrac{\sin^2\theta\cos\phi}{\sin\frac{\theta}{2}\cos\frac{\theta}{2}} - 1$

$\Rightarrow$ $2\cos\theta - 2\cos\theta\sin\phi = 2\sin\theta\cos\phi - 1$

$\Rightarrow$ $\sin(\theta+\phi) = \dfrac{1+2\cos\theta}{2}$

$\Rightarrow$ $\frac{1}{2} < \sin(\theta+\phi) < 1$

$\Rightarrow$ $\frac{\pi}{6} < \theta+\phi < \frac{5\pi}{6}$ or $\frac{13\pi}{6} < \theta+\phi < \frac{17\pi}{6}$

$\because$ $\theta \in \left(\frac{3\pi}{2}, \frac{5\pi}{3}\right) \Rightarrow \frac{13\pi}{6} < \theta+\phi < \frac{17\pi}{6}$

$\Rightarrow$ $\frac{\pi}{2} < \phi < \frac{4\pi}{3}$

52. Let S be the area of the region enclosed by $y = e^{-x^2}$, y = 0, x = 0, and x = 1. Then -

(A) $S \geq \frac{1}{e}$ (B) $S \geq 1 - \frac{1}{e}$ (C) $S \leq \frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (D) $S \leq \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

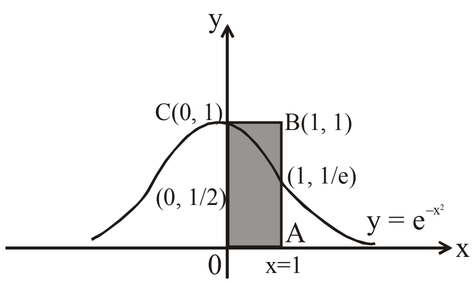
Ans. (A,B,D)

Sol. Area (OABC) = 1

Shaded area is S.

Clearly S < 1

and $\int_0^1 e^{-x^2}dx > \int_0^1 e^{-x}dx$



$\Rightarrow$ $S > 1 - \frac{1}{e}$ ($\therefore$ (B) is correct

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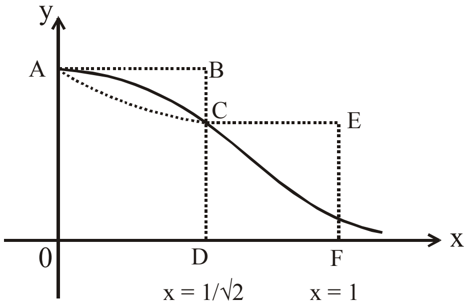
Again S ≥ Area (trapezium ACDO)



$\Rightarrow \quad S \geq \frac{1}{2}\left(1+\frac{1}{\sqrt{e}}\right)\left(\frac{1}{\sqrt{2}}\right)$

$\Rightarrow \quad S \geq \frac{1}{2\sqrt{2}}\left(1+\frac{1}{\sqrt{e}}\right)$

$\therefore$ C is wrong

Also S ≤ Sum of areas of rectangles ABDO and CEFD

$\Rightarrow \quad S \leq \frac{1}{\sqrt{2}}\times 1+\left(1-\frac{1}{\sqrt{2}}\right)\left(\frac{1}{\sqrt{e}}\right)$

$\Rightarrow \quad S \leq \frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

($\therefore$ (D) is correct)

53. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}, \frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of X_3 denotes respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true ?

(A) $P\left[X_1^c \mid X\right]=\frac{3}{16}$ (B) P[Exactly two engines of ship are functioning $\mid X]=\frac{7}{8}$

(C) $P[X \mid X_2]=\frac{5}{16}$ (D) $P[X \mid X_1]=\frac{7}{16}$

Ans. (B,D)

Sol. $P(X) = E_1E_2E_3 + E_1E_2\overline{E}_3 + E_1\overline{E}_2E_3 + \overline{E}_1E_2E_3$

$=\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{3}{4}+\frac{1}{2}\times\frac{3}{4}\times\frac{1}{4}+\frac{1}{2}\times\frac{1}{4}\times\frac{1}{4}$

$\Rightarrow P(X) = \frac{1}{4}$

$$P\left(\frac{X_1^c}{X}\right)=\frac{P\left(X_1^C \cap X\right)}{P(X)}=\frac{1/32}{1/4}=\frac{1}{8}$$

P(Exactly two engines are functioning |x)

$=\frac{7/32}{1/4}=\frac{7}{8}$

$$P\left(\frac{X}{X_2}\right)=\frac{P\left(X \cap X_2\right)}{P\left(X_2\right)}=\frac{5/32}{1/4}=\frac{5}{8}$$

$$P\left(\frac{X}{X_1}\right)=\frac{P\left(X \cap X_1\right)}{P\left(X_1\right)}=\frac{7/32}{1/2}=\frac{7}{16}$$

54. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line 2x – y = 1. The points of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}}, \frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}}, -\frac{1}{\sqrt{2}}\right)$

(C) $\left(3\sqrt{3}, -2\sqrt{2}\right)$ (D) $\left(-3\sqrt{3}, 2\sqrt{2}\right)$

Ans. (A,B)

Sol. Let parametric coordinates be P(3secθ, 2 tanθ)

Equation of tangent at point P will be

$$\frac{x\sec\theta}{3}-\frac{y\tan\theta}{2}=1$$

$\because$ tangent is parallel to 2x – y = 1

$$\Rightarrow \frac{2\sec\theta}{3\tan\theta}=2$$

$$\Rightarrow \sin\theta=\frac{1}{3}$$

$\therefore$ coordinates are

$$\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right) \text{ and } \left(-\frac{9}{2\sqrt{2}},-\frac{1}{2}\right)$$

55. If y(x) satisfies the differential equation y' – ytanx = 2x sec x and y(0) = 0, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$

(C) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Ans. (A,D)

Sol. $\frac{dy}{dx}-y\tan x=2x\sec x$

I.F. $=e^{\int -\tan x\,dx}=\cos x$

$\therefore$ Equation reduces to

$$y.\cos x=\int 2x.\sec x.\cos x dx$$

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$\Rightarrow$ y cosx = x^2 + C

$\because$ y(0) = 0 $\Rightarrow$ 0 = 0 + C

$\therefore$ y cosx = x^2

$\Rightarrow$ y(x) = x^2 secx

$\therefore\ y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{16}\sqrt{2}=\frac{\pi^2}{8\sqrt{2}}$ ($\therefore$ (A) is correct)

$y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}.2=\frac{2\pi^2}{9}$ ($\therefore$ (C) is wrong)

Also y'(x) = 2x secx + x^2 secx tanx

$\Rightarrow\ y'\left(\frac{\pi}{4}\right)=\frac{\pi}{2}.\sqrt{2}+\frac{\pi^2\sqrt{2}}{16}$ ($\therefore$ (B) is wrong)

and $y'\left(\frac{\pi}{3}\right)=2.\frac{\pi}{3}.2+\frac{\pi^2}{9}.2.\sqrt{3}$

$=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$ ($\therefore$ (D) is correct)

SECTION–III : Integer Answer Type

This section contains **5 questions**. The answer to each question is a **single digit integer,** ranging from 0 to 9 (both inclusive)

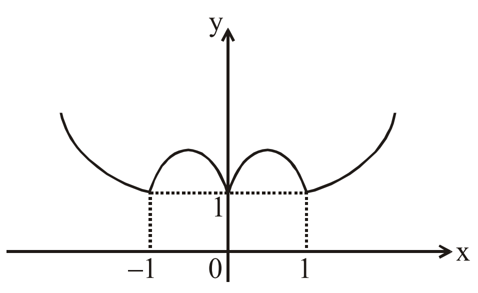
---

56. Let f : IR $\to$ IR be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Ans. 5

Sol. f(x) = |x| + |(x + 1) (x – 1)|

$$\Rightarrow\quad f(x)=\begin{cases} x^2-x-1 & x<-1\\ -x^2-x+1 & -1\le x<0\\ -x^2+x+1 & 0\le x<1\\ x^2+x-1 & x\ge 1\end{cases}$$



$\therefore$ f has 5 points where it attains either a local maximum or local minimum.

57. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}}}\right)$ is

Ans. 4

Sol. Let y = $\sqrt{4-\frac{1}{3\sqrt{2}}.\sqrt{4-\frac{1}{3\sqrt{2}}.\sqrt{4-\frac{1}{3\sqrt{2}}\ldots\ldots}}}$

$\Rightarrow \quad y^2 = 4 - \frac{1}{3\sqrt{2}}\,y \;\Rightarrow\; 3\sqrt{2}\,y^2 = 12\sqrt{2} - y$

$\Rightarrow \quad 3\sqrt{2}y^2 + y - 12\sqrt{2} = 0 \;\Rightarrow\; (3y - 4\sqrt{2})(\sqrt{2}y + 3) = 0$

$\Rightarrow \quad y = \frac{4\sqrt{2}}{3}\,;\; y = -\frac{3}{\sqrt{2}}$ (reject)

$\therefore \quad V = 6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}y\right)$

$= 6 + \log_{3/2}\left(\frac{1}{3\sqrt{2}}\cdot\frac{4\sqrt{2}}{3}\right) = 6 + \log_{3/2}\left(\frac{2}{3}\right)^2 = 6 - 2 = 4$

58. Let p(x) be a real polynomial of least degree which has a local maximum at x = 1 and a local minimum at x = 3. If p(1) = 6 and p(3) = 2, then p'(0) is

Ans. 9

Sol. Let $P'(x) = k(x-1)(x-3)$

$= k(x^2 - 4x + 3)$

$\Rightarrow P(x) = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + c$

$\because\; P(1) = 6$

$\Rightarrow \quad \frac{4k}{3} + c = 6 \qquad \ldots.(1)$

$P(3) = 2$

$\Rightarrow \quad c = 2 \qquad \ldots.(2)$

by (i) and (ii)

$k = 3$

$\therefore\; P'(x) = 3(x-1)(x-3)$

$\Rightarrow \quad P'(0) = 9$

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59. If $\vec{a}$, $\vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2+|\vec{b}-\vec{c}|^2+|\vec{c}-\vec{a}|^2=9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

Ans. 3

Sol. $\left|\vec{a}-\vec{b}\right|^2+\left|\vec{b}-\vec{c}\right|^2+\left|\vec{c}-\vec{a}\right|^2=9$

$\Rightarrow \quad 6-2\Sigma\vec{a}.\vec{b}=9$

$\Rightarrow \quad \Sigma\vec{a}.\vec{b}=-\dfrac{3}{2}$ ...(1)

$\left|\vec{a}+\vec{b}+\vec{c}\right|^2\geq 0$

$\Sigma\vec{a}^2+2\Sigma\vec{a}.\vec{b}\geq 0$

$\Sigma\vec{a}.\vec{b}\geq -\dfrac{3}{2}$

for equality $|\vec{a}+\vec{b}+\vec{c}|=0$

$\Rightarrow \vec{a}+\vec{b}+\vec{c}=0$

$5\vec{b}+5\vec{c}=-5\vec{a}$

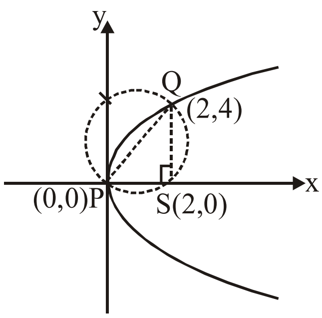
$2\vec{a}+5\vec{b}+5\vec{c}=-3\vec{a}$

$\left|2\vec{a}+5\vec{b}+5\vec{c}\right|=3\left|\vec{a}\right|=3$

60. Let S be the focus of the parabola $y^2=8x$ and let PQ be the common chord of the circle $x^2+y^2-2x-4y=0$ and the given parabola. The area of the triangle PQS is

Ans. 4

Sol. Focus of parabola S(2,0) points of intersection of given curves : (0,0) and (2,4).



Area (ΔPSQ) = $\dfrac{1}{2}.2.4=4$ sq. units