IIT-JEE 2012

EXAMINATION

(Held on 08-04-2012)

PAPER-2

(ANSWERS & SOLUTIONS)

Path to Success

ALLEN™

CAREER INSTITUTE

KOTA (RAJASTHAN)

Corporate Office

ALLEN CAREER INSTITUTE

"SANKALP", CP-6, INDRA VIHAR, KOTA-324005

PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003

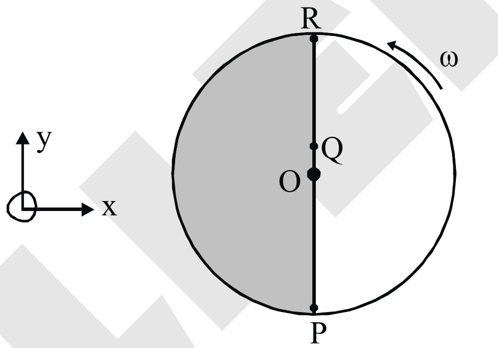
E-mail: info@allen.ac.in Website: www.allen.ac.in

PART - I : PHYSICS

SECTION–I : Single Correct Answer Type

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE is correct.**

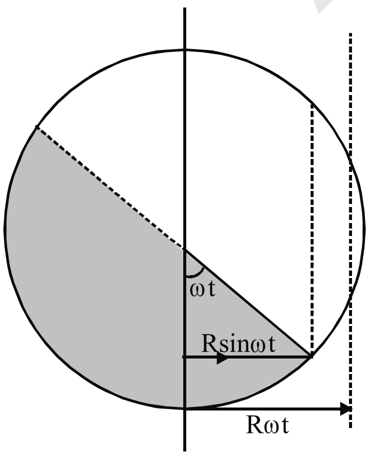
1. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre O. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles P and Q are simultaneously projected at an angle towards R. The velocity of projection is in the y-z plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then



(A) P lands in the shaded region and Q in the unshaded region.
(B) P lands in the unshaded region and Q in the shaded region.
(C) Both P and Q land in the unshaded region.
(D) Both P and Q land in the shaded region.

Ans. **(C)**

Sol.

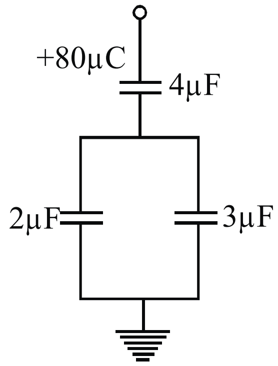


According to problem particle is to land on disc.
If we consider a time 't' then x component of displacement is $R\omega t$

$R\sin\omega t < R\omega t$

Thus particle P lands in unshaded region.
For Q, x-component is very small and y-component equal to P it will also land in unshaded region.

2. In the given circuit, a charge of +80 μC is given to the upper plate of the 4μF capacitor. Then in the steady state, the charge on the upper plate of the 3μF capacitor is



(A) + 32 μC (B) + 40μC (C) +48μC (D) + 80μC

Ans. (C)

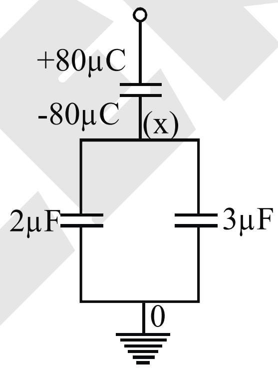
Sol. KCL at x

(x–0) 2 + (x–0) 3 – 80 = 0

⇒ 5x = 80

⇒ x =16 v

⇒ charge on 3μF = (x–0) (3μF) = 48 μC



3. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to required no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take R = 8.31 J/mol.K)

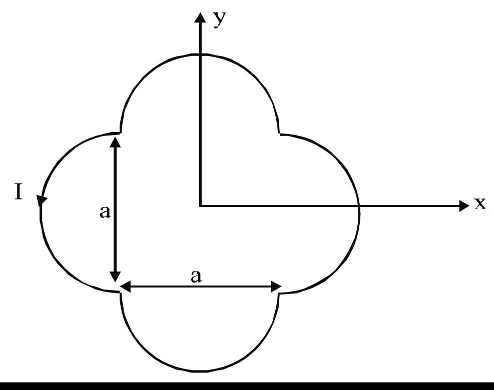
(A) 62 J (B) 104 J (C) 124 J (D) 208 J

Ans. (D)

Sol. $\Delta Q = \Delta U + \Delta W = \frac{f}{2} nR\Delta T + P\Delta V = \left(\frac{f}{2}+1\right) nR\Delta T$

$= \left(\frac{3}{2}+1\right)(2\times 8.31)(5) = 208 \text{ J}$

4. A loop carrying current I lies in the x-y plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is

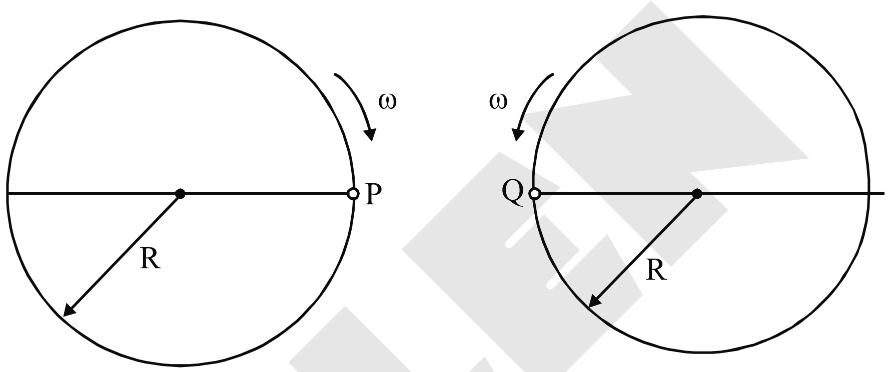


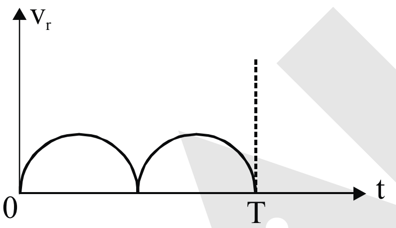
(A) $a^2 I\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (D) $(2\pi+1)a^2 I\hat{k}$

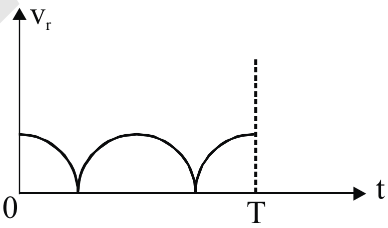
Ans. (B)

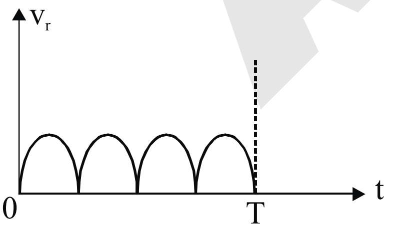
Sol. Magnetic moment $= iNA = (I)(1)\left[2\times\pi\left(\frac{a}{2}\right)^2 + a^2\right] = \left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

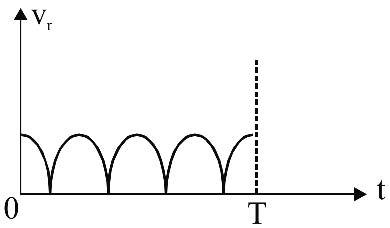
5. Two identical discs of same radius R are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time t =0, the points P and Q are facing each other as shown in the figure. The relative speed between the two points P and Q is v_r. In one time period (T) of rotation of the discs, v_r as a function of time is best represented by



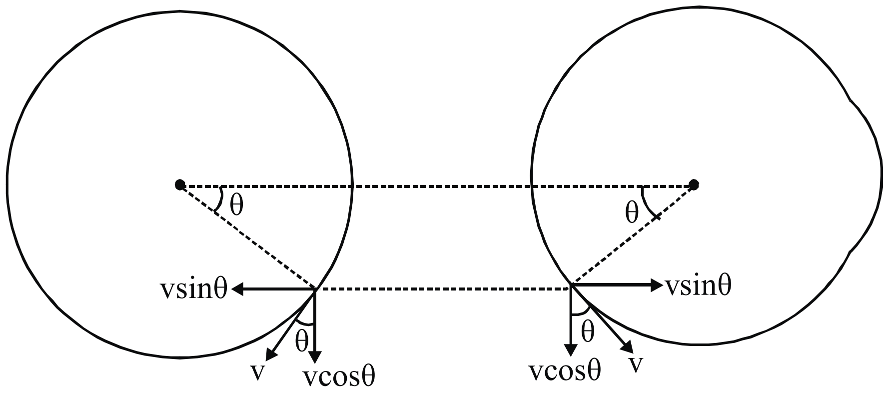
(A)


(B)


(C)


(D)


Ans. (A)

Sol.


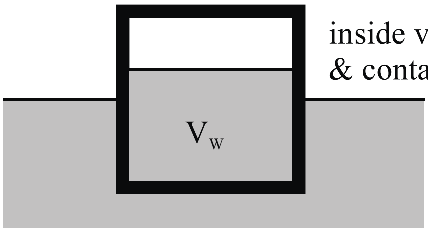
$V_{rel} = 2V\sin\theta = 2V\sin(\omega t)$

6. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If ρ_C is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(A) more than half-filled if ρ_C is less than 0.5
(B) more than half-filled if ρ_C is more than 1.0
(C) half-filled if ρ_C is more than 0.5
(D) less than half-filled if ρ_c is less than 0.5

Ans. (A)

Sol.



inside volume=V
& container material volume = V_c

For floating : $m_c g + m_w g = F_B$

$$\rho_C V_C g + 1 V_w g = 1\left(\frac{V}{2}+\frac{V_C}{2}\right) g \;\Rightarrow V_w = \frac{V}{2} + V_C\left(\frac{1}{2}-\rho_C\right)$$

if $\left(\rho_C < \frac{1}{2}\right)$, then $V_w > \frac{V}{2}$

7. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance column tube. When the first resonance occurs, the reading of the water level in the column is

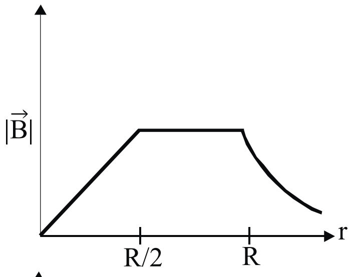
(A) 14.0 cm (B) 15.2 cm (C) 16.4 cm (D) 17.6 cm

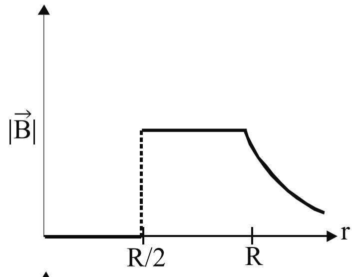
Ans. (B)

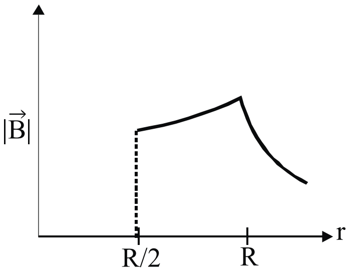
Sol. In resonance column experiment $\frac{\lambda}{4} = \ell_1 + e$ so, $\ell_1 = \frac{\lambda}{4} - e = \frac{v}{4f} - 0.3d$

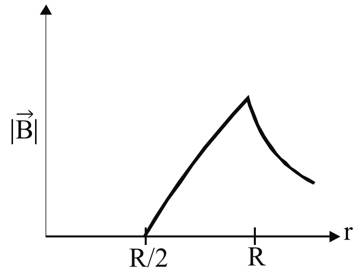
$$= \frac{336 \times 100 \text{ cm}}{4 \times 512} - (0.3)(4 \text{ cm}) = 16.4 - 1.2 = 15.2 \text{ cm}$$

8. An infinitely long hollow conducting cylinder with inner radius R/2 and outer radius R carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance r from the axis is best represented by

(A)


(B)


(C)


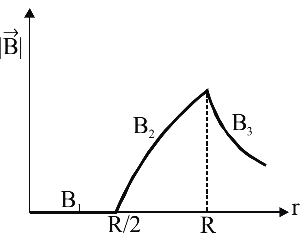
(D)


Ans. (D)

Sol. $B_1 = 0$ for $r < R/2$

$$B_2 = \left(\frac{\mu_0 Jr}{2} - \frac{\mu_0}{2}\frac{I}{r}\right) \text{ for } \frac{R}{2} \le r \le R =$$

$$B_3 = \frac{\mu_0 I}{2r} \qquad r \ge R$$



Section-II : Paragraph Type

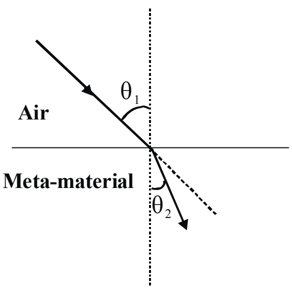
This section contains **6 multiple choice questions relating to three paragraphs with two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

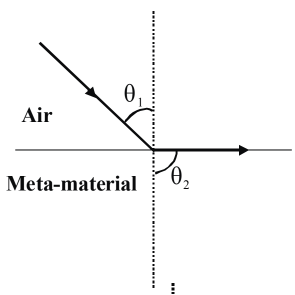
Paragraph for Questions 9 and 10

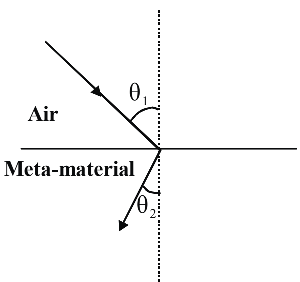
Most materials have the refractive index, n>1. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r \mu_r}$, where c is the speed of electromagnetic waves in vacuum, v its speed in the medium, ε_r and μ_r are the relative permittivity and permeability of the medium respectively.

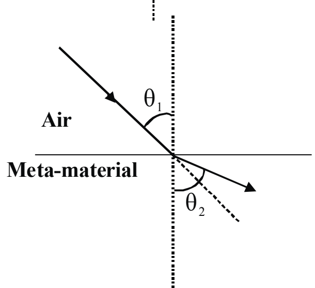
In normal materials, both ε_r and μ_r are positive, implying positive n for the medium. When both ε_r and μ_r are negative, one must choose the negative root of n. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behaviour, without violating any physical laws. Since n is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

9. For light incident from air on a meta-material, the appropriate ray diagram is

(A) 

(B) 

(C) 

(D) 

Ans. (C)

Sol. Snell's law : $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$ so, $\sin\theta_2 = \frac{n_1 \sin\theta_1}{n_2}$

For Air & meta material $\sin\theta_2 = \frac{1\sin\theta_1}{(-n)}$, θ_2 is (–ve)

10. Choose the correct statement.

(A) The speed of light in the meta-material is v = $c|n|$

(B) The speed of light in the meta-material is v = $\frac{c}{|n|}$

(C) The speed of light in the meta-material is v = c.

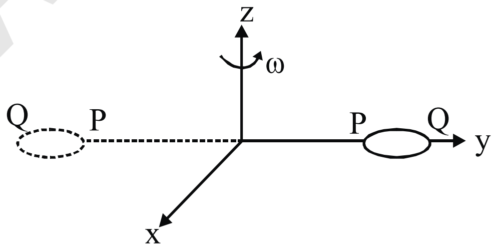
(D) The wavelength of the light in the meta-material (λ_m) is given by $\lambda_m = \lambda_{air}|n|$, where λ_{air} is the wavelength of the light in air.

Ans. (B)

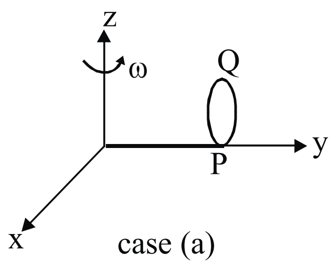
Sol. By definition of n : $V = \frac{C}{|n|}$

Paragraph for Questions 11 and 12

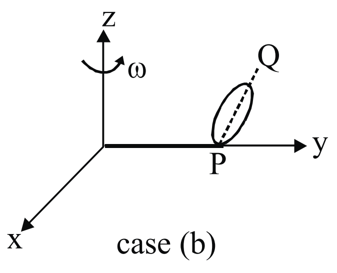
The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the z-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points P and Q). Both these motions have the same angular speed ω in this case.



Now consider two similar systems as shown in the figure : case (A) the disc with its face vertical and parallel to x-z plane; Case (B) the disc with its face making an angle of 45° with x-y plane and its horizontal diameter parallel to x-axis. In both the cases, the disc is welded at point P, and the systems are rotated with constant angular speed ω about the z-axis.



case (a)



case (b)

11. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2\omega}$ for both the cases.

(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b).

(C) It is ω for case (a); and $\sqrt{2\omega}$ for case (b).

(D) It is ω for both the cases.

Ans. **(D)**

Sol. Since body is rigid therefore ω is same about each point.

12. Which of the following statements about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is vertical for both the cases (a) and (b).

(B) It is vertical for case (a); and is at 45° to the x-z plane and lies in the plane of the disc for case (b).

(C) It is horizontal for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

(D) It is vertical for case (a); and is at 45° to the x-z plane and is normal to the plane of the disc for case (b).

Ans. **(A)**

Sol. Since body is rigid therefore all of its part rotate about same axis.

Paragraph for Questions 13 and 14

The β-decay process, discovered around 1900, is basically the decay of a neutron (n). In the laboratory, a proton (p) and an electron (e^-) are observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two-body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \vec{v}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino $(\vec{v}_e)$ to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied. From this calculation, the maximum kinetic energy of the electron is 0.8×10^6 eV. The kinetic energy carried by the proton is only the recoil energy.

13. If the anti-neutrino had a mass of 3 eV/c^2 (where c is the speed of light) instead of zero mass, what should be the range of the kinetic energy, K, of the electron?

(A) $0 \le K \le 0.8 \times 10^6 eV$
(B) $3.0eV \le K \le 0.8 \times 10^6 eV$
(C) $3.0eV \le K < 0.8 \times 10^6 eV$
(D) $0 \le K < 0.8 \times 10^6 eV$

Ans. **(D)**

Electron kinetic energy has continuous spectrum. Q-value of this reaction is very close to 0.8 MeV.

So $0 \le K < 0.8 \times 10^6 eV$

14. What is the maximum energy of the anti-neutrino?

(A) zero
(B) much less than 0.8×10^6 eV
(C) Nearly 0.8×10^6 eV
(D) Much larger than 0.8×10^6 eV

Ans. **(C)**

Sum of kinetic energy of electron and energy of neutrino remains nearly constant and proton carries away with negligible energy due to heavy mass.

Section III : Multiple Correct Answer (s) Type

This section contains **6 multiple questions**. Each question has four choices (A)_, (B), (C) and (D) out of which ONE or MORE are correct.

15. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is (are)

(A) The emf induced in the loop is zero if the current is constant.

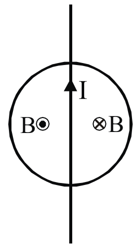
(B) The emf induced in the loop is infinite if the current is constant.

(C) The emf induced in the loop is zero if the current decreases at a steady rate.

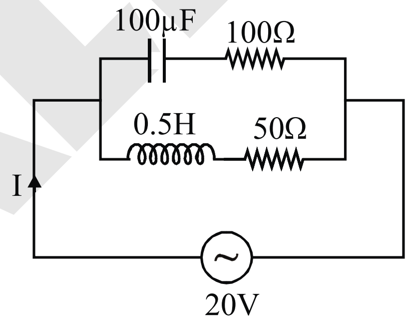
(D) The emf induced in the loop is finite if the current decreases at a steady rate.

Ans. **(A,C)**

Sol. Since flux of wire on the loop is zero therefore emf will not be induced.



16. In the given circuit, the AC source has ω = 100 rad/s. Considering the inductor and capacitor to be ideal, the correct choice (s) is(are)



(A) The current through the circuit, I is 0.3 A.
(B) The current through the circuit, i is 0.3 $\sqrt{2}$A.
(C) The voltage across 100Ω resistor = $10\sqrt{2}$V.
(D) The voltage across 50Ω resistor = 10V.

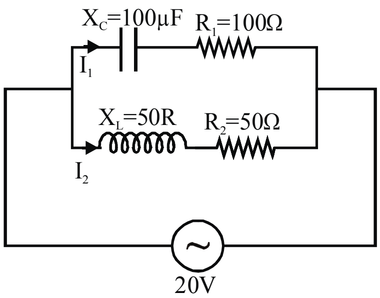
Ans. **(A,C)**

Sol. $I_1 = \frac{20}{z_1} = \frac{20}{100\sqrt{2}} = \frac{1}{5\sqrt{2}}$ A at 45° leading

$I_2 = \frac{20}{z_2} = \frac{20}{50\sqrt{2}} = \frac{\sqrt{2}}{5}$ A at 45° lagging

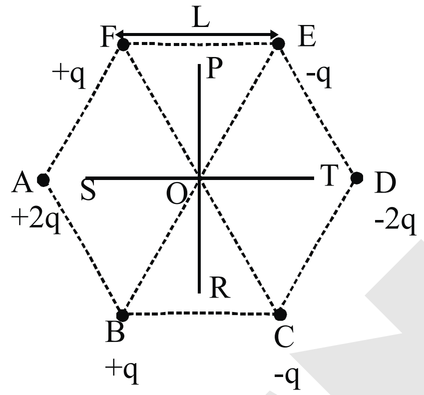
$I = \sqrt{I_1^2 + I_2^2} = \frac{1}{\sqrt{10}} A \approx 0.3A$

$V_{R_1} = I_1 R_1 = \frac{1}{5\sqrt{2}} \times 100 = 10\sqrt{2}V$



Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

17. Six point charges are kept at the vertices of a regular hexagon of side L and centre O, as shown in figure. Given that $K = \frac{1}{4\pi \epsilon_0} \frac{q}{L^2}$, which of the following statement(s) is(are) correct?



(A) The electric field at O is 6 K along OD.

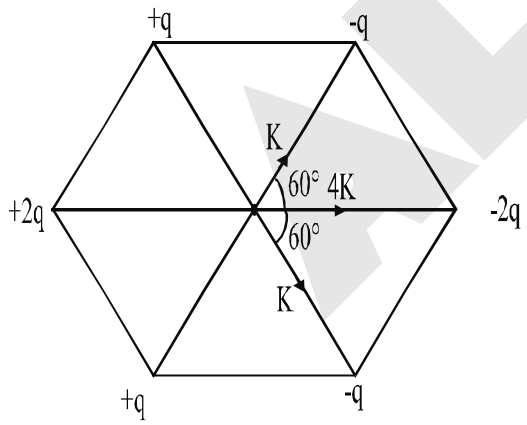
(B) The potential at O is zero.

(C) The potential at all points on the line PR is same.

(D) The potential at all points on the line ST is same.

Ans. **(A,B,C)**

Sol.



Resultant is 6 K along OD. For every point of POR there are two charges of opposite sign and equal magnitude at equal distance.

18. Two spherical planets P and Q have the same uniform density ρ, masses M_P and M_Q, and surface areas A and 4A, respectively. A spherical planet R also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets P, Q and R, are V_P, V_Q and V_R, respectively. Then

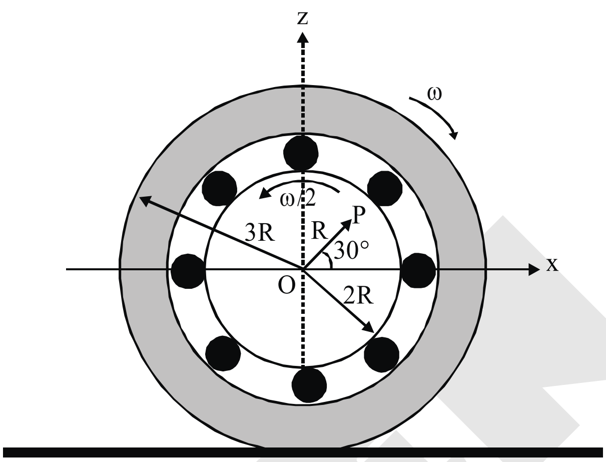
(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P/V_Q = 1/2$

Ans. **(B,D)**

Sol. Escape velocity = $\sqrt{\frac{2GM}{R}} \propto \sqrt{\frac{\frac{4}{3}\pi R^3}{R}} \propto \sqrt{Area}$

[since density of each planet is same]

19. The figure shows a system consisting of (i) a ring of outer radius 3R rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius 2R rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearing. The system is in the x-z plane. The point P on the inner disc is at a distance R from the origin, where OP makes an angle of 30° with the horizontal. Then with respect to the horizontal surface,



(A) the point O has a linear velocity $3R\omega\hat{i}$

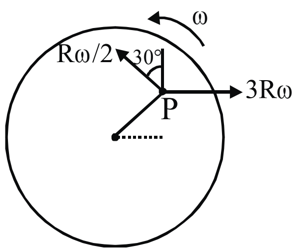
(B) the point P has a linear velocity $\frac{11}{4}R\omega\hat{i}+\frac{\sqrt{3}}{4}R\omega\hat{k}$

(C) the point P has a linear velocity $\frac{13}{4}R\omega\hat{i}-\frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point P has a linear velocity $\left(3-\frac{\sqrt{3}}{4}\right)R\omega\hat{i}+\frac{1}{4}R\omega\hat{k}$

Ans. (AB)

Velocity of the centre O = v = rω, $v=3R\omega\hat{i}$



$$v_P = 3R\omega\hat{i}-\frac{R\omega}{2}\sin 30°\hat{i}+\frac{R\omega}{2}\cos 30°\hat{k}$$

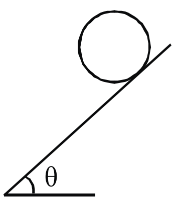
$$v_P = \frac{11R\omega}{4}\hat{i}+\frac{\sqrt{3}R\omega\hat{k}}{4}$$

20. Two solid cylinders P and Q of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder P has most of its mass concentrated near its surface, while Q has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(A) Both cylinders P and Q reach the ground at the same time.
(B) Cylinder P has larger acceleration than cylinder Q.
(C) Both cylinders reach the ground with same translational kinetic energy.
(D) Cylinder Q reaches the ground with larger angular speed.

Ans. (D)

Sol.



$a=\dfrac{g\sin\theta}{1+\dfrac{k^2}{R^2}}$ (where k is radius of curvature); $k_P > k_Q$

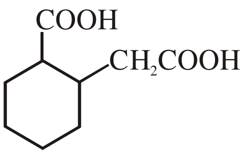
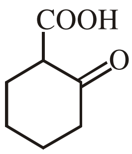
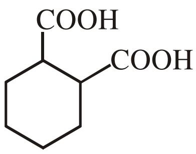
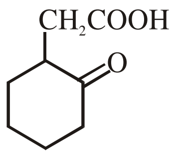
Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

PART - II : CHEMISTRY

SECTION–I : Single Correct Answer Type

This section contains **8 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE** is correct.

21. The compound that undergoes decarboxylation most readily under mild condition is :

(A) 
(B) 
(C) 
(D) 

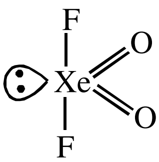
Ans. (B)

Sol. It is example of β-keto acid which undergoes decarboxylation most readily.

22. The shape of XeO_2F_2 molecule is

(A) Trigonal bipyramidal
(B) Square planar
(C) tetrahedral
(D) see-saw

Ans. (D)

Sol. 

See-saw

23. For a dilute solution containing 2.5 g of a non-volatile non-electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is 2°C. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is (take K_b=0.76 K kg mol^{-1})

(A) 724
(B) 740
(C) 736
(D) 718

Ans. (A)

Sol. $\Delta T_b = K_b \times m$ ....(i)

$$\frac{760 - P_S}{760} = m \times \frac{18}{1000} \text{ [for a dilute solution]} \quad ....(ii)$$

By eq. (i) / (ii)

$$\frac{2}{760 - P_S} \times 760 = \frac{0.76}{18} \times 1000$$

$P_S = 724$

24. $NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$ exhibits temperature dependent magnetic behavior (paramagnetic/diamagnetic). The coordination geometries of Ni^{2+} in the paramagnetic and diamagnetic states are respectively :

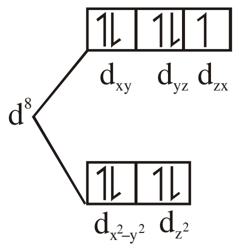
(A) tetrahedral and tetrahedral
(B) square planar and square planar
(C) tetrahedral and square planar
(D) square planar and tetrahedral

Ans. (C)

Sol.

For tetrahedral compound

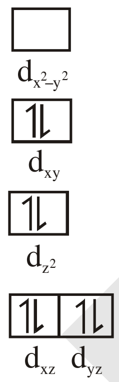
$Ni^{2+} = 3d^8$



Two unpaired electron.
Hence paramagnetic

For square planar compound

$Ni^{2+} = 3d^8$



No unpaired electron
Hence dimagnetic

25. The major product H of the given reaction sequence is :

$$CH_3-CH_2-CO-CH_3 \xrightarrow{^{\ominus}CN} \mathbf{G} \xrightarrow[\text{Heat}]{95\%\ H_2SO_4} \mathbf{H}$$

(A) $CH_3-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-COOH$

(B) $CH_3-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CN$

(C) $CH_3-CH_2-\overset{\displaystyle OH}{\overset{|}{\underset{\displaystyle CH_3}{\underset{|}{C}}}}-COOH$

(D) $CH_3-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CO-NH_2$

Ans. (B)

Sol. $CH_3-CH_2-\underset{\displaystyle O}{\underset{\|}{C}}-CH_3 \underset{}{\overset{\overset{\ominus}{CN}}{\rightleftharpoons}} CH_3-CH_2-\overset{\displaystyle OH}{\overset{|}{\underset{\displaystyle CN}{\underset{|}{C}}}}-CH_3$

$\Big\Updownarrow$ 95% H_2SO_4 (acts as dehydrating reagent)

$CH_3-CH=\underset{\displaystyle CH_3}{\underset{|}{C}}-CN+H_2O$

Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

26. The reaction of white phosphorus with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type ; the oxidation states of phosphorus in phosphine and the other product are respectively

(A) redox reaction ; –3 and –5

(B) redox reaction ; +3 and +5

(C) disproportionation reaction ; –3 and +5

(D) disproportionation reaction ; –3 and +3

Ans. NO OPTION IS CORRECT

Sol. $P_4 + NaOH\,(aq.) \longrightarrow \overset{-3}{P}H_3 + NaH_2\overset{+1}{P}O_2$

Oxidation state of P in $PH_3 = -3$

Oxidation state of P in $NaH_2PO_2 = +1$

Hence, (No option) is correct

27. Using the data provided, calculate the multiple bond energy (kJ mol^{-1}) of a C ≡ C bond in C_2H_2. That energy is (take the bond energy of a C–H bond as 350 kJ mol^{-1}.)

$2\,C(s) + H_2(g) \longrightarrow C_2H_2(g)$ $\quad \Delta H = 225$ kJ mol^{-1}

$2\,C(s) \longrightarrow 2C(g)$ $\quad \Delta H = 1410$ kJ mol^{-1}

$H_2(g) \longrightarrow 2H(g)$ $\quad \Delta H = 330$ kJ mol^{-1}

(A) 1165 (B) 837 (C) 865 (D) 815

Ans. (D)

Sol. $2C(s) + H_2(g) \longrightarrow H - C \equiv C - H(g) \quad \Delta H = 225$ kJ/mole

$225 = 2 \times \Delta H_{sub}\,C(s) + B.E._{H_2} - 2 \times B.E._{C-H} - 1 \times B.E._{C\equiv C}$

$225 = 2 \times 705 + 330 - 2 \times 350 - B.E._{C\equiv C}$

$225 = 1410 + 330 - 700 - (B.E.)_{C\equiv C}$

$\Rightarrow$ $B.E._{C\equiv C} = 815$ kJ/mole

28. In the cyanide extraction process of silver from argentite ore, the oxidizing and reducing agents used are :

(A) O_2 and CO respectively.

(B) O_2 and Zn dust respectively.

(C) HNO_3 and Zn dust respectively.

(D) HNO_3 and CO respectively.

Ans. (B)

Sol. $Ag_2S \xrightarrow[\text{in presence of air}]{NaCN} [Ag(CN)_2]^-$

$Ag_2S + 4NaCN \rightleftharpoons 2Na\,[Ag(CN)_2] + Na_2S$

$Na_2S + O_2 + H_2O \longrightarrow Na_2SO_4 + S\downarrow + NaOH$

Hence, oxidising agent is O_2.

$2[Ag(CN_2)]^- + Zn \longrightarrow 2Ag\downarrow + [Zn\,(CN)_4]^{2-}$

Hence, reducing agent is 'Zn'.

SECTION-II : Paragraph Type

This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question 29 and 30

Bleaching powder and bleach solution are produced on a large scale and used in several house-hold products. The effectiveness of bleach solution is often measured by iodometry.

29. Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is :

(A) Cl_2O (B) Cl_2O_7 (C) ClO_2 (D) Cl_2O_6

Ans. (A)

Sol. Bleaching powder is Ca(OCl)Cl.

So, the associated oxo acid is HOCl.

Hence, the anhydride of this acid is Cl_2O. ($2HOCl \xrightarrow[-H_2O]{} Cl_2O$)

30. 25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4 N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is

(A) 0.48 M (B) 0.96 M (C) 0.24 M (D) 0.024 M

Ans. (C)

Sol. Eq. of $CaOCl_2$ = Eq. of $Na_2S_2O_3$

$25 \times M \times 2 \quad = 48 \times 0.25$

$[2H^+ + \underset{(nf = 2)}{CaOCl_2} + 2e^- \longrightarrow Ca^{2+} + H_2O + 2Cl^-]$

$M = 0.24$

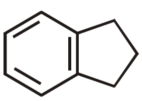
Paragraph for Question 31 and 32

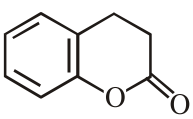
In the following reaction sequence, the compound J is an intermediate.

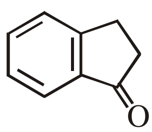
$$\mathbf{I} \xrightarrow[CH_3COONa]{(CH_3CO)_2O} \mathbf{J} \xrightarrow[\substack{(ii)\ SOCl_2 \\ (iii)\ anhyd.\ AlCl_3}]{(i)\ H_2,\ Pd/C} \mathbf{K}$$

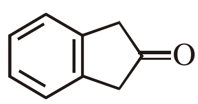
J ($C_9H_8O_2$) gives effervescence on treatment with $NaHCO_3$ and a positive Baeyer's test.

31. The compound **K** is :

(A) 

(B) 

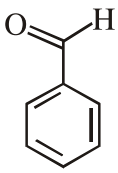
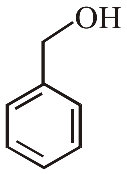
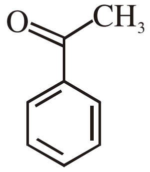
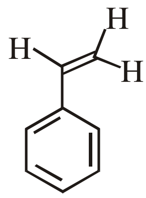
(C) 

(D) 

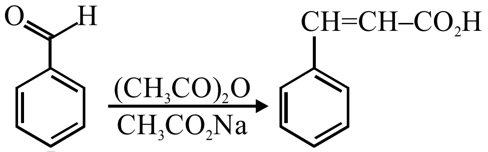
Ans. (C)

Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

32. The compound **I** is

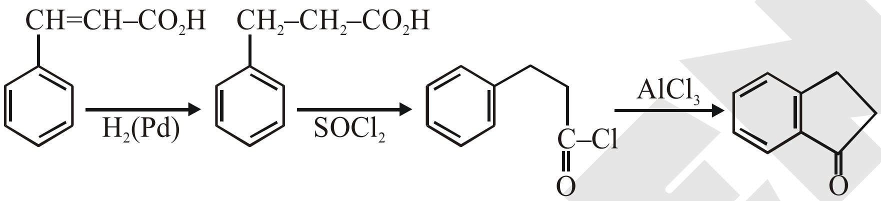
(A)  (B)  (C)  (D) 

Ans. (A)

Sol. 

I J ($C_9H_8O_2$)

J gives effervescence on treatment with $NaHCO_3$ due to presence of $-CO_2H$ group and positive Baeyer's test due to presence of unsaturation.



K
(Intramoleculer
Fridel craft reaction)

Paragraph for Question 33 and 34

The electrochemical cell shown below is a concentration cell.

M | M^{2+} (saturated solution of a sparingly soluble salt, MX_2) || M^{2+} (0.001 mol dm^{-3}) | M

The emf of the cell depends on the difference in concentrations of M^{2+} ions at the two electrodes. The emf of the cell at 298 K is 0.059V.

33. The value of ΔG (kJ mol^{-1}) for the given cell is (take If = 96500 C mol^{-1})

(A) –5.7 (B) 5.7 (C) 11.4 (D) –11.4.

Ans. (D)

Sol. $\because \Delta G = -nFE$

$\Delta G = -2 \times 96500 \times 0.059$ Joule

$= -11.4$ kJ/mole

34. The solubility product (K_{sp} ; $mol^3\,dm^{-9}$) of MX_2 at 298 K based on the information available for the given concentration cell is (take $2.303 \times R \times 298/F = 0.059$ V)

(A) 1×10^{-15} (B) 4×10^{-15} (C) 1×10^{-12} (D) 1×10^{-12}

Ans. (B)

Sol. $0.059 = 0 - \dfrac{0.059}{2}\log\dfrac{[M^{++}]}{0.001}$

$\dfrac{[M^{++}]}{0.001} = 10^{-2}$

$[M^{++}] = 10^{-5} = s$

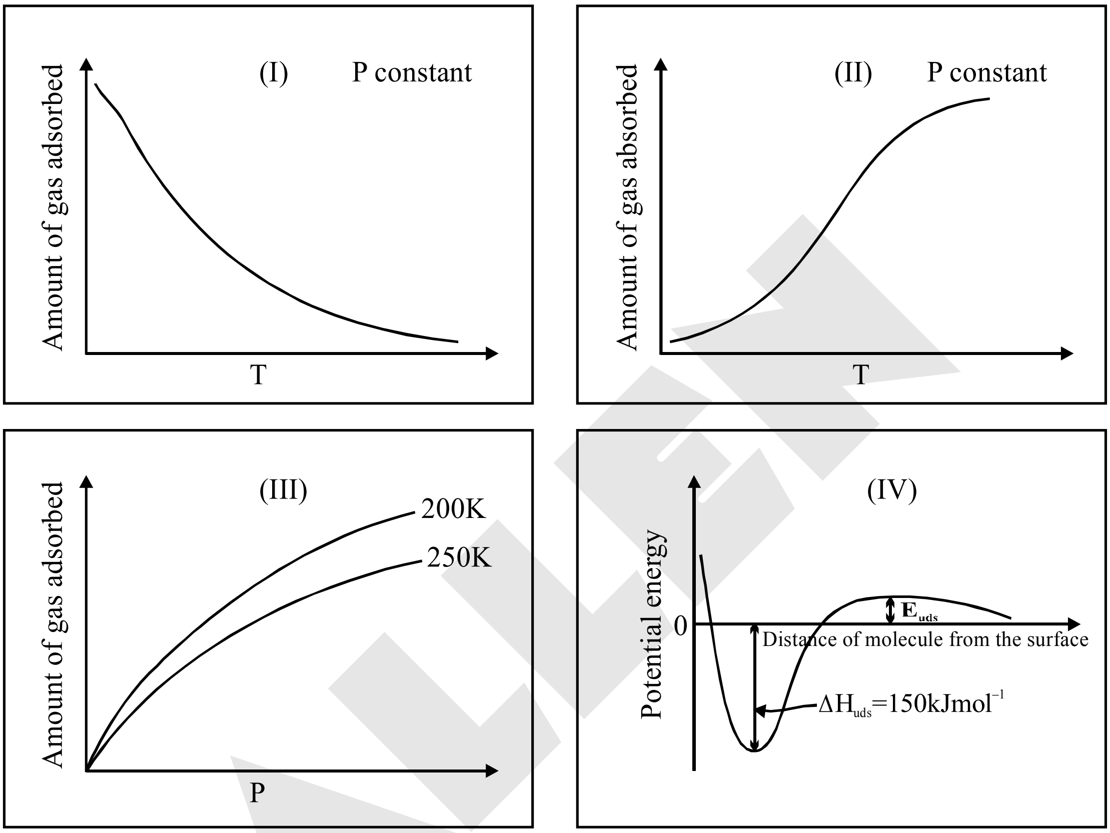
$K_{sp} = 4s^3 = 4 \times (10^{-5})^3$

$= 4 \times 10^{-15}$

SECTION-III : Multiple Correct Answser(s) Type

This section contains **5 multiple choice question**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

35. The given graphs / data **I, II, III and IV** represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about **I, II, III and IV** is (are) correct ?

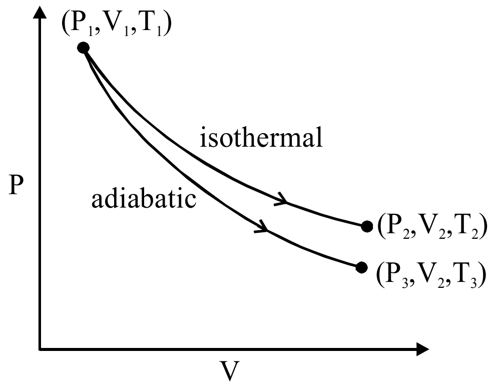


(A) **I** is physisorption and **II** is chemisorption (B) **I** is physisorption and **III** is chemisorption

(C) **IV** is chemisorption and **II** is chemisorption (D) **IV** is chemisorption and **III** is chemisorption

Ans. (A,C)

36. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct ?



(A) $T_1 = T_2$ (B) $T_3 > T_1$ (C) $w_{isothermal} > w_{adiabatic}$ (D) $\Delta U_{isothermal} > \Delta U_{adiabatic}$

Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

Ans. (A,D)

Sol. (A) Since process is isothermal.

(B) In adiabatic expansion temperature decreases.

(C) Although $|W_{isothermal,exp}| > |W_{adiabatic,exp}|$

Since expansion work is –ve, so option is wrong.

(D) $\because\ \Delta U_{isothermal} = 0\ \&\ \Delta U_{adiabatic} < 0$

$\Rightarrow$ $\Delta U_{isothermal} > \Delta U_{adiabatic}$

37. For the given aqueous reactions, which of the statement(s) is (are) true ?

$$\text{excess KI} + K_3[Fe(CN)_6] \xrightarrow{\text{dilute } H_2SO_4} \text{brownish-yellow solution}$$

$$\downarrow ZnSO_4$$

$$\text{white precipitate} + \underbrace{\text{brownish-yellow filtrate}}$$

$$\downarrow Na_2S_2O_3$$

$$\text{colourless solution}$$

(A) The first reaction is a redox reaction.

(B) White precipitate is $Zn_3[Fe(CN)_6]_2$.

(C) Addition of filtrate to starch solution gives blue colour.

(D) White precipitate is soluble in NaOH solution.

Ans. (A,C,D)

Sol. $2I^- + 2\,[Fe(CN)_6]^{3-} \longrightarrow 2[Fe(CN)_6]^{4-} + I_2$ (Redox reaction)

$I_2 + I^- \longrightarrow I_3^-$ (Brownish yellow filtrate or solution).

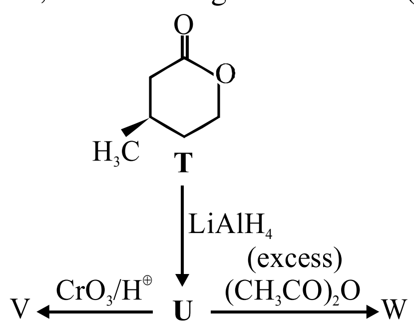
$[Fe(CN)_6]^{4-} + Zn \longrightarrow \underbrace{Zn_2[Fe(CN)_6]\downarrow \text{ or } K_2\,Zn_3\,[Fe(CN)_6]_2\downarrow}_{\text{white ppt. (soluble in NaOH)}}$

and $I_3^- + 2S_2O_3^{2-} \longrightarrow 3I^- + S_4O_6^{2-}$

$I_3^- \rightleftharpoons I^- + I_2$; starch $+ I_2 \longrightarrow$ deep blue solution.

Hence, answers are A,C, D

38. With reference to the scheme given, which of the given statement(s) about **T,U,V** and **W** is (are) correct?



(A) **T** is soluble in hot aqueous NaOH

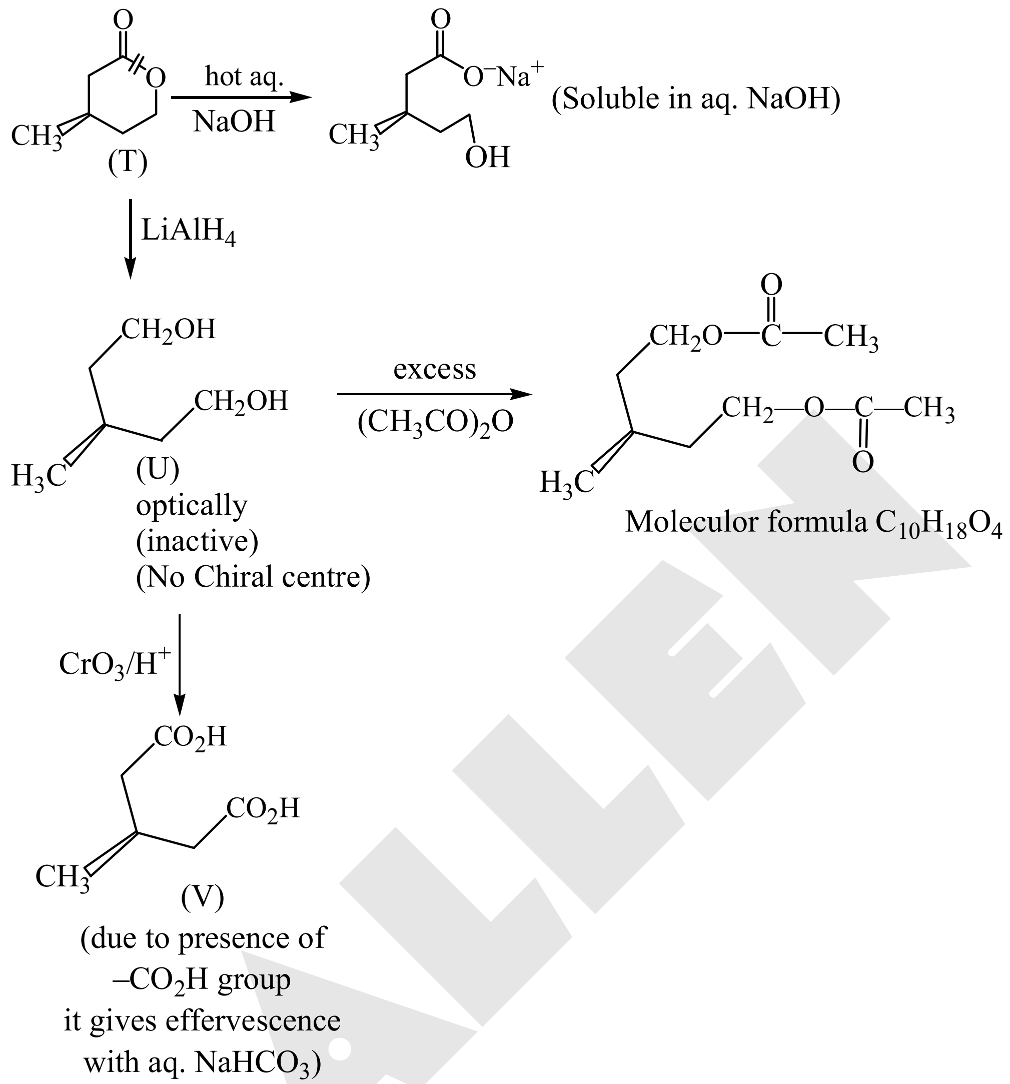
(B) **U** is optically active

(C) Molecular formula of **W** is $C_{10}H_{18}O_4$

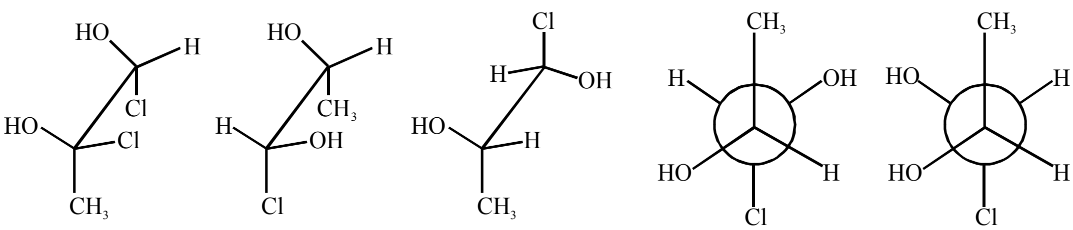
(D) V gives effervescence on treatment with aqueous $NaHCO_3$

Ans. (A,C,D)

Sol.



39. Which of the given statement(s) about N,O,P and Q with respect to M is (are) correct ?



M **N** **O** **P** **Q**

(A) M and N are non-mirror image stereoisomers

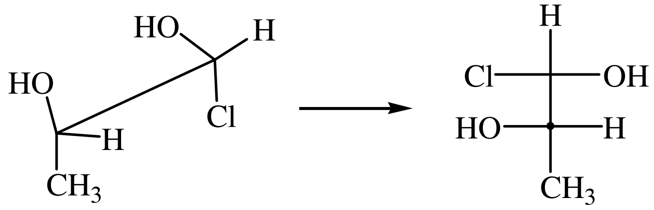
(B) M and O are identical

(C) M and P are enantiomers

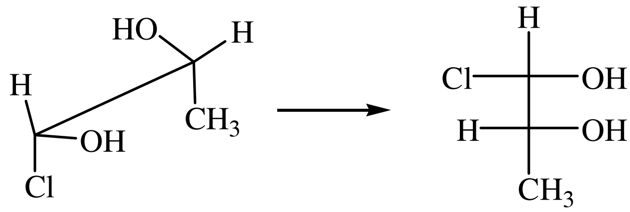
(D) M and Q are identical

Ans. (A,B,C)

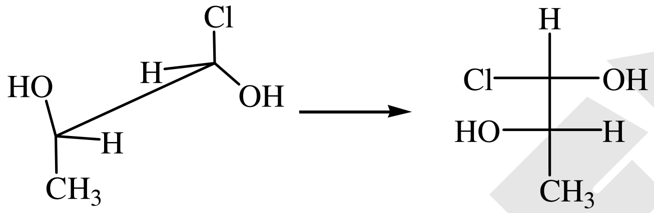
Sol. (M)



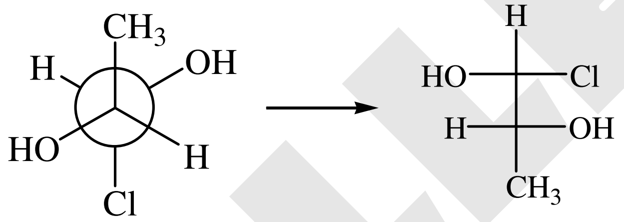
(N)



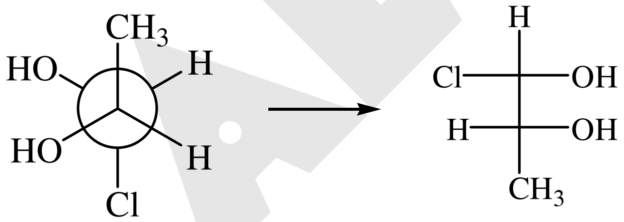
(O)



(P)



(Q)



MN ⟶ Diastereomers (non mirror image stereo isomers)

MO ⟶ Identical

MP ⟶ Enantiomer

MQ ⟶ Diastereomer

40. With respect to graphite and diamond, which of the statement(s) given below is (are) correct ?

(A) Graphite is harder than diamond.

(B) Graphite has higher electrical conductivity than diamond.

(C) Graphite has higher thermal conductivity than diamond.

(D) Graphite has higher C – C bond order than diamond.

Ans. (B,D)

Sol. Due to resonance the bond order is greater than 1 in case of graphite.

Diamond has higher thermal conductivity than graphite.

PART - III : MATHEMATICS

SECTION–I : Single Correct Answer Type

This section contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE is correct**.

---

41. Four fair dice D_1, D_2, D_3 and D_4, each having six faces numbered 1, 2, 3, 4, 5 and 6 are rolled simultaneously. The probability that D_4 shows a number appearing on one of D_1, D_2 and D_3 is -

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

Ans. (A)

Sol. $1-\frac{{}^6C_1.5^3}{6^4}=\frac{91}{216}$

42. If P is a 3×3 matrix such that $P^T = 2P + I$, where P^T is the transpose of P and I is the 3×3 identity matrix,

then there exists a column matrix $X=\begin{bmatrix}x\\y\\z\end{bmatrix}\neq\begin{bmatrix}0\\0\\0\end{bmatrix}$ such that

(A) $PX=\begin{bmatrix}0\\0\\0\end{bmatrix}$ (B) $PX = X$ (C) $PX = 2X$ (D) $PX = -X$

Ans. (D)

Sol. $P^T = 2P + I$

$\Rightarrow P = 2P^T + I$

$\Rightarrow P = 2(2P + I) + I$

$\Rightarrow P = 4P + 3I$

$\Rightarrow P = -I$

$\Rightarrow PX = -X$

43. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation

$\left(\sqrt[3]{1+a}-1\right)x^2+\left(\sqrt{1+a}-1\right)x+\left(\sqrt[6]{1+a}-1\right)=0$ where $a > -1$.

Then $\lim_{a\to0^+}\alpha(a)$ and $\lim_{a\to0^+}\beta(a)$ are

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and -1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

Ans. (B)

Sol. $\left(\left(1+\frac{a}{3}\right)-1\right)x^2+\left(\left(1+\frac{a}{2}\right)-1\right)x+\left(1+\frac{a}{6}-1\right)=0$

$a\left(\frac{x^2}{3}+\frac{x}{2}+\frac{1}{6}\right)=0 \Rightarrow 2x^2 + 3x + 1 = 0$

$\Rightarrow x=-\frac{1}{2},-1 \quad \Rightarrow \lim_{a\to0^+}\alpha(a)$ and $\lim_{a\to0^+}\beta(a)$ are $-\frac{1}{2}$ and -1

Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

44. The equation of a plane passing through the line of intersection of the planes x + 2y + 3z = 2 and x – y + z = 3 and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is

(A) 5x – 11y + z = 17 (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) x + y + z = $\sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

Ans. **(A)**

Sol. Let required plane be (x + 2y + 3z – 2) + λ(x – y + z – 3) = 0

$\because$ plane is at a distance $\frac{2}{\sqrt{3}}$ from the point (3,1,–1).

$$\Rightarrow \left|\frac{(3+2-3-2)+\lambda(3-1-1-3)}{\sqrt{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$$

$$\Rightarrow \lambda^2 = \frac{(1+\lambda)^2+(2-\lambda)^2+(3+\lambda)^2}{3}$$

$$\Rightarrow 3\lambda^2 = 3\lambda^2 + 2\lambda - 4\lambda + 6\lambda + 14$$

$$\Rightarrow \lambda = -\frac{7}{2}$$

$\therefore$ required plane is (x + 2y + 3z – 2) + $\left(-\frac{7}{2}\right)(x-y+z-3) = 0$

$\Rightarrow$ 5x – 11y + z = 17

45. Let a_1, a_2, a_3, ... be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer n for which $a_n < 0$ is

(A) 22 (B) 23 (C) 24 (D) 25

Ans. **(D)**

Sol. a_1,a_2,a_3...........be in H.P $\Rightarrow \frac{1}{a_1},\frac{1}{a_2},\frac{1}{a_3}$.... be in A.P.

in A.P. $T_1 = \frac{1}{a_1} = \frac{1}{5}$ and $T_{20} = \frac{1}{a_{20}} = \frac{1}{25}$

$\therefore T_{20} = T_1 + 19d$

$$\frac{1}{25} = \frac{1}{5} + 19d \Rightarrow d = -\frac{4}{19\times 25}$$

$T_n = T_1 + (n-1)d < 0$

$$\Rightarrow \frac{1}{5} - \frac{(n-1).4}{19\times 25} < 0 \qquad \Rightarrow \frac{1}{5} < \frac{4(n-1)}{25\times 19}$$

$$\Rightarrow \frac{5\times 19}{4} + 1 < n$$

$$\Rightarrow \frac{99}{4} < n$$

$\Rightarrow$ least positive integer n is 25.

46. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a}+\vec{b}|=\sqrt{29}$ and $\vec{a}\times(2\hat{i}+3\hat{j}+4\hat{k})=(2\hat{i}+3\hat{j}+4\hat{k})\times\vec{b}$, then a possible value of $(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k})$ is

(A) 0 (B) 3 (C) 4 (D) 8

Ans. (C)

Sol. $(\vec{a}+\vec{b})\times(2\hat{i}+3\hat{j}+4\hat{k})=0$

$\Rightarrow \quad \vec{a}+\vec{b}=\lambda(2\hat{i}+3\hat{j}+4\hat{k})$

$|\vec{a}+\vec{b}|=\sqrt{29} \Rightarrow |\lambda| = 1$

$\vec{a}+\vec{b}=(2\hat{i}+3\hat{j}+4\hat{k})$

$(\vec{a}+\vec{b}).(-7\hat{i}+2\hat{j}+3\hat{k}) = -14 + 6 + 12 = 4$

47. The value of the integral $\int\limits_{-\pi/2}^{\pi/2}\left(x^2+\ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is

(A) 0 (B) $\frac{\pi^2}{2}-4$ (C) $\frac{\pi^2}{2}+4$ (D) $\frac{\pi^2}{2}$

Ans. (B)

Sol. $\int\limits_{-\pi/2}^{\pi/2} x^2\cos x\,dx + \int\limits_{-\pi/2}^{\pi/2}\ell n\left(\frac{\pi+x}{\pi-x}\right)\cos x\,dx$

$= \int\limits_{-\pi/2}^{\pi/2} x^2\cos x\,dx = 2\int\limits_{0}^{\pi/2}\underset{\mathrm{I}}{x^2}\underset{\mathrm{II}}{\cos x}\,dx$

$= 2\left((x^2\sin x)_0^{\pi/2} - 2\int\limits_0^{\pi/2} x\sin x\,dx\right)$

$= 2\left(\frac{\pi^2}{4}-2\left(-(x\cos x)_0^{\pi/2}+\int\limits_0^{\pi/2}\cos x\,dx\right)\right)$

$= 2\left(\frac{\pi^2}{4}-2\int\limits_0^{\pi/2}\cos x\,dx\right)$

$= 2\left(\frac{\pi^2}{4}-2\right)=\frac{\pi^2}{2}-4$

48. Let PQR be a triangle of area Δ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where a, b and c are the lengths of the sides of the triangle opposite to the angles at P, Q and R respectively. Then $\frac{2\sin P-\sin 2P}{2\sin P+\sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

Ans. (C)

Sol. $\dfrac{2\sin P - 2\sin P\cos P}{2\sin P + 2\sin P\cos P} = \dfrac{(1-\cos P)}{(1+\cos P)}$

$= \tan^2\dfrac{A}{2} = \dfrac{\Delta^2}{s^2(s-a)^2} = \dfrac{\left((s-b)(s-c)\right)^2}{\Delta^2}$

$s = 4$

$= \left(\dfrac{\left(4-\frac{7}{2}\right)\left(4-\frac{5}{2}\right)}{\Delta}\right)^2 = \left(\dfrac{3}{4\Delta}\right)^2$

SECTION–II : Paragraph Type

This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph.** Each question has four choices (A), (B), (C) and (D), out of which **ONLY ONE** is correct.

Paragraph for Question 49 and 50

Let a_n denotes the number of all n-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let b_n = the number of such n-digit integers ending with digit 1 and c_n = the number of such n-digit integers ending with digit 0.

49. The value of b_6 is

(A) 7 (B) 8 (C) 9 (D) 11

50. Which of the following is correct ?

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq c_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

Solution for Q.49 & Q.50

For a_n

The first digit should be 1

For b_n

$\underbrace{1\_\,\_\,\_\,\_\,\_ \cdots \_\,\_1}_{(n-2\text{ Places})}$

Last digit is 1. so b_n is equal to number of ways of a_{n-1} (i.e. remaining (n – 1) places)

$b_n = a_{n-1}$

For c_n

Last digit is 0 so second last digit must be 1

So $c_n = a_{n-2}$

$b_n + c_n = a_n$

So $a_n = a_{n-1} + a_{n-2}$

Similarly $b_n = b_{n-1} + b_{n-2}$

49. **Ans.(B)**

$a_1 = 1,\ a_2 = 2$

So $a_3 = 3,\quad a_4 = 5\ \ a_5 = 8$

$\Rightarrow b_6 = a_5 = 8$

50. Ans.(A)

$a_n = a_{n-1} + a_{n-2}$

put $n = 17$

$a_{17} = a_{16} + a_{15}$ (A) is correct

$c_n = c_{n-1} + c_{n-2}$

So put $n = 17$

$c_{17} = c_{16} + c_{15}$ (B) is incorrect

$b_n = b_{n-1} + b_{n-2}$

put $n = 17$

$b_{17} = b_{16} + b_{15}$ (C) is incorrect

$a_{17} = a_{16} + a_{15}$

while (D) says $a_{17} = a_{15} + a_{15}$ (D) is incorrect

Paragraph for Question 51 and 52

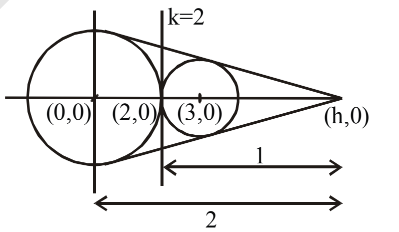
A tangent PT is drawn to the circle $x^2 + y^2 = 4$ at the point $P\left(\sqrt{3}, 1\right)$. A straight line L, perpendicular to PT is a tangent to the circle $(x-3)^2 + y^2 = 1$.

51. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x+\sqrt{3}y = 4$ (D) $x+2\sqrt{2}y = 6$

Ans. (D)

Sol. $h = \dfrac{2\times 3 - 1\times 0}{2-1} = 6$



equation of tangents from (6, 0) :

$y - 0 = m(x-6) \Rightarrow y - mx + 6m = 0$

use $p = r$

$\left|\dfrac{6m}{\sqrt{1+m^2}}\right| = 2 \Rightarrow 36m^2 = 4 + 4m^2$

$32m^2 = 4$

$m^2 = 1/8 \Rightarrow m = \pm\dfrac{1}{2\sqrt{2}}$

at $m = -\dfrac{1}{2\sqrt{2}}$

equation of tangent will be $x + 2\sqrt{2}y = 6$.

52. A possible equation of L is

(A) $x-\sqrt{3}y = 1$ (B) $x+\sqrt{3}y = 1$ (C) $x-\sqrt{3}y = -1$ (D) $x+\sqrt{3}y = 5$

Ans. (A)

Sol. Equation of tangent at P will be $\sqrt{3}x + y = 4$

Slope of line L will be $\frac{1}{\sqrt{3}}$

Let equation of L be : $y = \frac{x}{\sqrt{3}} + c$

$\Rightarrow \quad x - \sqrt{3}y + \sqrt{3}c = 0$

Now this L is tangent to 2^{nd} circle

So $\frac{3+\sqrt{3}c}{2} = \pm 1 \quad \Rightarrow \quad c = -\frac{1}{\sqrt{3}}$ or $c = -\frac{5}{\sqrt{3}}$

using $\qquad c = -\frac{1}{\sqrt{3}}$

$$y = \frac{x}{\sqrt{3}} - \frac{1}{\sqrt{3}} \Rightarrow x - \sqrt{3}y = 1.$$ Hence (A)

Paragraph for Question 53 and 54

Let $f(x) = (1-x)^2\sin^2x + x^2$ for all $x \in \mathbb{R}$, and let $g(x) = \int_1^x \left(\frac{2(t-1)}{t+1} - \ell nt\right) f(t)dt$ for all $x \in (1,\infty)$.

53. Consider the statements :

P : There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1 + x^2)$

Q : There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$

Then

(A) both **P** and **Q** are true
(B) **P** is true and **Q** is false
(C) **P** is false and **Q** is true
(D) both **P** and **Q** are false

Ans (C)

Sol. $f(x) = (1-x)^2 \sin^2x + x^2$

P : $f(x) + 2x = 2(1 + x^2)$

$\Rightarrow (1-x)^2 \sin^2x + x^2 + 2x = 2 + 2x^2$

$\Rightarrow (1-x)^2 \sin^2x - x^2 + 2x - 2 = 0$

$(1-x)^2\cos^2x + 1 = 0$

which is not possible.

$\therefore$ P is false.

Q : $2f(x) + 1 = 2x(1 + x)$

$2x^2 + 2(1-x)^2\sin^2x + 1 = 2x^2 + 2x$

$2(1-x)^2 \sin^2x - 2x + 1 = 0.$

Let $h(x) = 2(1-x)^2\sin^2x - 2x + 1$, clearly $h(1) = -1$

and $h(x) = 2(x^2 - 2x + 1)\sin^2x - 2x + 1$

$$= x^2\left[2\left(1 - \frac{2}{x} + \frac{1}{x^2}\right).\sin^2 x - \frac{2}{x} + \frac{1}{x^2}\right]$$

$\therefore h(x) \to \infty$ as $x \to \infty$.

$\therefore$ By intermediate value theorem

$h(x) = 0$ has a root which is greater than 1.

Hence Q is true.

54. Which of the following is true ?

(A) g is increasing on $(1,\infty)$

(B) g is decreasing on $(1,\infty)$

(C) g is increasing on (1,2) and decreasing on $(2,\infty)$

(D) g is decreasing on (1,2) and increasing on $(2,\infty)$

Ans. (B)

Sol. $g(x)=\int_1^x\left(\frac{2(t-1)}{(t+1)}-\ell nt\right)f(t)dt$

$$g'(x)=\left(\frac{2(x-1)}{x+1}-\ell nx\right)f(x)$$

$f(x)>0\ \forall\ x\in R$

Suppose.

$$h(x)=\frac{2(x-1)}{x+1}-\ell nx$$

$$h(x)=2-\left(\frac{4}{x+1}+\ell nx\right)$$

$$h'(x)=\frac{4}{(x+1)^2}-\frac{1}{x}$$

$$h'(x)=-\frac{(x-1)^2}{x(x+1)^2}$$

$h'(x)<0$

So h(x) is decreasing

so $h(x)<h(1).\quad \forall\ x>1$

$h(x)<0\qquad \forall\ x>1$

So $g'(x)=h(x)\,f(x)$

$g'(x)<0\ \forall\ x>1$

g(x) is decreasing in $(1,\infty)$.

SECTION–III : Multiple Correct Answer(s) Type

This section contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D), out of which **ONE or MORE are correct.**

---

55. If $f(x)=\int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x\in(0,\infty)$, then -

(A) f has a local maximum at x = 2

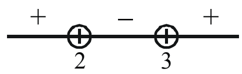
(B) f is decreasing on (2,3)

(C) there exists some $c\in(0,\infty)$ such that $f''(c)=0$

(D) f has a local minimum at x = 3

Ans. (A,B,C,D)

Sol. $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$



$\Rightarrow \quad f'(x) = e^{x^2}(x-2)(x-3)$

$\therefore \quad f'(2) = f'(3) = 0$

$\Rightarrow \quad f''(c) = 0$ for same $c \in (2, 3)$ (by Rolle's theorem)

56. For every integer n, let a_n and b_n be real numbers. Let function f : IR $\to$ IR be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for} \quad x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for} \quad x \in (2n-1, 2n) \end{cases}, \text{ for all integers n.}$$

If f is continuous, then which of the following holds(s) for all n ?

(A) $a_{n-1} - b_{n-1} = 0$ (B) $a_n - b_n = 1$ (C) $a_n - b_{n+1} = 1$ (D) $a_{n-1} - b_n = -1$

Ans. (B,D)

Sol. For f to be continuous :

$f(2n^-) = f(2n^+)$.

$\Rightarrow \quad b_n + \cos 2n\pi = a_n + \sin 2n\pi$

$\Rightarrow \quad b_n + 1 = a_n$

$\Rightarrow \quad a_n - b_n = 1$

($\therefore$ B is correct)

$$\text{Also } f(x) = \begin{bmatrix} b_n + \cos \pi x & (2n-1,\ 2n) \\ a_n + \sin \pi x & [2n,\ 2n+1] \\ b_{n+1} + \cos \pi x & (2n+1,\ 2n+2) \\ a_n + \sin \pi x & [2n+2,\ 2n+3] \end{bmatrix}$$

Again $f((2n + 1)^-) = f((2n + 1)^+)$

$\Rightarrow \quad a_n = b_{n+1} - 1$

$\Rightarrow \quad a_n - b_{n+1} = -1$

$\Rightarrow \quad a_{n-1} - b_n = -1$ ($\therefore$ D is correct)

57. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is(are)

(A) $y + 2z = -1$ (B) $y + z = -1$ (C) $y - z = -1$ (D) $y - 2z = -1$

Ans. **(B,C)**

Sol. (1, –1, 0); (–1, –1, 0)

For coplanarity of lines

$$\begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0 \quad \Rightarrow \quad 2(k^2 - 4) = 0$$

$\Rightarrow$ k = ±2

for k = 2

Normal vector $\vec{n} = \hat{j} - \hat{k}$

$\therefore$ Required plane : y – z = λ

$\because$ Passes through (1, –1, 0) $\Rightarrow$ λ = –1

$\therefore$ y – z = –1

for k = –2

$\vec{n} = \hat{j} + \hat{k}$

$\therefore$ Required plane : y + z = λ

$\because$ Passes through (1, –1, 0) $\Rightarrow$ λ = –1

$\therefore$ y + z = –1

58. If the adjoint of a 3×3 matrix P is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 2 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of P is (are)-

(A) –2 (B) –1 (C) 1 (D) 2

Ans. **(A,D)**

Sol. $|\text{adj}P| = \begin{vmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{vmatrix}$

$\Rightarrow |P|^2 = 4 \Rightarrow |P| = \pm 2$

59. Let $f : (-1,1) \to \text{IR}$ be such that $f(\cos 4\theta) = \dfrac{2}{2 - \sec^2\theta}$ for $\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$. Then the value(s) of $f\left(\dfrac{1}{3}\right)$ is (are)-

(A) $1 - \sqrt{\dfrac{3}{2}}$ (B) $1 + \sqrt{\dfrac{3}{2}}$ (C) $1 - \sqrt{\dfrac{2}{3}}$ (D) $1 + \sqrt{\dfrac{2}{3}}$

Corporate Office : ALLEN CAREER INSTITUTE, "SANKALP", CP-6, INDRA VIHAR, KOTA-324005
PHONE : +91 - 744 - 2436001, Fax : +91-744-2435003, E-mail: info@allen.ac.in Website: www.allen.ac.in

Ans. **(A,B)**

Sol. $\because \quad \theta \in \left(0, \frac{\pi}{4}\right) \cup \left(\frac{\pi}{4}, \frac{\pi}{2}\right)$

$\Rightarrow \quad 2\theta \in \left(0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right)$

Now $f(\cos 4\theta) = \dfrac{2}{2-\sec^2\theta} = \dfrac{1+\cos 2\theta}{\cos 2\theta} = 1 + \dfrac{1}{\cos 2\theta}$ ....(i)

Let $\cos 4\theta = \frac{1}{3}$

$\Rightarrow \quad 2\cos^2 2\theta - 1 = \frac{1}{3}$

$\Rightarrow \quad \cos 2\theta = \pm\sqrt{\frac{2}{3}}$

$\Rightarrow$ From (i)

$f\left(\frac{1}{3}\right) = 1 \pm \sqrt{\frac{3}{2}}$

$\Rightarrow$ (A, B) are correct

60. Let X and Y be two events such that $P(X|Y) = \frac{1}{2}, P(Y|X) = \frac{1}{3}$ and $P(X \cap Y) = \frac{1}{6}$. Which of the following is(are) correct ?

(A) $P(X \cup Y) = \frac{2}{3}$

(B) X and Y are independent

(C) X and Y are not independent

(D) $P(X^C \cap Y) = \frac{1}{3}$

Ans. **(A,B)**

Sol. P(X $\cap$ Y) = P(X), P(Y/X)

$\Rightarrow$ P(X) $= \frac{1}{2}$

Also P(X $\cap$ Y) = P(Y).P(X/Y)

$\Rightarrow \quad P(Y) = \frac{1}{3}$

$\Rightarrow$ P(X $\cap$ Y) = P(X).P(Y)

$\Rightarrow$ X,Y are independent

P(X $\cup$ Y) = P(X) + P(Y) − P(X $\cap$ Y)

$= \frac{1}{3} + \frac{1}{2} - \frac{1}{6} = \frac{2}{3}$

$P(X^C \cap Y) = P(Y) - P(X \cap Y) = \frac{1}{3} - \frac{1}{6} = \frac{1}{6}$

$\Rightarrow$ (A, B) are correct