School of Mathematics, IISER-TVM

Syllabus for PhD. Admission Test

1. Test will be of 2 hours duration.

2. Test paper will have two parts, Part A and Part B.

3. Part A will consists of fill in the blank type questions and all the questions must be answered.

4. Part B will consists of 6 descriptive type questions of which 4 has to be answered.

Linear Algebra: Finite dimensional vector spaces; Linear transformations and their matrix representations, rank; systems of linear equations, eigen values and eigen vectors, minimal polynomial, Cayley-Hamilton Theroem, diagonalisation, Hermitian, Skew-Hermitian and unitary matrices; Finite dimensional inner product spaces, Gram-Schmidt orthonormalization process, self-adjoint operators.

Abstract Algebra: Groups, subgroups, normal subgroups and homomorphism theorems, automorphisms; cyclic groups, permutation groups, Cayley's theorem, Sylow's theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, Euclidean domains, Principle ideal domains and unique factorization domains. Fields, finite fields

Real Analysis: Real valued functions of a real variable; Continuity and differentiability; Sequences and series of functions, uniform convergence, power series, Fourier series, functions of several variables, maxima, minima; Riemann integration, multiple integrals, line, surface and volume integrals, theorems of Green, Stokes and Gauss; metric spaces, completeness, Weierstrass approximation theorem, compactness; Lebesgue measure, measurable functions; Lebesgue integral, Fatou's lemma, dominated convergence theorem.

Complex Analysis: Algebra of complex numbers, the complex plane, polynomials, Power series, transcendental functions such as exponential, trigonometric and hyperbolic functions Analytic functions, conformal mappings, bilinear transformations; complex integration: Cauchy's integral theorem and formula; Liouville's theorem, maximum modulus principle; Taylor and Laurent's series; residue theorem and applications for evaluating real integrals.

Functional Analysis: Normed Linear Spaces; Banach spaces, Hahn-Banach extension theorem, open mapping and closed graph theorems, principle of uniform boundedness; Hilbert spaces, orthonormal bases, Riesz representation theorem, bounded linear operators.

Numerical Analysis: Numerical solution of algebraic and transcendental equations: bisection, secant method, Newton-Raphson method, fixed point iteration; interpolation: error of polynomial interpolation, Lagrange, Newton interpolations; numerical differentiation; numerical integration: Trapezoidal and Simpson rules, Gauss Legendre quadrature, method of undetermined parameters; least square polynomial approximation; numerical solution of systems of linear equations: direct methods (Gauss elimination, LU decomposition); iterative methods (Jacobi and Gauss-Seidel); matrix eigenvalue problems: power method, numerical solution of ordinary differential equations: initial value problems: Taylor series methods, Euler's method, Runge-Kutta methods.

Ordinary Differential Equations: First order ordinary differential equations, existence and uniqueness theorems, systems of linear first order ordinary differential equations, linear ordinary differential equations of higher order with constant coefficients; linear second order ordinary differential equations with variable coefficients; method of Laplace transforms for solving ordinary differential equations, series solutions; Legendre and Bessel functions and their orthogonality.

Partial Differential Equations: Linear and quasilinear first order partial differential equations, method of characteristics; second order linear equations in two variables and their classification; Cauchy, Dirichlet and Neumann problems; solutions of Laplace, wave and diffusion equations in two variables; Fourier series and Fourier transform and Laplace transform methods of solutions for the above equations.

Topology: Basic concepts of topology, product topology, connectedness, compactness, countability and separation axioms, Urysohn's Lemma.

Probability and Statistics: Probability space, conditional probability, Bayes theorem, independence, Random variables, joint and conditional distributions, standard probability distributions and their properties, expectation, conditional expectation, moments; Weak and strong law of large numbers, central limit theorem; Sampling distributions, maximum likelihood estimators, Testing of hypotheses, standard parametric tests based on normal, X^2 , t, F – distributions; Linear regression; Interval estimation.

Suggested Books for Reading

1. Hoffman, K and Kunze, R, Linear Algebra, Prentice Hall of India Pvt Ltd., New Delhi, 1978
2. Herstein, I.N, Topic in Algebra, 2e, Vikas Publishing House Pvt. Ltd, NewDelhi, 1976
3. Rudin, W. Principles of Mathematical Analysis, 3e, International Edition, McGraw-Hill, 1976
4. Apostol,T.M, Calculus - Vol. 1 & 2, 2nd Edn., Wiley India, 2003.
5. Royden, H.L, Real Analysis, 3rd edition, Prentice Hall of India, 1995.
6. Churchill, R.V and Brown, J.W, Complex Variables and Applications, 5th Edition, McGraw-Hill, 1990
7. Simmons, G.F., Introduction to Topology and Modern Analysis, Tata McGraw Hill,2003

8. Atkinson, K.E, Introduction to Numerical Analysis, 2nd Edition, John Wiley, 1989
9. Coddington, E.A and Levinson, N. Theory of Ordinary Differential Equations, Tata McGraw Hill, 1990.
10. Sneddon,I.N, Elements of Partial Differential Equations, McGraw Hill, 1957.
11. Rohatgi, V.K and Md. Ehsane Saleh, A.K, An Introduction to Probability and Statistics, Wiley Student Edition, 2e, 2006.