N5

I B.Tech(ccc) Regular Examinations, December 2007
MATHEMATICS-I
(Common to Civil Engineering, Electrical & Electronic Engineering,
Mechanical Engineering, Electronics & Communication Engineering and
Computer Science & Engineering)

Time: 3 hours   Max Marks:100

Answer any FIVE Questions
All Questions carry equal marks
⋆ ⋆ ⋆ ⋆ ⋆

1. (a) Test the following series for convergence or divergence.
    $$\sum_{n=1}^{\infty} \left[\sqrt{n^2 + n + 1} - \sqrt{n^2 - n + 1} \right]$$ [6]

    (b) Test whether the following series is absolutely convergent.
    $$\sum_{n=1}^{\infty} \frac{Cos\, n\pi}{n^2 + 1}$$ [6]

    (c) State Rolle's theorem and verify Rolle's theorem for the following function
    $\frac{Sinx}{e^x}$ in $(0, \pi)$ [8]

2. (a) If U=$Sin^{-1}\left(\frac{x}{y}\right)$ + $\tan^{-1}$ (y/x) show that $x\frac{\partial \mu}{\partial x} + y\frac{\partial \mu}{\partial y} = 0$ [8]

    (b) Find the radius of curvature at any point on the curve y = c cosh$\frac{x}{C}$ [12]

3. (a) Trace the Cissoid of Diocles : y^2 (2a -x) = x^3.

    (b) Find the entire perimeter of the cardiod r = a (1 + cosθ).   [10+10]

4. (a) Solve the differential equation: x$\frac{dy}{dx}$ + y = x^3y^6.   [10]

    (b) Solve the differential equation: $(D^2 + 4)y$ = sin t + 1/3 sin3t + 1/5 sin5t,
    y(0) = 1, y $'$(0) = 3/35.   [10]

5. (a) Find the Laplace Transformations of the following functions: t e^{-t} sin 2t . [6]

    (b) Evaluate $\iint \frac{r\, dr\, d\theta}{\sqrt{a^2 + r^2}}$ over one loop of the lemniscate r^2 = a^2 Cos2θ.   [8]

    (c) Evaluate $\int\limits_{0}^{5} \int\limits_{0}^{x^2} x(x^2 + y^2)\, dxdy$ [6]

6. (a) Prove that div($\mathbf{A}$x$\mathbf{B}$)=$\mathbf{B}$.curl$\mathbf{A}$ - $\mathbf{A}$.curl$\mathbf{B}$.

    (b) Find the directional derivative of ϕ (x,y,z) = $4xy^2$ + $2x^2yz$ at the point A(1,
    2, 3) in the direction of the line AB where B = (5,0,4).   [10+10]

7. (a) Find the rank of the matrix by reducing it to the normal form.   [10]
    $$\begin{bmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{bmatrix}$$

(b) Find whether the following set of equations are consistent if so, solve them.

[10]

$$3x + 7y + 5z = 4; \quad 26x + 2y + 3z = 9; \quad 2x + 10y + 7z = 5.$$

8. Diagonalize the matrix $A = \begin{bmatrix} 8 & -8 & -2 \\ 4 & -3 & -2 \\ 3 & -4 & 1 \end{bmatrix}$ [20]

★ ★ ★ ★ ★