NORTH MAHARASHTRA UNIVERSITY
JALGAON

SYLLABUS

FOR

M.Sc. STATISTICS
(Semester I and II)
With specialization in Industrial Statistics

WITH EFFECT FROM ACADEMIC YEAR 2007-2008

NORTH MAHARASHTRA UNIVERSITY, JALGAON
Syllabus for M.Sc. (Statistics)
With specialization in Industrial Statistics

(With effect from Academic Year 2007-2008)

Syllabus Structure

Semester-I

Course Code	Title of the Course	Contact hours/week			Distribution of Marks for Examination					
					Internal		External		Total	
		Th(L)	Pr	Total	Th	Pr	Th	Pr	Th	Pr
ST-101	Real Analysis	04	--	04	40	--	60	--	100	--
ST-102	Linear Algebra	04	--	04	40	--	60	--	100	--
ST-103	Sample Surveys and Statistics for National Development	04	--	04	40	--	60	--	100	--
ST-104	Distribution Theory	04	--	04	40	--	60	--	100	--
ST-105	Computer Programming in C++ and Numerical Methods	04	--	04	40	--	60	--	100	--
ST-106	Practicals- I	--	06	06	--	40	--	60	--	100

Semester-II

Course Code	Title of the Course	Contact hours / week			Distribution of Marks for Examination					
					Internal		External		Total	
		Th(L)	Pr	Total	Th	Pr	Th	Pr	Th	Pr
ST-201	Probability Theory	04	--	04	40	--	60	--	100	--
ST-202	Stochastic Processes	04	--	04	40	--	60	--	100	--
ST-203	Multivariate Analysis	04	--	04	40	--	60	--	100	--
ST-204	Parametric Inference	04	--	04	40	--	60	--	100	--
ST-205	Linear Models and Regression Analysis	04	--	04	40	--	60	--	100	--
ST-206	Practicals-II	--	06	06	--	40	--	60	--	100

Th: Theory **Pr: Practicals** **L: Lectures**

General Instructions to Teachers and Paper Setters/ Examiners

1. Each Theory Course requires 60 lectures each of one hour.
2. Each Practical Course requires 90 laboratory periods each of one hour.
3. Numbers of Lectures/periods to be devoted for each topic and minimum number of marks to be allotted out of 60 for main topics are mentioned in parentheses in the detailed syllabi.
4. Teacher should follow syllabus as well as time schedule given in the syllabus for all topics. Variation up to 4 to 5 hours (more or less) may be acceptable.
5. Each external examination theory question paper should contain 5 questions each of 12 marks (all questions will be compulsory with internal choices). Each external practical examination question paper should contain 5 questions each of 20 marks and student has to answer any 3 questions. Paper Setter may consider sub parts such as ((a), (b), (c)…) in each question.
6. Question paper should generally be uniformly distributed over the syllabus.

Examination Pattern:

Number of Internal Tests and Time duration:

Concern Teacher in consultation with Head of the Department may conduct 2 or 3 tests of 40 marks with time duration 2 Hrs for Internal Examination of all Theory and Practical courses. Head of the Department will declare detailed Time-Table well in advance.

External Examination:

University will conduct external examinations at the end of each semester. Each course will have examination of 60 marks of duration 3 Hrs. Practical examination will be conducted before Theory examination. Time-Table for external examinations will be declared by University office well in advance.

Standard of Passing:

To pass any course, the candidate has to secure at least 40% marks in the internal as well as in the external examinations.

The student failed in Internal or External or in both examinations shall have to appear for subsequent Internal or External or both Examinations respectively for that course.

The student having the backlog of any course(s) from first year of M.Sc. can be admitted to second year of M.Sc.

ST-101: REAL ANALYSIS

- **The Real Number System: (8 Marks)**
  - Introduction, The field axioms, the order axioms, Geometric representation of real numbers, Intervals, Integers, The unique factorization theorem for integers. (2L)
  - Rational numbers, Irrational numbers, Upper bounds, Lower bounds, Least upper bound (supremum), Greatest lower bound (infimum) of the sets of real numbers. (2L)
  - The completeness axiom, some properties of the supremum and infimum, Archimedean property of the real number system. (2L)
  - Rational numbers with finite decimal representation, Finite decimal approximations to real numbers, Infinite decimal representation of real numbers. (1L)
  - Absolute values and the triangle inequality, The Cauchy-Schwarz inequality, Plus and minus infinity and the extended real number system R^*. (1L)

- **Basic Notions of Set Theory: (6 Marks)**
  - Ordered pairs, Cartesian product of two sets, Relations and functions. Further terminology concerning functions, One-to-one functions and inverses, Composite functions. (2L)
  - Similar (equinumerous) sets, Finite and infinite sets, Countable and uncountable sets, Uncountability of real number system. (2L)
  - Set algebra, countable collections of countable sets and related results. (2L)

- **Elements of Point Set Topology: (5 Marks)**
  - Introduction to n-dimensional Euclidean space, Open and closed intervals (rectangles), Open and closed sets on the real line, limit points of a set, Compact set. (2L)
  - The Bolzano-Weierstrass theorem, Heine-Borel theorem for real line R (without proof). (1L)

- **Sequences and Series of Real Numbers: (15 Marks)**
  - Introduction and examples of sequences of real number. (1L)
  - Convergence of sequences, limit of a sequence, limit superior and limit inferior of a real-valued sequences, Monotone sequences of real numbers. (2L)
  - Cauchy sequences and related results. (2L)
  - Infinite series, Alternating series. (1L)
  - Convergence of Series, Absolute and conditional convergence. (1L)
  - Test for convergence of series with positive terms (Comparison test and limit comparison test). (1L)
  - The geometric series. (1L)
  - The integral test. (1L)
  - The big O(h) and little o(h) notation. (1L)
  - The Ratio test and Root test, Abel's test. (2L)

- **Limit and Continuity: (6 Marks)**
    - Limits of functions. (1L)
    - Continuous functions. (1L)
    - Uniform continuity. (1L)
    - Discontinuities. (1L)
    - Continuity and compactness. (2L)
    - Monotone function and discontinuities. (1L)

- **Sequences of Functions: (6 Marks)**
    - Introduction and examples of sequences of real-valued functions. (1L)
    - Pointwise convergence of sequences of functions. (1L)
    - Definition of uniform convergence, Uniform convergence and continuity. (2L)
    - Power series and radius of convergence. (1L)

- **Differentiation and functions of several variables. (6 Marks)**
    - The Derivative of a Real Function. (1L)
    - Maxima-minima of function, Mean value theorems. (2L)
    - The Continuity and Derivatives. (1L)
    - Derivatives of higher order, Taylor's theorem (without proof). (1L)
    - Functions of several variables, constrained maxima-minima functions. (2L)

- **Integrals: (8 Marks)**
    - Riemann and Riemann- Stieltjes integrals, integration by parts, mean value theorem. (3L)
    - Multiple integrals and their evaluation by repeated integration. (2L)
    - Change of variables in multiple integration. (2L)
    - Improper Riemann – Stieltjes integrals: Improper integrals of first and second kind for one variable, uniform convergence of improper integrals. (2L)
    - Differentiation under the sign of integral Leibnitz rule. (2L)

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Rudin, Walter (1976). Principles of Mathematical Analysis, (McGraw Hill).

Malik, S. C. (2005). Principles of Real Analysis, (New Age Inter-national (P) Ltd.).

Bartle, R. G. (1976). Elements of Real Analysis, (Wiley).

ST-102: LINEAR ALGEBRA

- **Preliminaries: (2 Marks)**
  - Binary operations, Groups, Polynomials. (2L)
- **Vector Spaces (VS) : (12 Marks)**
  - Definition of VS, Subspaces, Linear span of set, examples of VS over real and complex fields. (3L)
  - Linear span of a set, Span of a set as a smallest subspace containing the set, Generating set of VS, Results on span of a set. (2L)
  - Intersection and union of sub spaces, Completion theorem. (2L)
  - Linear dependence and linear independence of set of vectors, Necessary and sufficient condition for linear dependence of set of vectors. (2L)
  - Steinitz exchange theorem, Maximal linearly independent set, minimal generating sets. (2L)
  - Basis of VS, Dimension, Extension of linearly independent set to a basis (algorithm and theorem), relation between dimensions of subspaces; one of which is subset of other. (4L)
  - Sum of 2 sets, modular law. (1L)
- **Algebra of Matrices: (12 Marks)**
  - Linear transformations and matrices, Addition, Scalar multiple and composition of linear transformation, The corresponding operations on matrices, Elementary properties of matrix operations, Upper and lower triangular matrices, Trace of a matrix and related results. (3L)
  - Row and column spaces, Rank of a matrix, Left inverse, Right inverse and inverse of a matrix, properties of inverse, Upper bound for rank of product of matrices, Rank cancellation laws. (3L)
  - Nullity of matrix, null space of a matrix, relation of rank of null space of a matrix with rank of a matrix. (2L)
  - Lower bound for rank of product of 2 matrices, Rank of sum of matrices. (1L)
  - Partitioned matrix, Elementary matrix, Determinant of a matrix, Its elementary properties, Determinant and inverse of partitioned matrix, Kronecker product. (3L)
- **Linear Equations of Systems of Equations: (8 Marks)**
  - Consistent and inconsistent system of equations, Homogeneous systems and existence of nontrivial solution for it, General linear systems, Solution of systems of equations. (3L)
  - Generalized inverse of a matrix and its properties, Moore-Penrose generalized inverse, Solution of systems of equations. (3L)

- **Inner Product and Orthogonality: (10 Marks)**
  - VS with inner product, Normed vector spaces, Cauchy-Schawrz inequality, Orthogonality and linear independence. (2L)
  - Orthonormal basis, Expression of any vector in VS as a linear combination of elements of orthonomal basis. (2L)
  - Gram-Schmidt orthogonalization process, Extension of any orthogonal set to orthonomal basis of VS, Examples. (2L)
  - Orthogonal and unitary matrices and their properties. (2L)
- **Eigen Values: (8 Marks)**
  - Characteristic polynomial and characteristic equation of a matrix, Characteristic roots, their properties. (2L)
  - Eigen values and eigen vectors, Eigenspaces, Geometric and algebraic multiplicity of an eigen value, Relation between the 2 multiplicities, Simple and regular eigen values, Properties of eigen values. (3L)
  - Cayley-Homilton theorem and minimal polynomial, Singular values and singular vectors. (2L)
  - Spectral decomposition of real symmetric matrix, singular value decomposition, Jordan decomposition. (2L)
- **Quadratic Forms (QF): (8 Marks)**
  - Real QF, Classification, Rank and signature, reduction of any QF to diagonal form. (2L)
  - Definiteness of a matrix, equivalence of nonnegative definite matrix and variance-covariance matrix, Simultaneous reduction of two QF. (3L)
  - Extrema of QF, Maxima and Minima of ratio of two QF. (2L)

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Searle, S. R. (1982). Matrix Algebra Useful for Statistics. (John Wiley and Sons Inc.).

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Bellman, R.(1970). Introduction to Matrix Analysis, (2^{nd} ed.Tata McGraw Hill).
Biswas, S.(1984). Topics in Algebra of Matrices, (Academic Publictions).
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Hoffman, K. and Kunze, R. (1971). Linear Algebra, (2^{nd} Ed.Prentice Hall, Inc.)
Rao, C.R. and Mitra, S.K. (1971). Generlized Inverse of Matrices and its Applications, (John Wiley and Sons Inc.).

ST-103: SAMPLE SURVEYS AND STATISTICS FOR NATIONAL DEVELOPMENT

- **Sample Surveys:**

  - **Preliminaries: (6 Marks)**
    - Objectives of sample survey, planning for sample survey. (1L)
    - Basic issue related to estimation [biased and unbiased estimator, mean square error (MSE)] and confidence interval (2L)
    - Concept of sampling distribution of statistic (2L)
    - Sampling and non-sampling errors (1L)

  - **Review of basic methods of sample selection from finite population. (10 Marks)**
    - Simple random sampling with replacement, Simple random sampling without replacement, Systematic sampling and related results on estimation of population total, mean and proportion. (5L)
    - Stratified sampling: Formation of strata and number of strata, Allocation problems and estimation problems. (5L)

  - **Unequal Probability Sampling Designs: (8 Marks)**
    - Inclusion probabilities, Horwitz-Thompson estimator and its properties. (3L)
    - PPSWR, PPSWOR methods (including Lahiri's scheme) and related estimators of a finite population mean (Heansen-Horwitz and Desraj estimators for a general sample size and Murthy's estimator for a sample of size 2). (5L)
    - Midzuno sampling design, πps design. (3L)

  - Use of supplementary information for estimation, Ratio and Regression estimators based on SRSWOR method of sampling, Their properties and MSEs. (5 Marks,5L)
  - The Jackknife technique. (2 Marks,2L)
  - Cluster sampling, Estimator of population mean and its properties. (4 Marks,3L)
  - Two-stage sampling with equal number of second stage units. (2 Marks,2L)
  - Double sampling and its uses in ratio and regression estimation. (3 Marks,3L)
  - Randomized response technique, Warner's model; related and unrelated questionnaire methods. (4 Marks, 3L)

- **Statistics for National Development:**
  - **Economic Development: (6 Marks)**
    - Growth in per capita income and distributive justice. (1L)
    - Indices of development. (1L)
    - Human Development indexes. (1L)
    - Estimation of national income-product approach, income approach and expenditure approach. (2L)

- Population growth in developing and developed countries, Population projection using Leslie matrix, Labour force projection. (2 Marks, 2L)
- Measuring inequality in incomes, Lorenz curve, Gini coefficient, Theil's measure. (2 Marks, 2L)

- **Poverty measurement: (6 Marks)**
  - Different issues related to poverty. (2L)
  - Measures of incidence and intensity. (2L)
  - Combined measures e.g. Indices due to Kakwani, Sen etc. (2L)

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CSO. National Accounts Statistics- Sources and Health.
Keyfitz, N. (1977). Applied Mathematical Demography, (Springer Verlag).
UNESCO. Principles for Vital Statistics Systems, Series M-12.
Sen, A. (1997). Poverty and Inequality.
Datt R., Sundharam, K. P. M. (Revised edition). Indian Economy, (Sultan Chand & company Ltd.)

ST-104: DISTRIBUTION THEORY

- **Brief review of basic distribution theory: (5 Marks)**
  - Random experiment and its sample space, events. (1L)
  - Probability axioms. (1L)
  - Random variables, Discrete random variables, Continuous random variables. (1L)
  - P.d.f., p.m.f., c.d.f. of random variables. (1L)
  - M.g.f., p.g.f., c.g.f., characteristic function of random variables. (1L)
  - Moments: raw moments, Central moments, Factorial moments. (1L)

- **Standard discrete and continuous distributions: (8 Marks)**
  - Bernoulli, Binomial, Geometric, Negative Binomial, Poisson, Hypergeometric distributions. (2L)
  - Exponential, Normal, Gamma, Beta, Uniform, Chi-square, Lognormal, Weibull, Cauchy distributions. (2L)
  - M.g.f, p.g.f., c.g.f., characteristic function, Moments of above distributions. (2L)
  - Properties of above distributions. (2L)

- **Joint, Marginal and Conditional distributions: (10 Marks)**
  - Concept of random vectors, Joint, Marginal and conditional distributions Variance-covariance matrix. (1L)
  - Joint p.m.f. of discrete random variables, Joint p.d.f. of continuous random variables. (1L)
  - Marginal and conditional density using joint density. (1L)
  - Conditional expectation and variance. (1L)
  - Independence of random variables. (1L)
  - Bivariate normal distribution; Joint p.d.f. Marginal p.d.f.s, Conditional p.d.f., Joint m.g.f., Some properties. (2L)
  - Bivariate exponential distribution: joint p.d.f., Marginal p.d.f.s, properties. (1L)
  - Multivariate normal distribution: joint p.d.f., Marginal p.d.f., Conditional p.d.f., Joint m.g.f. (2L)
  - Multinomial distribution: joint p.m.f., Marginal p.m.f., Conditional p.m.f., Joint m.g.f. (2L)

- **Functions of random variables and their distributions: (10 Marks)**
  - Function of random variables. (1L)
  - Joint density of functions of random variables using jacobian of transformation. (3L)
  - Convolution of random variables. (1L)

- **Compound, Truncated and Mixture Distributions: (3 Marks)**
  - Concept, applications, examples and problems. (3L)

- **Regression: (3 Marks)**
  - Linear and multiple regression, Regression Function, Best linear regression function. (3L)
- **Correlation: (3 Marks)**
  - Multiple and Partial Correlation. (2L)
- **Sampling Distributions: (6 Marks)**
  - Introduction, Sampling distribution of statistics from univariate normal random samples. (2L)
  - Non-central Chi-square, t and F- distributions and their properties. (5L)
- **Quadratic forms under Normality: (6 Marks)**
  - Distribution of linear and quadratic forms in i.i.d. Standard normal variables (Technique based on m.g.f.). (2L)
  - Independence of two linear forms, Independence of two quadratic forms and independence of linear form and quadratic form. (2L)
  - Fisher Cochran's theorem. (2L)
- **Order Statistics: (6 Marks)**
  - Distribution of r^{th} order statistics, Joint distribution of several order statistics and their functions. (4L)
  - Distribution of function of order statistics. (2L)
  - Extreme values and their asymptotic distributions (statement only) with applications. (2L)

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Cramer H. (1946). Mathematical Methods of Statistics, (Princeton).

ST-105: COMPUTER PROGRAMMING IN C++ AND NUMERICAL METHODS

Computer Programming in C++ (40 Marks)

- Principles of Object-Oriented Programming. (3L)
- Beginning with C++. (3L)
- Tokens, Expressions and Control Structures. (6L)
- Functions in C++. (3L)
- Classes and Objects. (5L)
- Constructors and Destructors. (5L)
- Operator Overloading and Type Conversions. (5L)
- Inheritance: Extending Classes. (3L)
- Pointers, Virtual Functions and Polymorphism. (3L)
- Managing Console I/O Operations. (3L)
- Working with Files (including linking to databases). (4L)

Numerical Methods: (20 Marks)

- **Errors in Numerical Calculations**
  Introduction, Errors and their Analysis. A general error formula, error in series approximation. (3L)
- **Iterative methods**
  Introduction, the method of successive bisection, Newton-Raphson method. (3L)
- Interpolation (2L)
- Solution of Simultaneous Algebraic Equations
  Introduction, Direct method, Matrix Inversion Method, The Gauss elimination method, Pivoting, The Gauss-Seidel iterative method, The Eigen value Problem. (3L)

- Numerical Integration
  Introduction, Simpson's 1/3 rule, Trapezoidal Rule, Quadrature Rule, Simpson's 3/8 Rule, Errors in integration formulae, Monte Carlo integration. (6L)

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R.A. Thisted (1988). Elements of Statistical Computing, (Chapman and Hall).

Ross, S. (2005). Introduction to Probability Models, (6th Ed. Academic Press).

ST-106: PRACTICALS-I

A. Introduction to MS-DOS, WIDOWS-XP and different Statistical Software Packages (4 Marks) (10 Hrs.: First 2 weeks)

(Introduction can be done through following simple practical)

1. Classification, tabulation and frequency tables.
2. Bar graphs, histogram.
3. Stem-and- Leaf plots, Box plots.
4. Summary statistics.
5. Two-way tables and plots.
6. Scatter diagram correlation coefficient.

B. Practicals based on Linear Algebra. (Using software packages) (10 Hours, 8 Marks)

1. Checking linearly dependence/independence of set of vector using system of linear equations. (1L)
2. Getting vectors in row/column space and null space of the given matrix. (1L)
3. Computation of inverse of a given matrix.
   - Natural inverse.
   - G-inverse, left and right inverse
   - MP-inverse (1L)
5. Computing higher order powers of a given matrix using spectral decomposition. (1L)
6. Diagonalization and triangulation of a given matrix. (1L)
7. Gram-Schmidt orthonormalization, forming an orthogonal matrix of specified order using Gram-Schmidt orthogonalization. (2L)
8. Checking and demonstrating the definiteness of the given matrix, getting vectors from eigen-space, algebraic and geometric multiplicity of an eigen value etc. (1L)
9. Demonstration of maxima and minima of
   - Quadratic forms in normed vectors.
   - Ratio of two quadratic forms in normed vectors. (2L)

C. Practicals based on the Sampling Theory and Statistics for National Development. (Using software packages) (26 Hours, 20 Marks)

1. Model Sampling and Estimation (4L)
   - Drawing simple random samples from a given finite population using SRSWR and SRSWOR.
   - Estimating the population total, mean and proportion using the sample drawn.
   - Estimating the variance of the estimator obtained above using the sample drawn.
   - Confidence interval for population total, mean and proportion.
   - Comparison of two estimators.
   - Minimum sample size requirement.

2. Stratified Random Sampling (4L)
   - Various kinds of allocation and estimation of population total and mean with S.E.
   - Post stratification.
3. Using Auxiliary Information (4L)
   - Ratio method of estimation
   - Regression method of estimation.
4. H-T estimator and PPS, π PS designs (2L)
5. Double Sampling. (2L)
6. Systematic Sampling (2L)
7. Cluster Sampling (2L)
8. Two stage sampling (2L)
9. Randomized Response Technique (1L)
10. Practical based on Estimation of national income, Income inequality, Poverty measurement. (3L)

D. Practical based on Distribution Theory. (Using software packages) (4 Marks)

1. Generating random samples from discrete, continuous and mixture distributions (2L)
2. Fitting of standard distributions and tests for goodness of fit. (2L)

E. Practicals based on Computer Programming C++ and Numerical Methods. (40 Hrs, 24 Marks)

1. Writing programs to calculate different summary statistics (mean median, mode, variance, standard deviation, order statistics, range and quantiles) based on the given n observations. (3L)
2. Programs to compute the c.d.f.'s of standard probability distributions. (Binomial, Poisson, Geometric, Hyper Geometric, Negative Binomial) (9L)
3. Drawing random samples from standard distributions (Binomial, Poisson, Geometric, Exponential, Normal, Gamma, Beta, Discrete, Mixture) (6L)
4. Drawing a random sample of size n using SRSWR and SRSWOR. (2L)
5. Write a program to define a specified class. Use member function, friend function and overload the specified operators to perform the following tasks. (9L)
   - To create an object of type class.
   - To modify the value of the element of object of a class.
   - To perform unary/binary operations on object.
6. Programs based on the numerical methods. (7L)
   - Bisection method, Newton-Raphson Method
   - Numerical Integration by Simpson's rules
7. Program to compute c.d.f. of normal distribution. (2L)
8. Computing expectations of complicated functions. (2L)

F. Assignment Problem to be solved by students.

1. Preparing frequency distribution of given data.
2. Calculation of p-value for standard Normal distribution (for given Z value)
3. Calculation of regression and correlation coefficients.
4. Sketching p.d.f of the given distribution for various parameters. (Using software)

ST-201: PROBABILITY THEORY

- **Sets and Classes of Events: (6 Marks)**
  - Random experiment, Sample space and events. (1L)
  - Algebra of sets. (1L)
  - Sequence of sets, limit supremum and limit infimum of sequence of sets. (2L)
  - Classes of sets, Sigma-fields (σ-fields), Minimal fields, Minimal σ-field, Partition. (3L)
  - Borel fields in R^1 and R^k, Monotone field. (2L)

- **Random Variables: (6 Marks)**
  - Point function and set function, Inverse function. (2L)
  - Measurable function, Borel function, induced σ-field, Function of a function, Borel function of measurable function. (2L)
  - Real and vector-valued random variable. (2L)
  - σ-field induced by a sequence of random variables. (1L)
  - Limits of Random variable. (2L)

- **Measure and Probability Measure: (6 Marks)**
  - Measure (Definition and simple properties). (1L)
  - Probability measure, Properties of a measure. (1L)
  - Probability space (finite, countable) Continuity of a probability measure. (1L)
  - Extension of probability measure, Caratheodory Extension theorem (without proof). (1L)
  - Probability space induced by r.v. X, Distribution of Borel functions of r.v. (1L)
  - Other measures: Generalized Probability measure, Conditional Probability measure, Counting measure, Lebesgue measure. (2L)

- **Distribution Functions: (6 Marks)**
  - Distribution functions of a r.v. and its properties. (1L)
  - Jordan decomposition theorem, Mixture of distribution functions. (2L)
  - Distribution functions of vector valued r.v.s. (1L)
  - Empirical distribution functions. (1L)

- **Expectation and Moments: (8 Marks)**
  - Integration of measurable function with respect to a measure. (1L)
  - Expectation of a r.v. (Definition for simple, Nonnegative and arbitrary r.v.), Properties of expectation, Expectation of Complex r.v. (3L)
  - Moments, Moment generating function. (1L)
  - Moment inequalities: C_r-inequality, Holder inequality, Schwarz's inequality, Minkowski's inequality, Jensen's inequality, Liapounov's inequality, Basic inequality, Markov inequality, Chebyshev's inequality. (3L)

- **Convergence of Sequence of Random variables: (8 Marks)**
  - Convergence in distribution, Convergence in probability, Almost sure convergence and convergence in quadratic mean and their inter-relations. (5L)
  - Monotone convergence theorem, Fatou's Lemma, Dominated convergence theorem. (3L)

- **Characteristic function: (4 Marks)**
  - Definition and simple properties, Some inequalities. (2L)
  - Uniqueness theorem (statement only), Levy's continuity thereon (Statement only). (1L)

- **Independence: (5 Marks)**

  Independence of two events, Independence of n>2 events, sequence of independent events, independent classes of events, independence of r.v.s, Borel zero-one law. (4L)

- **Law of large numbers: (5 Marks)**
  - Weak laws of large numbers (WLLN), Khintchine's WLLN, Kolmogorov's strong law of large number (Statement only) and their applications. (4L)

- **Central limit theorem (CLT): (6 Marks)**
  - CLT for a sequence of independent r.v.s. under Lindeberg's condition, CLT for i.i.d. r.v.s. and its applications. (3L)

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ST-202: STOCHASTIC PROCESSES

- **Introduction: Stochastic Processes, Markov chains (14 Marks)**
  - Review of conditional probability and Expectation with problems. (2L)
  - Introduction to Stochastic Processes, Classification of Stochastic Processes according to state space and time domain. (1L)
  - Finite and countable state space Markov chains (Definitions and examples). (2L)
  - Chapman-Kolmogorov equations, Calculation of n-step transition probability and its limit. (2L)
  - Stationary distribution of Markov chains. (2L)
  - Classification of states, Period of the state, Transient and recurrent Markov chain and related results. (4L)
  - Random walk and gambler's ruin problem. (2L)
  - First passage time and other problems with applications. (2L)
  - Applications of Marker Chains in Social, Biological and Physical Sciences. (1L)
- **Branching Process: (6 Marks)**
  - Golton-Watson branching process. (2L)
  - Probability of ultimate extinction, Distribution of population size. (2L)
  - Applications. (1L)
- **Discrete state space continuous time Markov Chain. (12 Marks)**
  - Definition and examples. (1L)
  - Markov Pure jumps processes. (2L)
  - Kolmogorov's differential equations. (2L)
  - Poisson process (Definitions, properties and applications). (3L)
  - Birth and death processes, Machine repairmen problem. (2L)
  - Wiener process as a limit of random walk. (2L)
- **Simple Queuing Systems: (6 Marks)**
  - $M|M|1$, $M|M|s$, $M|M|\infty$ queuing systems and their applications. (2L)
  - Stationary solution for $M|M|1$, $M|M|s$, $M|M|\infty$. using birth and death process approach. (2L)
- **Renewal Theory: (10 Marks)**
  - Renewal process (Definition and examples) (2L)
  - Elementary renewal theorem and its applications (2L)
  - Statement and uses of key renewal theorem (1L)
  - Renewal reward process, Regenerative Process, Semi-Markov process. (2L)
  - Age of renewal process and residual life in renewal processes and their distributions. (2L)

- **MCMC Algorithm:(2 Marks)** (2L)
- **Stationary Process: (2 Marks)**
  - Weakly Stationary and strongly stationary processes. (2L)

- **Inference in Markov chains: (8 Marks)**
- Estimation of transition probabilities, estimation of functions of transition probabilities in Markov chains, Testing of order of a Markov chains, Parametric models and their goodness of fit. (8L)

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ST-203: MULTIVARIATE ANALYSIS

- **Introduction to Bivariate Distributions: (4 Marks)**
  - Bivariate normal, Bivariate Poisson, Bivariate Exponential, Bivariate binomial, Bivariate negative binomial. (3L)
- **Multivariate Normal Distribution(MVND): (16 Marks)**
  - Singular and nonsingular MVND, Mean vector and variance covariance matrix. (3L)
  - Characteristic function of MVND. (1L)
  - Additivity property of MVND. (1L)
  - Distribution of linear combinations of individual elements of a vector having MVND, Marginal distributions, Conditional distributions (2L)
  - Necessary and sufficient condition for independence of $\underline{X}^{(1)}$ and $\underline{X}^{(2)}$ (two components of $\underline{X}$). (2L)
  - Central and noncentral χ^2 distribution, their characteristic function, χ distribution. (2L)
  - Distribution of quadratic forms of normal random variables. (2L)
  - Necessary and sufficient condition for a quadratic form to have χ^2 distribution, condition for independence of two quadratic forms, applications. (2L)
  - Random sampling from MVND, Unbiased and maximum likelihood estimators of parameters of MVND, their sampling distributions, independence. (3L)
  - Sample correlation coefficients, their maximum likelihood estimators (mle), Correlation matrix and its mle. (1L)
- **Wishart distribution: (10 Marks)**
  - Wishart matrix, Derivation of Wishart distribution in canonical case and in general case. (3L)
  - Characteristic function of Wishart distribution, Additive property of Wishart distribution, Moments of Wishart distribution. (3L)
  - Properties of Wishart distribution. (4L)
  - Necessary and sufficient condition for XAX' to have Wishart distribution and its application. (2L)
- **Hotelling's T^2 and its applications: (10 Marks)**
  - Hotelling's T^2 statistic as a generalization of square of Student's statistic. (1L)
  - Derivation of Hotelling's T^2 statistic from Likelihood Ratio Test, Application of union-intersection principle to obtain Hotelling's T^2 statistic, Invariance of T^2 statistic under scale transformation. (2L)
  - Distance between two populations, Mahalnobis D^2 statistic and its relation with Hotelling's T^2 statistic. (1L)
  - Application of Hotelling's T^2 **:**
    Test of equality of mean vector for one or more multivariate normal population, Test of equality of components of a mean vector of MVND, Two sample problem. (2L)
  - Rao's U-statistic and its distribution. (1L)

- **Correlation and regression: (4 Marks)**
  - Sample correlation coefficient, its null and non-null distribution, Partial and multiple correlation coefficient, Their sampling distributions and maximum likelihood estimators. (2L)
  - Multiple linear regression models. (2L)

- **Discriminant Analysis: (4 Marks)**
  - Cluster Analysis and classification problem. (2L)
  - Classification and discrimination procedure for discrimination between two multivariate normal populations, sample discriminant function, Probabilities of misclassification and their estimation, Optimum error rate, Test associated with discriminant function. (4L)

- **Principal Components: (4 Marks)**
  - Introduction and need, population principal components, Finding i^{th} principal component, correlation of i^{th} principal component with k^{th} element of vector $\underline{X}$, principal component when Σ has special structure, Sample principal components. (3L)

- **Canonical Correlation: (4 Marks)**
  - Concept of canonical correlation as generalization of multiple correlation, Geometrical interpretation and its use, Definition of canonical correlation and canonical variables, Existence of canonical variables, Canonical correlation as a maximum root of characteristic equation of a matrix, Sample canonical correlation and canonical variables. (3L)

- **Multivariate Analysis of Variance (MANOVA): (4 Marks)**
  - MANOVA for one way and two ways classified data, Wilk's Λ criteria. (3L)

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ST-204: PARAMETRIC INFERENCE

- **Introduction: (4 Marks)**
  - Introduction of Parametric models, Point estimation, Tests of hypotheses and Interval estimation. (1L)
  - Joint distribution of a sample and sampling distribution of a Statistic. (2L)
  - Likelihood function; examples from standard discrete and continuous models (such as Bernoulli, Poisson, Negative Binomial, Normal, Exponential, Gamma, Pareto etc.) Plotting likelihood functions for these models up to two parameters. (2L)

- **Sufficiency: (10 Marks)**
  - Information in data about the parameters and variation in likelihood function, concept of no information. (1L)
  - Sufficiency, Fisher's concept of sufficiency, Sufficient Statistic, Neyman Factorizability criterion, Likelihood equivalence, Minimal sufficient Statistic. (4L)
  - Invariance property of sufficiency under one-one transformation of sample space. (1L)
  - Exponential families and Pitman families. (3L)
  - Fisher information for one and several parameters models. (2L)

- **Methods of Estimation: (8 Marks)**
  - Maximum Likelihood method. (3L)
  - Methods of moments and percentiles. (2L)
  - Unbiased Estimation. (1L)

- **Minimum Variance Unbiased Estimation: (12 Marks)**
  - UMVUE, Rao-Blackwell Theorem. (2L)
  - Completeness property of family of distributions. (3L)
  - Lehmann-Scheffe-Rao-Blackwell Theorem and its applications. (2L)
  - Necessary and sufficient condition for UMVUE. (1L)
  - Cramer-Rao lower bound approach. (2L)

- **Tests of Hypotheses: (15 Marks)**
  - Concepts of critical regions. (1L)
  - Test functions. (1L)
  - Two kinds of errors, Size function, Power function, Level of the test. (2L)
  - Introduction of null and alternative hypotheses with examples. (1L)
  - Most powerful (MP) and Uniformly Most Powerful (UMP) test in the class of size α tests. (1L)
  - Neyman-Pearson Lemma, MP test for simple null against simple alternative hypothesis. (2L)

- UMP tests for simple null hypothesis against one-sided alternatives and for one-sided null against one-sided alternatives in one parameter exponential family. (3L)
- Extensions of these results of Pitman family when only upper or lower end depends on the parameter. (2L)
- MLR property and extension of the above results to the distributions with MLR property. (2L)
- Non-existence of UMP test for simple null against two sided alternatives in one parameter exponential family. (2L)

- **Interval Estimation: (5 Marks)**

  - Confidence level, construction of confidence intervals using pivots, Shortest expected length confidence interval. (3L)
  - Uniformly most accurate one-sided confidence interval and its relation to UMP test for one-sided null against one-sided alternative hypotheses. (2L)

- **Bayesian Estimation: (6 Marks)**

  - Prior distribution, loss function, principle of minimum expected posterior loss, quadratic and other common loss functions, conjugate prior distributions, common examples. (6L)

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ST-205: LINEAR MODELS AND REGRESSION ANALYSIS

- **General Linear Model: (15 Marks)**

  - Gauss-Markov set up, Least square estimation, Normal equations, Consistency of system of normal equations and their solution. (3L)
  - Estimability of linear parametric function, necessary and sufficient condition for estimability, Best Linear Unbiased Estimator (BLUE). (2L)
  - Gauss-Markov theorem, Variances and covariances of BLUE's. (2L)
  - Estimation space, Error space, their ranks, Orthogonality of estimation space and error space. (2L)
  - Simultaneous estimates of linear parametric function, Estimation of error variance, Estimation with correlated observations. (3L)
  - Least square estimates with restriction on parameters, Method of generalized least squares. (3L)

- **Interval Estimation and Test of Hypothesis: (15 Marks)**

  - Under the normality assumption, Distribution of error sum of squares, Regression sum of squares and distribution of BLUE's, their independence. (2L)
  - Distribution of conditional error sum of squares, Distribution of sum of squares due to null hypothesis. (3L)
  - Test of hypothesis for one or more than one estimable linear parametric function, Test of hypothesis of equality of all estimable functions to zero, Testing of sub hypothesis for full rank model, Power of F-test. (3L)
  - Simultaneous confidence interval for n linearly independent estimable parametric functions. (2L)
  - One way and two way classified data, multiple comparison tests due to Tukey-Scheffe. (4L)

- **Regression Analysis: (30 Marks)**

  - Simple linear regression in Gauss-Markov set up. (2L)
  - Multiple regression model, Estimation of regression coefficients, Regression analysis of variance, Fitted values and residuals. (3L)
  - Polynomial regression, Orthogonal polynomials, Response analysis using orthogonal polynomials. (4L)
  - Residuals and their plots as tests for departure from assumptions such as fitness of the model, Normality, Homogeneity of variances and detection of outliers. (4L)
  - Remedial measures and validation, Multi-collinearity, Ridge regression, Robust regression principal component regression subset selection of explanatory variables, Mallows Cp statistic. (7L)
  - Introduction to non-linear regression models, Least square estimation in non-linear regression, Model building and diagnostics. (4L)
  - Logistic Regression: Logit transform, ML estimation, Test of hypotheses, Wald test, LR test, score test. (7L)

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Cook, R.D. and Weisberg, S. (1982). Residual and Influence in Regression, (Chapman and Hall).

Draper, N.R.and Smith, H. (1998). Applied Regression Analysis, (3^{rd} Ed.Wiley).

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ST-206: PRACTICALS-II

A. Practicals based on Probability Theory. (5 Hours, 4 Marks)

1. Plotting c.d.f. (1L)
2. Applications and verification WLLN. (2L)
3. Applications and verification of CLT. (2L)

B. Practicals based on Stochastic Processes. (12 Hours, 8 Marks)

1. Calculation of n-step transition probabilities and limiting distribution in Markov chain. (2L)
2. Realization of Markov chain. (2L)
3. Realization of Branching process. (2L)
4. Simulation of Poisson process. (1L)
5. Simulation of Random Walk. (1L)
6. Simulation of Renewal process. (1L)
7. Simulation of $M \mid M \mid 1$ queuing system. (1L)
8. Estimation of transition probability of Markov chain using realization. (1L)
9. MCMC Techniques (1L)

C. Practicals based on Multivariate Analysis. (28 Hours, 18 Marks)

1. Model sampling from bivariate distributions (Bivariate exponential, Bivariate Poisson, Bivariate Poisson, Bivariate negative binomial, Bivariate binomial) (4L)
2. Model sampling from multivariate normal distribution (including conditional distribution) (4L)
3. Estimation of $\underline{\mu}, \Sigma$-matrix, correlation coefficient, multiple correlation coefficients. Test of significance of multiple and partial correlation coefficients. (4L)
4. Applications of Hotelling's T^2. (6L)
5. Discriminant Analysis and Classification problem. (4L)
6. Principal components. (2L)
7. Canonical Correlation. (2L)
8. MANOVA (2L)

D. Practicals based on Parametric Inference. (14 Hours, 10 Marks)

1. Sampling distribution of Statistics/ Estimators (3L)
2. Plotting likelihood functions for standard probability distributions. (3L)
3. Moment Estimation, Maximum Likelihood Estimation (for discrete, continuous, mixture, truncated distributions.) (4L)
4. Power of the test, MP test, UMP test (for continuous, mixture and truncated distributions), Minimum sample size needed to attain given power. (4L)

E. Practicals based on Linear Models and Regression Analysis.
(30 Hours, 20Marks)

1. Linear Estimation. (4L)
2. Analysis of CRD, RBD, LSD. (6L)
3. Test of hypotheses for one and more than one linear parametric functions. (4L)
4. Multiple Regression:
   - Estimation of regression coefficient, Fitting of multiple linear regression.(2L)
   - Testing of hypothesis concerning regression coefficient. (2L)
   - Testing of significance of association between the dependent and independent variables. (2L)
   - Lack of fit test, Extra sum of squares principle. (2L)
5. Orthogonal Polynomials: Fitting of orthogonal polynomials. (2L)
6. Residual Analysis. (2L)
7. Non-linear regression. (2L)
8. Logistic Regression. (2L)