15. (a) Show that $T \in A_F(v)$ is unitary if and only if, it takes an orthonormal basis of V into orthonormal basis of V.

(b) Show that the multiplication group of non-zero elements of a finite field is cyclic.

Register Number :

Name of the Candidate :

2 0 8 3

M.Sc. DEGREE EXAMINATION, 2011

(MATHEMATICS)

(FIRST YEAR)

(PAPER - I)

110. ALGEBRA

May] [Time : 3 Hours

Maximum : 100 Marks

SECTION - A (8× 5 =40)

Answer any EIGHT questions.
Each question carries FIVE marks.

1. Show that HK is a subgroup of G if and only if, HK = KH.

2. Show that the number of conjugate classes in S_n is P(n), the number of partitions of *n*.

3. Show that if F is a field, its only ideals are {0} and F itself.

Turn Over

4. If f(x) , g(x) are non-zero elements in F(x) then, show that

$$\deg [f(x)g(x)] = \deg f(x) + \deg g(x).$$

5. Show that every abelian group G is a module over the ring of integers.

6. Show that $a \in K$ is a root of $p(x) \in F(x)$, where $F \subset K$, then K(x), $(x-a) \mid p(x)$.

7. Show that homomorphic image of a solvable group is solvable.

8. State and prove Bessel's inequality.

9. If S, T are nilpotent linear transformations which commute, prove that ST, S+T are nilpotent linear transformations.

10. For every prime numbers *p* and every positive integer *m,* show that there exists a field having p^m elements.

SECTION - B (3×20 = 60)

Answer any THREE questions.
Each question carries TWENTY marks.

11. (a) Let $\phi ; G \to \overline{G}$ be a onto homomorphism with kernal K. Show that

$$\frac{G}{K} \cong \overline{G}.$$

(b) State and prove First Sylow's theorem.

12. State and prove second isomorphism theorem for ring theory.

13. (a) Show that if

$$\dim_F V = m, \text{ then}$$

$$\dim_F \mathrm{Hom}(V,V) = m^2.$$

(b) Show that if V is a finite dimensional inner product space and W a subspace of V then, $V = W \oplus W^+$.

14. (a) Show that $a \in K$ is algebraic over F if and only if, F(a) is a finite extension of F.

(b) Show that K is a normal extension of F if and only if, K is the splitting field of some polynomial over F.