MATHEMATICS

This section contains 8 multiple choice questions numbered 1 to 8. Each question has 4 choices (a), (b), (c) and (d), out of which **ONLY ONE** choice is correct.

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1. If every pair of equation among the equations $x^2 + px + qr = 0, x^2 + qx + rp = 0$ and $x^2 + rx + pq = 0$ has a common root then the sum of the three common roots is

(a) $-\frac{1}{2}$ (b) 0

(c) –1 (d) 1

2. The number of integral solutions of $x + y + z = 0$, where $x \geq -5$, $y \geq -5$, $z \geq -5$ is

(a) 135 (b) 136

(c) 455 (d) 105

3. A triangle with vertices represented by complex numbers z_0, z_1, z_2 has opposite side lengths in ratio $2 : \sqrt{6} : \sqrt{3} - 1$ respectively. Then

(a) $(z_2 - z_0)^4 = -9(7 + 4\sqrt{3})(z_1 - z_0)^4$ (b) $(z_2 - z_0)^4 = 9(7 + 4\sqrt{3})(z_1 - z_0)^4$

(c) $(z_2 - z_0)^4 = (7 + 4\sqrt{3})(z_1 - z_0)^4$ (d) none of these

4. Let the function $f(x)$ be defined as follows:

$$f(x) = \begin{cases} x^3 + x^2 - 10x & , \quad -1 \leq x < 0 \\ \cos x & , \quad 0 \leq x < \frac{\pi}{2} \\ 1 + \sin x & , \quad \frac{\pi}{2} \leq x \leq \pi \end{cases}$$

. Then $f(x)$ has

(a) a local minimum at $x = \frac{\pi}{2}$ (b) a local maximum at $x = \frac{\pi}{2}$

(c) absolute minimum at $x = -1$ (d) absolute maximum at $x = \pi$

5. If $f(x) = \begin{cases} [x], & \text{if } -3 \leq x < 0 \\ 2x + 1, & \text{if } 0 \leq x \leq 2 \end{cases}$ and $g(x) = f(|x|) + |f(x)|$, then in [–3, 2]

(a) $\lim_{x \to 0^+} g(x) = 2$ (b) $\lim_{x \to 0^-} g(x) = 0$

(c) $g(x)$ is discontinuous at three points (d) none of these

6. Let $f(x)$ be a continuous function for all x, such that $(f(x))^2 = \int_0^x f(t) \cdot \frac{2\sec^2 t}{4 + \tan t}\, dt$ and $f(0) = 0$, then

(a) $f\left(\frac{\pi}{4}\right) = \log\frac{5}{4}$ (b) $f\left(\frac{\pi}{4}\right) = \frac{3}{4}$

(c) $f\left(\frac{\pi}{2}\right)=2$ (d) none of these

7. The equation of the smallest circle passing through the intersection of line $x + y = 1$ and the circle $x^2+y^2=9$ is

(a) $x^2+y^2+x+y-8=0$ (b) $x^2+y^2-x-y-8=0$

(c) $x^2+y^2-x+y-8=0$ (d) none of these

8. If $\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$ $(a > b)$ and $x^2-y^2=c^2$ cut each other at right angles, then

(a) $a^2+b^2=2c^2$ (b) $b^2-a^2=2c^2$

(c) $a^2-b^2=2c^2$ (d) $a^2b^2=2c^2$

Passage-I

$A(3, 7)$ and $B(6, 5)$ are two points. $C:\ x^2+y^2-4x-6y-3=0$ is a circle.

9. The chords in which the circle C cuts the members of the family S of circle passing through A and B are concurrent at

(a) (2, 3) (b) $\left(2, \frac{23}{3}\right)$

(c) $\left(3, \frac{23}{2}\right)$ (d) (3, 2)

10. Equation of the member of the family of circles S that bisects the circumference of C is

(a) $x^2+y^2-5x-1=0$ (b) $x^2+y^2-5x+6y-1=0$

(c) $x^2+y^2-5x-6y-1=0$ (d) $x^2+y^2+5x-6y-1=0$

11. If O is the origin and P is the center of C, then difference of the squares of the lengths of the tangents from A and B to the circle C is equal to

(a) $(AB)^2$ (b) $(OP)^2$

(c) $\left|(AP)^2-(BP)^2\right|$ (d) none of these

Passage-II

In a parallelogram $OABC$, vectors $\vec{a}, \vec{b}, \vec{c}$ are respectively the position vectors of vertices A, B, C with reference to O as origin. A point E is taken on the side BC which divides it in the ratio of 2 : 1. Also, the line segment *AE* intersects the line bisecting the angle *O* internally in point *P*. If *CP*, when extended meets *AB* in point *F*. Then

12. The position vector of point *P* is

(a) $\frac{3|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$ (b) $\frac{|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$

(c) $\frac{2|\vec{a}||\vec{c}|}{3|\vec{c}|+2|\vec{a}|}\left\{\frac{\vec{a}}{|\vec{a}|}+\frac{\vec{c}}{|\vec{c}|}\right\}$ (d) none of these

13. The position vector of point *F* is

(a) $\vec{a}+\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (b) $\vec{a}+\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$

(c) $\vec{a}+\frac{1}{2}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (d) $\vec{a}+\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

14. The vector $\overrightarrow{AF}$ is given by

(a) $\frac{1}{3}\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$ (b) $\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

(c) $\frac{2|\vec{a}|}{|\vec{c}|}\vec{c}$ (d) $-\frac{|\vec{a}|}{|\vec{c}|}\vec{c}$

15. The period of the function $f(x)=\cos(\sin|x|+\cos|x|)$ is

(a) $\frac{\pi}{2}$ (b) π (c) 1 (d) none of these

16. $\lim_{x\to 0^+}[1+[x]]^{\frac{2}{x}}$, where $[\cdot]$ is greatest integer function, is equal to

(a) 0 (b) 1 (c) e^2 (d) does not exist

17. $P(A)=\frac{3}{7}, P(B')=\frac{1}{2}, P(A'\cap B')=\frac{1}{14}$ then $P(A\cap B)$ is equal to

(where A', B' are complement of *A*, *B*)

(a) $\frac{1}{14}$ (b) $\frac{3}{8}$ (c) 0 (d) none of these

18. If the medians *AD* and *BE* of a ΔABC intersect at right angles then (a^2+b^2) is equal to

(a) $5c^2$ (b) $4c^2$ (c) $3c^2$ (d) $2c^2$

19. $\lim\limits_{x \to I^-} \dfrac{e^{\{x\}} - \{x\} - 1}{\{x\}^2}$ equals, where $\{\cdot\}$ is fractional part function and I is an integer

(a) $\dfrac{1}{2}$ (b) $e - 2$

(c) I (d) does not exist

20. The differential equation of the system of circles touching the x-axis at origin is

(a) $(x^2 - y^2)\dfrac{dy}{dx} + 2xy = 0$ (b) $(x^2 - y^2)\dfrac{dy}{dx} - 2xy = 0$

(c) $(x^2 + y^2)\dfrac{dy}{dx} + 2xy = 0$ (d) a second order differential equation

MORE THAN ONE

21. Let $g(x) = [1 + |x|]$, $x \in (-1, 0) \cup (0, 1)$, then the range of $\ln[|g(x)|]$, (where [.] denotes the greatest integer function) is

(a) finite set (b) R

(c) natural number (d) $\{0\}$

22. Let $f : R \to R; f(x) = [x]^2 + [x + 1] - 3$, (where [.] denotes the greatest integer function), then

(a) $f(x)$ is many-one-into (b) $f(x) = 0$ for infinite $x \in R$

(c) $f(x) = 0$ for only two real values (d) $f(x) = 0$ for only finite real values

23. The function $f : R \to (-1, 1)$ defined by $f(x) = \dfrac{e^x - e^{-x}}{e^x + e^{-x}}$, then

(a) $f(x)$ is a non-bijective function (b) $f(x)$ is a bijective function

(c) $f^{-1}(x) = \dfrac{1}{2}\log\left(\dfrac{1+x}{1-x}\right)$ (d) $f(x)$ is many-one-onto function

24. Let $f(x) = \cos(\sqrt{p}\, x)$, where $p = [a]$, ([.] denotes the greatest integer function), is a periodic function with period π, then

(a) $a = \dfrac{9}{2}$ (b) $a = 4$ (c) $a = 2$ (d) $a = \dfrac{5}{2}$

25. If $f = \sin|\cos x|$, $g = \cos|\sin y|$, then

(a) least value of $f + g$ is cos 1 (b) greatest value of $f + g$ is sin 1

(c) period of g is $\dfrac{\pi}{2}$ (d) greatest value of g is 1

26. Let f and g be functions from the interval $[0, \infty)$ to the interval $[0, \infty)$, f being an increasing function and g being a decreasing function, then

(a) $f(g(x)) \geq f(g(0))$ (b) $g(f(x)) \leq g(f(0))$

(c) $f(g(2)) \leq f(g(0))$ (d) none of these

27. If $f(x)$ is a polynomial function satisfying the condition $f(x).f\left(\frac{1}{x}\right)=f(x)+f\left(\frac{1}{x}\right)$ and $f(2)=9$, then

(a) $2f(4)=3f(6)$ (b) $14f(1)=f(3)$ (c) $9f(3)=2f(6)$ (d) $f(10)=f(11)$

28. Let $f(x)=e^{[\sin x-\cos x+1]}$, where [.] denotes greatest integer function, then

(a) least value of $f(x)$ is e^{-1} (b) greatest value of $f(x)$ is e^{-1}

(c) $f\left(\frac{\pi}{2}\right)$ is e (d) $f(x)$ takes only four values

29. $\lim\limits_{n\to\infty} {}^nC_x \left(\frac{m}{n}\right)^x \left(1-\frac{m}{n}\right)^{n-x}$ equals to

(a) $\frac{m^x}{x!}.e^{-m}$ (b) $\frac{m^x}{x!}.e^{m}$ (c) e^0 (d) $\frac{m^{x+1}}{me^m x!}$

30. Given the function $f(x)=\frac{1}{(1-x)}$, the points of discontinuity of the composite function $y=f^{3n}(x)$, where $f^n(x)=fof......of$ (n times) are

(a) 0 (b) 1 (c) $3n$ (d) 2

SUBJECTIVE

1. n has 4 digits, which are consecutive integers in decreasing order (from left to right). Find the sum of the possible remainders when n is divided by 37.

2. The set A consists of m consecutive integers with sum 2m. The set B consists of 2m consecutive integers with sum m. The difference between the largest elements of A and B is 99. Find m.

3. P is a convex polyhedron with 26 vertices, 60 edges and 36 faces. 24 of the faces are triangular and 12 are quadrilaterals. A *space diagonal* is a line segment connecting two vertices which do not belong to the same face. How many space diagonals does P have?

4. A square X has side 2. S is the set of all segments length 2 with endpoints on adjacent sides of X. The midpoints of the segments in S enclose a region with area A. Find 100A to the nearest whole number.

5. A and B took part in a two-day maths contest. At the end both had attempted questions worth 500 points. A scored 160 out of 300 attempted on the first day and 140 out of 200 attempted on the second day, so his two-day success ratio was 300/500 = 3/5. B's attempted figures were different from A's (but with the same two-day total). B had a positive integer score on each day. For each day B's success ratio was less than A's. What is the largest possible two-day success ratio that B could have achieved?

6. An integer is *snakelike* if its decimal digits $d_1d_2...d_k$ satisfy $d_i < d_{i+1}$ for i odd and $d_i > d_{i+1}$ for i even. How many snakelike integers between 1000 and 9999 have four distinct digits?

7. Find the coefficient of x^2 in the polynomial (1-x)(1+2x)(1-3x)...(1+14x)(1-15x).

8. A *regular n-star* is the union of n equal line segments P_1P_2, P_2P_3, ... , P_nP_1 in the plane such that the angles at P_i are all equal and the path $P_1P_2...P_nP_1$ turns counterclockwise through an angle less than 180^o at each vertex. There are no regular 3-stars, 4-stars or 6-stars, but there are two non-similar regular 7-stars. How many non-similar regular 1000-stars are there?

9. ABC is a triangle with sides 3, 4, 5 and DEFG is a 6 x 7 rectangle. A line divides ABC into a triangle T_1 and a trapezoid R_1. Another line divides the rectangle into a triangle T_2 and a trapezoid R_2, so that T_1 and T_2 are similar, and R_1 and R_2 are similar. Find the smallest possible value of area T_1.

10. A circle radius 1 is randomly placed so that it lies entirely inside a 15 x 36 rectangle ABCD. Find the probability that it does not meet the diagonal AC.