

Code No: X0521

R07

SET - 1

II B.Tech I Semester, Supplementary Examinations, May - 2012

PROBABILITY AND STATISTICS

(Com. to CSE, IT)

Time: 3 hours

Max Marks: 80

Answer any **FIVE** Questions
All Questions carry **Equal** Marks

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- A Single card is drawn from an ordinary deck of 52 cards. Find the probability that the card is i) a king ii) a face card iii) a red card iv) a red face card.
 - If A and B are two events, than prove that $P(A \cup B) = P(A) + P(B) - P(A \cap B)$.
 - State and prove Baye's Theorem. (6M+5M+5M)
 - Find k in the Probability function $f(x) = k \binom{3}{x}$, $x = 0, 1, 2, 3$ and Sketch f and the distribution function F .
 - A random variable X has density function

$$f(x) = \frac{k}{x^2 + 1}, \text{ where } -\infty < x < \infty.$$

Find: i) The constant k ,
 ii) The distribution function $F(x)$. (8M+8M)
 - Find the mean and variance of a random variable X that is binomially distributed.
 - If the heights of 300 students are normally distributed with mean 68.0 inches and standard deviation 3.0 inches, how many students have heights
 - Greater than 72 inches,
 - Less than or equal to 64 inches,
 - Between 65 and 71 inches inclusive
 (8M+8M)
 - A population consists of the four numbers 3, 7, 11, 15. Consider all possible samples of size two that can be drawn with replacement from this population.

Find i) The population mean,
 ii) The population standard deviation,
 iii) The mean of the sampling distribution of means,
 iv) The standard deviation of the sampling distribution of means. (16M)
 - Define unbiased estimator. Show that S^2 is an unbiased estimator of the parameter σ^2
 - The contents of 7 similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Find a 95% confidence interval for the mean of all such containers, assuming an approximate normal distribution. (8M+8M)

6. a) A random sample of 100 recorded deaths in the United States during the past year showed an average life span of 71.8 years. Assuming a population standard deviation of 8.9 years, does this seem to indicate that the mean life span today is greater than 70 years? Use a 0.05 level of significance.
- b) In a random sample of 100 tube lights produced by company A, the mean lifetime (mlt) of tube light is 1190 hours with standard deviation of 90 hours. Also in a random sample of 75 tube lights from company B the mean lifetime is 1230 hours with standard deviation of 120 hours. Is there a difference between the mean lifetimes of the two brands of tube lights at a significance level of 0.05? (8M+8M)
7. a) From a random sample of 10 pigs fed on diet A, the increases in weight in a certain period were 10, 6, 16, 17, 13, 12, 8, 14, 15, 9 lbs. For another random sample of 12 pigs fed on diet B, the increases in the same period were 7, 13, 22, 15, 12, 14, 18, 8, 21, 23, 10, 17 lbs. Test whether diets A and B differ significantly as regards their effect on increases in weight? Use a 0.05 level of significance.
- b) Referring to the data of following table, test the hypothesis that the opinions concerning the proposed abortion law the same within each political affiliation. Use a 0.05 level of significance. (8M+8M)

Political Affiliation				
Abortion Law	Democrat	Republican	Independent	Total
For	82	70	62	214
Against	93	62	67	222
Undecided	25	18	21	64
Total	200	150	150	500

8. a) Derive the average number of customers in the system. In (M/M/1): (∞ /FCFS) model.
- b) Assume that both arrival rate and service rate following Poisson distribution.
- The arrival rate and service rate are 25 and 35 customers/hour respectively, at a single window in RTC reservation counter. Find ρ, L_s, L_q, W_s, W_q . (6M+10M)

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SET - 2

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- ~~~~~
- Suppose three companies X, Y, Z produce T.V's. X produce twice as many as Y while Y and Z produce the same number. It is known that 2% of X, 2% of Y and 4% of Z are defective. All the TV's produced are put into one shop and then one TV is chosen at random. Suppose a TV chosen is defective, what is the probability that this TV is produced by company X?
 - Three balls are drawn successively from the box containing 6 red balls, 4 white balls and 5 blue balls. Find the probability that they are drawn in the order red, white and blue if each ball is i) replaced, ii) not replaced. (8M+8M)
 - Let X be the number of years before a certain kind of pump needs replacement. Let X has the Probability function $f(x) = kx^3$ $x = 0, 1, 2, 3, 4$. Find k . Sketch f and the distribution function. F.
 - The distribution function for a random variable X is $F(x) = \begin{cases} 1 - e^{-2x}, & x \geq 0 \\ 0, & x < 0 \end{cases}$
Find i) the density function,
ii) $P(X > 2)$,
iii) $P(-3 < X \leq 4)$. (8M+8M)
 - Find the mean and variance of a random variable X that is Poisson distributed.
 - Find the mean and standard deviation of a normal distribution in which 7% of the items are under 35 and 89% are under 63. (8M+8M)
 - Let U_1 be a variable that stands for any of the elements of the population 3, 7, 8 and U_2 be a variable that stands for any of the elements of the population 2, 4. Compute: i) μ_{U_1}

ii) μ_{U_2} iii) $\mu_{U_1 + U_2}$ iv) $\mu_{U_1 - U_2}$ v) σ_{U_1} vi) σ_{U_2} vii) $\sigma_{U_1 + U_2}$ viii) $\sigma_{U_1 - U_2}$ Verify that vii) $\mu_{U_1 + U_2} = \mu_{U_1} + \mu_{U_2}$ viii) $\mu_{U_1 - U_2} = \mu_{U_1} - \mu_{U_2}$ (16M)
 - Define unbiased estimator. Show that \bar{X} is an unbiased estimator of the parameter μ .
 - The average zinc concentration recovered from a sample of zinc measurements in 36 different locations is found to be 2.6 grams per milliliter. Find the **95%** and **99%** confidence intervals for the mean zinc concentration in the river. Assume that the population standard deviation is 0.3. (6M+10M)

6. a) A manufacturer of sports equipment has developed a new synthetic fishing line that he claims has mean breaking strength of 8 kilograms with a standard deviation of 0.5 kilogram. Test the hypothesis that $\mu = 8$ kilograms against the alternative that $\mu \neq 8$ kilograms if a random sample of 50 lines is tested and found to have a mean breaking strength of 7.8 kilograms. Use a 0.01 level of significance.
- b) To test the effects a new pesticide on rice production, a farm land was divided into 60 units of equal areas, all portions having identical qualities as to soil, exposure to sunlight, etc. The new pesticide is applied to 30 units while old pesticide to the remaining 30. Is there reason to believe that the new pesticide is better than old pesticide if the mean number of kgs of rice harvested/unit using new pesticide is 496.31 with s.d.of 17.08kgs while for old pesticide is 485.41 kgs and 14.73kgs. Test at a level of significance 0.05. (8M+8M)

7. a) Two samples of sodium vapor bulbs were tested for length of life and the following results were returned:

	Size	Sample mean	Sample S.D
Type I	8	1234 hrs	36 hrs
Type II	7	1036 hrs	40 hrs

Is the difference in the means significant to generalize that type I is superior to type II regarding length of life? Use a 0.01 Level of significance.

- b) A sample poll of 300 voters from district A and 200 voters from district B showed that 56% and 48%, respectively, were in favor of a given candidate. At a level of significance of 0.05, test the hypothesis that the candidate is preferred in district A. (8M+8M)

8. a) Explain briefly the main characteristics of Queuing system?
- b) Explain Traffic intensity?
- c) Explain (M/ M/ 1), (∞ / FCFS) Queuing model. (6M+5M+5M)

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1. a) Define Probability of an Event.
 - i) Prove that the impossible event has probability zero.
 - ii) If A^c is the complement A of then prove that $P(A^c) = 1 - P(A)$.
 b) What is the Probability of getting a total of **7** or **11** when a pair fair dice are tossed?
 c) Box I contains 1 white, 2 red, 3 green balls, Box II contains 2 white, 3 red, 1 green ball, Box III contains 3 white, 1 red, 2 green balls. Two balls are drawn from a box chosen at random. These are found to be one white and one red. Determine the probability that the balls are drawn came from box I. (6M+5M+5M)
2. a) Suppose that pair of fair dice are to be tossed, and let the random variable X denote the sum of the points. Obtain the probability distribution for X .
 b) If the Probability density of a random variable is given by

$$f(x) = \begin{cases} x & \text{for } 0 < x < 1 \\ 2 - x & \text{for } 1 \leq x < 2. \\ 0 & \text{elsewhere} \end{cases}$$
 Find the Probabilities that a random variable having this Probability density will take on a value i) between 0.3 and 0.9 ; ii) between 0.8 and 1.5. (8M+8M)
3. a) If a random variable X has a Poisson distribution such that $P(X=1) = P(X=2)$, Find i) Mean of the distribution. ii) $P(X=4)$.
 b) Calculate the mean and S.D of a normal distribution in which 31% are under 45 and 8% are over 64. (8M+8M)
4. A population consists of the four numbers **1, 5, 6, 8**. Consider all possible samples of size two that can be drawn with replacement from this population.
 Find i) The population mean,
 ii) The population standard deviation,
 iii) The mean of the sampling distribution of means,
 iv) The standard deviation of the sampling distribution of means. (16M)
5. a) Explain briefly the following
 - i) Point Estimation ii) Interval Estimation iii) Bayesian Estimation
 b) In nine determinations of the melting point of tin, a chemist obtained a mean of 230.86 degrees Celsius with a standard deviation of 0.17 degree. If he uses this mean to estimate the actual melting point of tin, what can the chemist assert with 95% confidence about the maximum error?
 c) The contents of 7 similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Find a 99% confidence interval for the mean of all such containers, assuming an approximate normal distribution. (6M+5M+5M)

6. a) Explain briefly the following

- i) Null Hypothesis,
- ii) Alternative hypothesis
- iii) Type I error
- iv) Type II error

b) An examination was given to two classes consisting of 40 and 50 students, respectively. In the first class the mean grade was 74 with a standard deviation of 8, while in the second class the mean grade was 78 with a standard deviation of 7. Is there a significant difference between the performances of the two classes at a level of significance of 0.05? (8M+8M)

7. a) A test of the breaking strengths of 6 ropes manufactured by a company showed a mean breaking strength of 7750 lb and a strength deviation of 145 lb, whereas the manufacturer claimed a mean breaking strength of 8000 lb. Can we support the manufacturer's claim at a level of significance of 0.05?

b) Fit a Poisson distribution to the following data and test for its goodness of fit at level of significance 0.05. (6M+10M)

x	0	1	2	3	4	5
f	275	138	75	7	4	1

8. a) Derive the average number of customers in the Queue. In (M/M/1): (∞ /FCFS) model.

b) The arrival rate of customers at a counter in a bank follows Poisson distribution with a mean of 45/hour; service rate of the clerk follows Poisson distribution with a mean of 60/hour. Find the probability of having 0, 5, 10 customers in the system. Find L_s, L_q, W_s, W_q . (8M+8M)

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1. a) Define Probability of an Event. Write the axioms of Probability.
b) What is the Probability of getting a total of **8** or **10** when a pair fair dice are tossed?
c) Suppose a student dormitory in a college consists of the following: 30 % are freshmen of whom 10% own a car, 40 % are Sophomores of whom 20% own a car, 20 % are Juniors of whom 40% own a car, 10 % are Seniors of whom 60% own a car. A student is randomly selected from the dormitory,
i) Find the Probability that the student owns a car.
ii) If the student owns a car; find Probability that a student is Junior? (4M+4M+8M)
2. a) Determine the discrete probability distribution of a discrete random variable X which denotes the minimum of the two numbers that appear when a pair of fair dice is thrown once.
b) Consider the density function $f(x) = \begin{cases} k\sqrt{x}, & 0 < x < 1 \\ 0, & \text{otherwise} \end{cases}$.
i) Evaluate k,
ii) Find $F(x)$ and use it to evaluate $P(0.3 < X < 0.6)$. (8M+8M)
3. a) Out of 800 families with 5 children each, how many would you expect to have
i) 3 boys ii) 5 girls iii) either 2 or 3 boys. Assume equal probabilities for boys and girls.
b) Let X be normally distributed with mean $\mu = 8$ and standard deviation $\sigma = 4$ Find
i) $P(5 \leq X \leq 10)$. ii) $P(10 \leq X \leq 15)$. iii) $P(X \geq 15)$. iv) $P(X \leq 5)$. (8M+8M)
4. A population consists of the four numbers **2, 3, 4, 5**. Consider all possible samples of size two that can be drawn without replacement from this population.
Find a) The population mean,
b) The population standard deviation,
c) The mean of the sampling distribution of means,
d) The standard deviation of the sampling distribution of means, (16M)
5. a) Define unbiased estimator. Show that S^2 is an unbiased estimator of the parameter σ^2 .
b) The contents of 7 similar containers of sulfuric acid are 9.8, 10.2, 10.4, 9.8, 10.0, 10.2, and 9.6 liters. Find a 95% and a 99% confidence interval for the mean of all such containers, assuming an approximate normal distribution. (8M+8M)

6. a) Explain briefly the following
 i) Level of significance ii) Critical region iii) Left One tailed test iv) Right one tailed test
 b) The IQ s (intelligence quotients) of 16 students from one area of a city showed a mean of 107 with a standard deviation of 10, while the IQs of 14 students from another area of the city showed a mean of 112 with a standard deviation of 8. Is there a significant difference between the IQs of the two groups at a 0.01 level of significance? (8M+8M)

7. a) Mean life time (mlt) of computers manufactured by a company is 1120 hours with standard deviation of 125 hours. Test the hypothesis that mean lifetime of computers has not changed if a sample of 8 computers has a mlt of 1070 hours? Use 0.05 level of significance.
 b) Fit a Binomial distribution to the following data and test for its goodness of fit at level of significance 0.05. (6M+10M)

No. of Heads	0	1	2	3	4	5
No.of Tosses(Frequency)	38	144	342	287	164	25

8. The containers from railway goods wagons are unloaded at a single platform of a railway goods yard. The arrival rate of wagons is 8 wagons per day and service rate is 14 wagons per day. Assuming the arrival rate and service rate to follow Poisson distribution, determine the following:
 a) Utilization of railway goods yard
 b) Average number of waiting wagons in the queue
 c) Average number of waiting wagons in the system
 d) Average waiting time per wagon in the queue
 e) Expected waiting time per wagon in the system (16M)