DESIGN OF QUESTION PAPER

MATHEMATICS

CLASS XII

Time : 3 Hours **Max. Marks : 100**

Weightage of marks over different dimensions of the question paper shall be as follows:

A. Weightage to different topics/content units

S.No.	Topics	Marks
1.	Relations and Functions	10
2.	Algebra	13
3.	Calculus	44
4.	Vectors & three-dimensional Geometry	17
5.	Linear programming	06
6.	Probability	10
	Total	**100**

B. Weightage to different forms of questions

S.No.	Forms of Questions	Marks for each question	No. of Questions	Total marks
1.	Very Short Answer questions (VSA)	01	10	10
2.	Short Answer questions (SA)	04	12	48
3.	Long answer questions (LA)	06	07	42
	Total		**29**	**100**

C. Scheme of Options

There will be no overall choice. However, internal choice in any four questions of four marks each and any two questions of six marks each has been provided.

D. Difficulty level of questions

S.No.	Estimated difficulty level	Percentage of marks
1.	Easy	15
2.	Average	70
3.	Difficult	15

Based on the above design, separate sample papers along with their blue prints and Marking schemes have been included in this document. About 20% weightage has been assigned to questions testing higher order thinking skills of learners.

CBSE SAMPLE PAPER - I

CLASS XII MATHEMATICS

BLUE PRINT - I

S. No.	Topics	VSA	SA	LA	Total
1. (a)	Relations and Functions	1(1)	4(1)	-	
(b)	Inverse Trigonometric Functions	1(1)	4(1)	-	10(4)
2. (a)	Matrices	2(2)	-	6(1)	
(b)	Determinants	1(1)	4(1)	-	13(5)
3. (a)	Continuity and differentiability	-	8(2)	-	
(b)	Applications of derivatives	-	4(1)	6(1)	
(c)	Integration	2(2)	4(1)	6(1)	
(d)	Applications of Integrals	-	-	6(1)	
(e)	Differential Equations	-	8(2)	-	44(11)
4. (a)	Vectors	2(2)	4(1)	-	
(b)	3-dimensional Geometry	1(1)	4(1)	6(1)	17(6)
5.	Linear - Programming	-	-	6(1)	6(1)
6.	Probability	-	4(1)	6(1)	10(2)
	Total	10(10)	48(12)	42(7)	100(29)

SAMPLE PAPER - I
MATHEMATICS
CLASS - XII

Time : 3 Hours **Max. Marks : 100**

General Instructions

1. All questions are compulsory.
2. The question paper consists of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators in not permitted. You may ask for logarithmic tables, if required.

SECTION A

1. Give an example to show that the relation R in the set of natural numbers, defined by $R=\{(x, y), x, y \in N, x \leq y^2\}$ is not transitive.

2. Write the principal value of $\cos^{-1}(\cos\frac{5\pi}{3})$.

3. Find x, if $\begin{pmatrix} 5 & 3x \\ 2y & z \end{pmatrix} = \begin{pmatrix} 5 & 4 \\ 12 & 6 \end{pmatrix}^T$.

4. For what value of a, $\begin{pmatrix} 2a & -1 \\ -8 & 3 \end{pmatrix}$ is a singular matrix?

5. A square matrix A, of order 3, has |A| = 5, find |A.adjA|.

6. Evaluate $\int 5^x dx$

7. Write the value of $\int_{-\pi/2}^{\pi/2} \sin^5 x \, dx$.

8. Find the position vector of the midpoint of the line segment joining the points $A(5\hat{i}+3\hat{j})$ and $B(3\hat{i}-\hat{j})$.

9. If $\vec{a} = 2\hat{i} - \hat{j} + 3\hat{k}$ and $\vec{b} = (6\hat{i} + \lambda\hat{j} + 9\hat{k})$ and $\vec{a} \parallel \vec{b}$, find the value of λ.

10. Find the distance of the point (a,b,c) from x-axis.

SECTION B

11. Let N be the set of all natural numbers and R be the relation in NxN defined by (a,b) R (c,d) if ad=bc. Show that R is an equivalence relation.

12. Prove that $\tan^{-1}\left(\frac{1}{4}\right) + \tan^{-1}\left(\frac{2}{9}\right) = \frac{1}{2}\cos^{-1}\left(\frac{3}{5}\right)$.

OR

Solve for x : $\sin^{-1}(1-x) - 2\sin^{-1}x = \frac{\pi}{2}$.

13. Using properties of determinants, prove that :

$$\begin{vmatrix} a^2+1 & ab & ac \\ ab & b^2+1 & bc \\ ca & cb & c^2+1 \end{vmatrix} = (1+a^2+b^2+c^2).$$

14. For what values of a and b, the function f defined as :

$$f(x) = \begin{cases} 3ax+b, & \text{if } x<1 \\ 11, & \text{if } x=1 \\ 5ax-2b, & \text{if } x>1 \end{cases}$$ is continuous at $x=1$

15. If $x^y+y^x = a^b$, find $\frac{dy}{dx}$.

OR

If $x=a(\cos t + t \sin t)$ and $y = b(\sin t - t \cos t)$, find $\frac{d^2y}{dx^2}$.

16. Find the intervals in which the following function is strictly increasing or strictly decreasing :

$f(x) = 20-9x+6x^2-x^3$

OR

For the curve $y=4x^3-2x^5$, find all points at which the tangent passes through origin.

17. Evaluate : $\int \frac{\sin x + \cos x}{\sqrt{\sin x \cos x}} dx$

OR

Evaluate : $\int e^x \left(\frac{x^2+1}{(x+1)^2} \right) dx$

18. Form the differential equation of the family of circles having radii 3.

19. Solve the following differential equation:

$$\sqrt{1+x^2+y^2+x^2y^2} + xy \frac{dy}{dx} = 0.$$

20. If the sum of two unit vectors is a unit vector, show that the magnitude of their difference is $\sqrt{3}$.

21. Find whether the lines $\vec{r} = (\hat{i} - \hat{j} - \hat{k}) + \lambda\,(2\hat{i} + \hat{j})$ and $\vec{r} = (2\hat{i} - \hat{j}) + \mu(\hat{i} + \hat{j} - \hat{k})$ intersect or not. If intersecting, find their point of intersection.

22. Three balls are drawn one by one without replacement from a bag containig 5 white and 4 green balls. Find the probability distribution of number of green balls drawn.

SECTION C

23. If $A = \begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}$, find A^{-1} and hence solve the following system of equations :

$$\begin{aligned} 2x+y+3z &= 3 \\ 4x-y &= 3 \\ -7x + 2y + z &= 2 \end{aligned}$$

OR

Using elementary transformations, find the inverse of the matrix :

$$\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$$

24. If the lengths of three sides of a trapezium, other than the base are equal to 10cm each, then find the area of trapezium when it is maximum.

25. Draw a rough sketch of the region enclosed between the circles $x^2+y^2 = 4$ and $(x-2)^2 + y^2 = 1$. Using integration, find the area of the enclosed region.

26. Evaluate $\int_1^2 (x^2+x+2)dx$ as a limit of sums.

OR

Evaluate $\int_0^1 \sin^{-1}(x\sqrt{1-x} - \sqrt{x}\,\sqrt{1-x^2})dx,\ 0 \le x \le 1$

27. Find the coordinates of the foot of the perpendicular and the perpendicular distance of the point (1, 3, 4) from the plane 2x-y+z+3=0. Find also, the image of the point in the plane.

28. An aeroplane can carry a maximum of 200 passengers. A profit of Rs. 1000 is made on each excutive class ticket and a profit of Rs. 600 is made on each economy class ticket. The airline reserves at least 20 seats for the executive class. However, at least 4 times as many passengers prefer to travel by economy class, than by the executive class. Determine how many tickets of each type must be sold, in order to maximise profit for the airline. What is the maximum profit? Make an L.P.P. and solve it graphically.

29. A fair die is rolled. If 1 turns up, a ball is picked up at random from bag A, if 2 or 3 turns up, a ball is picked up at random from bag B, otherwise a ball is picked up from bag C. Bag A contains 3 red and 2 white balls, bag B contains 3 red and 4 white balls and bag C contains 4 red and 5 white balls. The die is rolled, a bag is picked up and a ball is drawn from it. If the ball drawn is red, what is the probability that bag B was picked up?

MARKING SCHEME

MATHEMATICS CLASS - XII

SAMPLE PAPER 1

SECTION A

1. $(8, 3) \in R$, $(3, 2) \in R$ but $(8, 2) \notin R$.

2. $\frac{\pi}{3}$

3. $x = 4$

4. $a = \frac{4}{3}$

5. 125

6. $\frac{5^x}{\log 5} + c$

7. Zero.

8. $4\hat{i} + \hat{j}$

9. $\lambda = -3$

10. $\sqrt{b^2+c^2}$ (1 mark each for correct answer for Qs. 1 to 10)

SECTION B

11. For any $(a, b) \in N \times N$, $ab = ba$

$\Rightarrow (a, b) R (a, b)$. Thus R is reflexive. 1

Let $(a, b) R (c, d)$ for any $a, b, c, d \in N$

$\therefore ad = bc$

$\Rightarrow cb = da \Rightarrow (c, d) R (a, b)$

$\therefore$ R is symmetric 1

Let $(a, b) R (c, d)$ and $(c, d) R (e, f)$ for $a, b, c, d, e, f \in N$

then $ad = bc$ and $cf = de$

$\Rightarrow adcf = bcde$ or $af = be \Rightarrow (a, b) R (e, f)$

$\therefore$ R is transitive 1½

Since, R is reflexive, symmetric and transitive, hence R is an equivalence relation. ½

12. L.H.S = $\tan^{-1}\left(\dfrac{\frac{1}{4}+\frac{2}{9}}{1-\frac{1}{4}\cdot\frac{2}{9}}\right) = \tan^{-1}\left(\dfrac{17}{34}\right) = \tan^{-1}\left(\dfrac{1}{2}\right)$ 1½

$= \dfrac{1}{2}\left(2\tan^{-1}\left(\dfrac{1}{2}\right)\right) = \dfrac{1}{2}\cos^{-1}\left[\dfrac{1-\left(\frac{1}{2}\right)^2}{1+\left(\frac{1}{2}\right)^2}\right]$ 1½

$= \dfrac{1}{2}\cos^{-1}\left(\dfrac{1-\frac{1}{4}}{1+\frac{1}{4}}\right) = \dfrac{1}{2}\cos^{-1}\dfrac{3}{5} = \text{RHS}$ 1

OR

$\sin^{-1}(1-x)-2\sin^{-1}x = \dfrac{\pi}{2} \Rightarrow \sin^{-1}(1-x) = \dfrac{\pi}{2}+2\sin^{-1}x$ ½

$\Rightarrow (1-x) = \sin(\dfrac{\pi}{2} + 2\sin^{-1}x) = \cos(2\sin^{-1}x)$ ½

$\Rightarrow (1-x) = \cos(2\alpha)$ where $\sin^{-1}x = \alpha$ or $x = \sin\alpha$ ½

$\Rightarrow (1-x) = 1-2\sin^2\alpha = 1-2x^2, \quad \therefore 2x^2-x = 0$ 1

$\Rightarrow x(2x-1) = 0$ ½

$\therefore x = 0, \dfrac{1}{2}$

Since $x = \dfrac{1}{2}$ does not satisfy the given equation $\therefore$ $x=0$ 1

13. $\text{LHS} = \dfrac{1}{abc}\begin{vmatrix} a(a^2+1) & a^2b & a^2c \\ ab^2 & b(b^2+1) & b^2c \\ ac^2 & bc^2 & c(c^2+1) \end{vmatrix} = \dfrac{abc}{abc}\begin{vmatrix} a^2+1 & a^2 & a^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$ 1

$R_1 \rightarrow R_1+R_2+R_3$

$= \begin{vmatrix} 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 & 1+a^2+b^2+c^2 \\ b^2 & b^2+1 & b^2 \\ c^2 & c^2 & c^2+1 \end{vmatrix}$ 1

$$= (1+a^2+b^2+c^2)\begin{vmatrix}1 & 1 & 1\\ b^2 & b^2+1 & b^2\\ c^2 & c^2 & c^2+1\end{vmatrix}$$ 1

$C_2 \longrightarrow C_2 - C_1$

$C_3 \longrightarrow C_2 - C_1$

$$= (1+a^2+b^2+c^2)\begin{vmatrix}1 & 0 & 0\\ b^2 & 1 & 0\\ c^2 & 0 & 1\end{vmatrix}$$ 1

$= (1+a^2+b^2+c^2)\cdot 1 = (1+a^2+b^2+c^2)$ 1

14. $\lim_{x \to 1^-} f(x) = 3a+b$, RHL $= \lim_{x \to 1^+} f(x) = 5a-2b$, $f(1) = 11$ 2

$\therefore\ 3a+b = 11,\quad 5a-2b = 11$ 1

Solving to get $a = 3,\ b = 2$ 1

15. Put $x^y = u$ and $y^x = v$ $\therefore\ u+v = a^b \Rightarrow \frac{du}{dx} + \frac{dv}{dx} = 0$ ½

$\log u = y \log x \Rightarrow \frac{1}{u}\frac{du}{dx} = \frac{y}{x} + \log x \cdot \frac{dy}{dx} \therefore \frac{du}{dx} = y\cdot x^{y-1} + x^y \cdot \log x \cdot \frac{dy}{dx}$ 1

$\log v = x \log y \Rightarrow \frac{1}{v}\cdot\frac{dv}{dx} = \frac{x}{y}\frac{dy}{dx} + \log y \therefore \frac{dv}{dx} = xy^{x-1}\cdot\frac{dy}{dx} + y^x \cdot \log y$ 1

$\therefore y\cdot x^{y-1} + x^y \log x \frac{dy}{dx} + xy^{x-1}\frac{dy}{dx} + y^x \log y = 0$ 1

$\Rightarrow \frac{dy}{dx} = -\frac{y\cdot x^{y-1} + y^x \cdot \log y}{x^y \cdot \log x + x\,y^{x-1}}$ ½

OR

$\frac{dx}{dt} = a(-\sin t + t\cos t + \sin t) = at\cos t$ 1

$\frac{dy}{dt} = b(\cos t + t\sin t - \cos t) = bt\sin t$ 1

$\therefore \frac{dy}{dx} = \frac{b}{a} \cdot \tan t \Rightarrow \frac{d^2y}{dx^2} = \frac{b}{a} \sec^2 t \cdot \frac{dt}{dx}$ 1

$\therefore \frac{d^2y}{dx^2} = \frac{b}{a} \sec^2 t \cdot \frac{1}{at \cos t} = \frac{b \sec^3 t}{a^2 t}$ 1

16. $f^1(x) = 0 \Rightarrow -9+12x-3x^2 = 0 \Rightarrow -(3)(x-1)(x-3) = 0$ 1

$\therefore x = 1, \quad x = 3$

$\therefore$ The intervals are $(-\infty, 1), (1, 3), (3, \infty)$ 1

Getting f(x) to be stricktly decreasing in $(-\infty, 1) \cup (3, \infty)$ 1

and stricktly increasing in $(1, 3)$ 1

OR

Let (x_1, y_1) be a point on the given curve, the tangent at which passes through origin.

$\therefore$ Slope of tangent $= \frac{y_1}{x_1}$ ------- (i) ½

also, $\frac{dy}{dx} = 12x^2-10x^4 \Rightarrow$ slope of tangent $= 12x_1^2-10x_1^4$ --------- (ii) ½

$\Rightarrow \frac{y_1}{x_1} = 12x_1^2-10x_1^4$ or $y_1 = 12x_1^3-10x_1^5 \Rightarrow 4x_1^3-2x_1^5 = 12x_1^3 - 10x_1^5$ 1

solving to get $x_1 = 0$ or $1-x_1^2 = 0$ i.e. $x_1 = \pm 1$ 1

Hence the required points are (0, 0), (1, 2) and (-1, -2) 1

17. Putting $(\sin x - \cos x) = t$ to get $(\cos x + \sin x)\, dx = dt$ 1

and $\sin^2 x + \cos^2 x - 2\sin x \cos x = t^2$ or $\sin x \cos x = \frac{1}{2}(1-t^2)$ ½

$\therefore$ Given integral $= \sqrt{2} \int \frac{dt}{\sqrt{1-t^2}} = \sqrt{2} \cdot \sin^{-1} t + c$ 1½

$= \sqrt{2} \sin^{-1} (\sin x - \cos x) + c$ 1

OR

$\int e^x \cdot \frac{x^2+1}{(x+1)^2} dx = \int e^x \cdot \frac{[(x+1)^2-2x]}{(x+1)^2} dx$ 1

$$= \int e^x dx - 2\int e^x \cdot \frac{(x+1-1)}{(x+1)^2}\, dx$$ 1

$$= e^x - 2\int \left[\frac{1}{x+1} - \frac{1}{(x+1)^2}\right] e^x dx$$ 1

$$= e^x - 2\cdot\frac{e^x}{x+1} + c \qquad [\text{using } \int e^x (f(x)+f^1(x))\, dx = e^x f(x)+c$$ 1

18. The equation of the family of circles is $(x-a)^2+(y-b)^2 = 9$ ----------------(i) ½

$$\Rightarrow 2(x-a) + 2(y-b)\frac{dy}{dx} = 0 \text{ or } (x-a) = -(y-b)\frac{dy}{dx} \qquad \text{----------------(ii)}$$ 1

$$\Rightarrow 1+(y-b)\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^2 = 0 \Rightarrow (y-b) = -\frac{[1+\left(\frac{dy}{dx}\right)^2]}{\frac{d^2y}{dx^2}} \qquad \text{----------------(iii)}$$ 1

from (ii), $(x-a) = + \dfrac{[1+\left(\frac{dy}{dx}\right)^2]}{\frac{d^2y}{dx^2}} \cdot \dfrac{dy}{dx}$ ½

putting in (i) to get $\left[\dfrac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 \left(\dfrac{dy}{dx}\right)^2 + \left[\dfrac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 = 9$ ½

or $\left[\dfrac{1+\left(\frac{dy}{dx}\right)^2}{\frac{d^2y}{dx^2}}\right]^2 \left[\left(\dfrac{dy}{dx}\right)^2 + 1\right] = 9 \Rightarrow \left[1+\left(\dfrac{dy}{dx}\right)^2\right]^3 = 9\left(\dfrac{d^2y}{dx}\right)^2$ ½

19. Given differential equation can be written as

$$\sqrt{(1+x^2)(1+y^2)} + xy\frac{dy}{dx} = 0$$ ½

$$\Rightarrow \frac{\sqrt{1+x^2}}{x}dx + \frac{y}{\sqrt{1+y^2}}dy = 0$$ ½

$$\therefore \int \frac{y}{\sqrt{1+y^2}}\,dy = -\int \frac{\sqrt{1+x^2}}{x^2}\,xdx$$ ½

Putting $1+y^2 = u^2$ and $1+x^2 = v^2$ to get $ydy = udu$ and $xdx = vdv$ ½

$$\therefore \int \frac{u\,du}{u} = -\int \frac{v\cdot vdv}{v^2-1} = -\int \frac{v^2-1+1}{v^2-1}dv = -\int\left(1+\frac{1}{v^2-1}\right)dv$$ 1

$$u = -v-\frac{1}{2}\log\left|\frac{v-1}{v+1}\right|+c \text{ or } \sqrt{1+y^2} = -\sqrt{1+x^2}-\frac{1}{2}\log\left|\frac{\sqrt{1+x^2}-1}{\sqrt{1+x^2}+1}\right|+c$$ 1

20. Let $\hat{a}$, $\hat{b}$ and $\hat{c}$ be unit vectors such that $\hat{a}+\hat{b} = \hat{c}$ 1

$$\therefore \left|\hat{a}+\hat{b}\right| = 1 \Rightarrow 1 = \left|\hat{a}+\hat{b}\right|^2 = \hat{a}^2+\hat{b}^2+2\hat{a}\cdot\hat{b} = 2+2\hat{a}\cdot\hat{b}$$ 1

$\Rightarrow 2(\hat{a}\cdot\hat{b}) = 1-2 = -1$ ----------------------------------(i) ½

Now $\left|\hat{a}-\hat{b}\right|^2 = \hat{a}^2+\hat{b}^2-2\hat{a}\cdot\hat{b} = 1+1-(-1) = 3$ 1½

$\Rightarrow \left|\hat{a}-\hat{b}\right| = \sqrt{3}$ ½

21. Given lines are $\vec{r} = (1+2\lambda)\hat{i} + (-1+\lambda)\hat{j} - \hat{k}$ and ½

$$\vec{r} = (2+\mu)\hat{i} + (-1+\mu)\hat{j} - \mu\hat{k}$$ ½

If lines are intersecting, then for some value of λ and μ,

$1+2\lambda = 2+\mu$, --(i) $-1+\lambda = -1+\mu$ --(ii) $-1 = -\mu$ --(iii) 1

Solving (ii) and (iii) to get $\lambda = 1$, $\mu = 1$, which satisfy (i) hence the line are intersecting 1

and point of intersection is (3, 0, -1) 1

22. Let X denotes the random variable, 'number of green balls,

X :	0	1	2	3	1
P(X) :	$\frac{5c_3}{9c_3}$	$\frac{5c_2\cdot 4c_1}{9c_3}$	$\frac{5c_1\cdot 4c_2}{9c_3}$	$\frac{4c_3}{9c_3}$	1

$$= \frac{5}{42} \qquad \frac{10}{21} \qquad \frac{5}{14} \qquad \frac{1}{21}$$ 2

SECTION C

23. $|A| = 2(-1) - 1(4) + 3(1) = -3 \neq 0 \quad A^{-1} = \frac{1}{|A|} \text{ adj } A$ 1

The cofactors are

$$A_{11} = -1, \; A_{12} = -4, \; A_{13} = 1$$

$$A_{21} = 5, \; A_{22} = 23, \; A_{23} = -11$$ 2

$$A_{31} = 3, \; A_{32} = 12, \; A_{33} = -6$$

$$\therefore A^{-1} = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}$$ ½

Given equations can be written as

$$\begin{pmatrix} 2 & 1 & 3 \\ 4 & -1 & 0 \\ -7 & 2 & 1 \end{pmatrix}\begin{pmatrix} x \\ y \\ z \end{pmatrix} = \begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} \text{ or } A \cdot X = B$$ 1

$$\therefore X = A^{-1} \cdot B$$

$$\therefore \begin{pmatrix} x \\ y \\ z \end{pmatrix} = -\frac{1}{3}\begin{pmatrix} -1 & 5 & 3 \\ -4 & 23 & 12 \\ 1 & -11 & -6 \end{pmatrix}\begin{pmatrix} 3 \\ 3 \\ 2 \end{pmatrix} = \begin{pmatrix} -6 \\ -27 \\ 14 \end{pmatrix}$$ 1

$$\therefore x = -6, \; y = -27, \; z = 14$$ ½

OR

Let $A = \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix}$ then $\begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 3 & -2 & 4 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} A$ ½

$R_3 \rightarrow R_3 - 3R_1 \Rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 0 & 2 & -3 \\ 0 & +1 & -2 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ -3 & 0 & 1 \end{pmatrix} A$ 1

$R_2 \leftrightarrow R_3 \Rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 2 & -3 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 0 & 1 \\ 0 & 1 & 0 \end{pmatrix} A$ 1

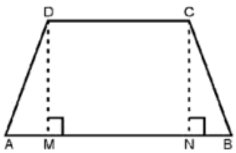
$R_3 \rightarrow R_3 - 2R_2 \Rightarrow \begin{pmatrix} 1 & -1 & 2 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 & 0 \\ -3 & 0 & 1 \\ 6 & 1 & -2 \end{pmatrix} A$ 1

$R_1 \rightarrow R_1 + R_2 \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ -3 & 0 & 1 \\ 6 & 1 & -2 \end{pmatrix} A$ 1

$R_2 \rightarrow R_2 + 2R_3 \Rightarrow \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{pmatrix} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix} A$ 1

Hence $A^{-1} = \begin{pmatrix} -2 & 0 & 1 \\ 9 & 2 & -3 \\ 6 & 1 & -2 \end{pmatrix}$ ½

24.



AD = DC = BC = 10cm.

$\Delta\ ADM \cong \Delta BCN \therefore AM = BN = x$ (say)

$\therefore DM = \sqrt{10^2 - x^2}$ 1

Area (A) $= \frac{1}{2}(10+10+2x)\sqrt{100-x^2} = (10+x)\sqrt{100-x^2}$ 1

Let $S = A^2 = (10+x)^2 (100-x^2)$

$\frac{ds}{dx} = 0 \Rightarrow (10+x)^2 (-2x) + (100-x^2)\, 2(10+x) = 0$ ½

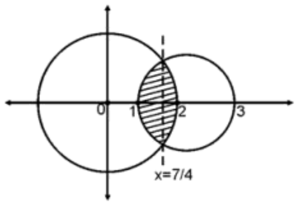
$(10+x)^2 (-2x+20-2x) = 0 \Rightarrow x = 5$ 1

$\frac{d^2s}{dx^2} = (10+x)^2 (-4) + (20-4x)\, 2(10+x) < 0$ at $x = 5$ 1

$\therefore$ for Maximum area, $x = 5$ ½

Maximum area $= 15\sqrt{75} = 75\sqrt{3}\ cm^2$ 1

25. Correct figure 1



Solving $x^2+y^2 = 4$ and $(x-2)^2+y^2 = 1$

we get $x = \frac{7}{4}$ 1

$\therefore$ Required area $= 2\left[\int_{7/4}^{2} \sqrt{4-x^2}\, dx + \int_{1}^{7/4} \sqrt{1-(x-2)^2}\, dx\right]$ 1

$= 2\left[\left[\frac{x}{2}\sqrt{4-x^2} + 2\sin^{-1}\frac{x}{2}\right]_{\frac{7}{4}}^{2} + \left[\frac{x-2}{2}\sqrt{1-(x-2)^2} + \frac{1}{2}\sin^{-1}\cdot(x-2)\right]_{1}^{\frac{7}{4}}\right]$ 1

$= 2\left[\pi - \frac{7}{8}\,\frac{\sqrt{15}}{4} - 2\sin^{-1}\frac{7}{8} + \left(-\frac{1}{8}\cdot\frac{\sqrt{15}}{4} + \frac{1}{2}\sin^{-1}\left(-\frac{1}{4}\right) + \frac{1}{2}\cdot\frac{\pi}{2}\right)\right]$ 1

$= \frac{5\pi}{2} - \frac{\sqrt{15}}{2} - \sin^{-1}\left(\frac{1}{4}\right) - 4\sin^{-1}\left(\frac{7}{8}\right)$ sq.u 1

26. Here $f(x) = (x^2+x+2)$, $h = \frac{b-a}{n} = \frac{1}{n}$ ½

$\int_{1}^{2} f(x)dx = \lim_{n\to\infty} \frac{1}{n}\cdot[f(1) + f(1+h) + f(1+2h) + \ldots\ldots\ldots\ldots + f[1+(n-1)h]]$ 1

$= \lim_{n\to\infty} \frac{1}{n}\cdot\left[4 + (4+3h+h^2) + (4+6h+4h^2) + \ldots\ldots\ldots\ldots + (4+(n-1)3h+(n-1)^2h^2\right]$ 2

$= \lim_{n\to\infty} \frac{1}{n}\left[4n + \frac{3}{n}\cdot\frac{n(n-1)}{2} + \frac{1}{n^2}\frac{n(n-1)(2n-1)}{6}\right]$ 1½

$= \lim_{n\to\infty}\left[4 + \frac{3}{2}\left(1-\frac{1}{n}\right) + \frac{1}{6}\left(1-\frac{1}{n}\right)\left(2-\frac{1}{n}\right)\right]$ ½

$= 4 + \frac{3}{2} + \frac{1}{3} = \frac{24+9+2}{6} = \frac{35}{6}$ ½

OR

put $x = \sin\alpha$ and $\sqrt{x} = \sin\beta$ ½

$\therefore \sin^{-1}[x\sqrt{1-x} - \sqrt{x}\sqrt{1-x^2} = \sin^{-1}\ [\sin\alpha \cos\beta - \cos\alpha \sin\beta]$

$= \sin^{-1}[\sin(\alpha-\beta)] = \alpha-\beta = \sin^{-1}x - \sin^{-1}\sqrt{x}$ ½

$\therefore$ Given integral $= \int_0^1 (\sin^{-1}x - \sin^{-1}\sqrt{x})dx = \int_0^1 \sin^{-1}x\, dx - \int_0^1 \sin^{-1}\sqrt{x}\, dx$ 1

$= \left[x \cdot \sin^{-1}x\right]_0^1 - \frac{1}{2}\int_0^1 \frac{2x}{\sqrt{1-x^2}}\, dx - \left[x \cdot \sin^{-1}\sqrt{x}\right]_0^1 + \int_0^1 \frac{1}{\sqrt{1-x}}\, \frac{1}{2\sqrt{x}} \cdot x\, dx$ 1

$= \frac{\pi}{2} + \left[\sqrt{1-x^2}\right]_0^1 - \frac{\pi}{2} + \frac{1}{2}\int_0^1 \frac{\sqrt{x}}{\sqrt{1-x}}\, dx$ 1

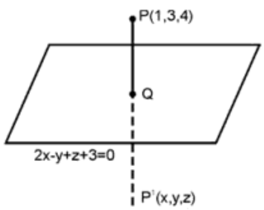
$= -1 + \frac{1}{2}\int_1^0 \frac{-\sqrt{1-t^2} \cdot 2t dt}{t}$ $\left[1 - x = t^2,\ dx = -2t\ dt\right]$ 1

$= -1 + \int_0^1 \sqrt{1-t^2} dt = 1 + \left[\frac{t}{2}\sqrt{1-t^2} + \frac{1}{2}\sin^{-1}t\right]_0^1 = \left(-1 + \frac{\pi}{4}\right)$ 1

27. Let Q be the foot of perpendicular from P to the plane and P^1 (x, y, z) be the image of P in the plane.

$\therefore$ The equations of line through P and Q is

$\frac{x-1}{2} = \frac{y-3}{-1} = \frac{z-4}{1}$ 1



The coordinates of Q (for some value of λ) are

$(2\lambda+1,\ -\lambda+3,\ \lambda+4)$ ½

Since Q lies on the plane, $\therefore 2(2\lambda+1) - 1(-\lambda+3) + (\lambda+4) + 3 = 0$ 1

Solving to get $\lambda = -1$ ½

$\therefore$ coordinates of foot of perpendicular (Q) are (-1, 4, 3) ½

Perpendicular distance (PQ) = $\sqrt{(2)^2 + (-1)^2 + (1)^2} = \sqrt{6}$ units ½

Since Q is mid point of PP`

$\therefore \frac{x+1}{2} = -1,\ \frac{y+3}{2} = 4,\ \frac{z+4}{2} = 3 \Rightarrow x = -3,\ y = 5,\ z = 2$ 1

$\therefore$ Image of P is (-3, 5, 2) 1

28. Let, number of executive class tickets to be sold, be x and that of economy class be y.

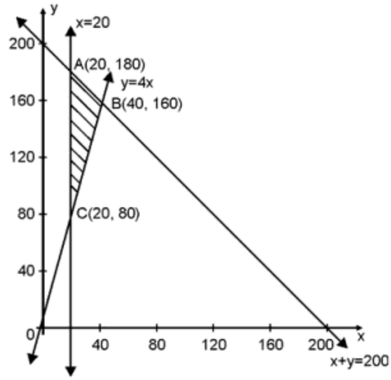
$\therefore$ LPP becomes : Maximise Profit (P) = 1000x + 600y 1

Subject to : $x \geq 0, y \geq 0$

$x + y \leq 200$ 1½

$y \geq 4x$ or $4x - y \leq 0$

$x \geq 20$



For correct graph 2

Getting vertices of feasible region as
A(20, 180), B(40, 160), C(20, 80) ½

Profit at A = Rs. 128000

Profit at B = Rs. 136000

Profit at C = Rs. 68000

$\therefore$ Max profit = Rs. 136000 for 40 executive and 160 economy tickets 1

29. Let the events be defined as :

E_1 : Bag A is selected

E_2 : Bag B is selected 1

E_3 : Bag C is selected

A : A red ball is selected

$\therefore P(E_1) = \frac{1}{6},\ P(E_2) = \frac{2}{6} = \frac{1}{3}$ and $P(E_3) = \frac{3}{6} = \frac{1}{2}$ 1

$P\left(A/E_1\right) = \frac{3}{5},\ P\left(A/E_2\right) = \frac{3}{7}$ and $P\left(A/E_3\right) = \frac{4}{9}$ 1

$$P\left(E_2/A\right) = \frac{P(E_2)\,P\left(A/E_2\right)}{P(E_1)P\left(A/E_1\right) + P(E_2)\,P\left(A/E_2\right) + P(E_3)\,P\left(A/E_3\right)} = \frac{\frac{1}{3}\cdot\frac{3}{7}}{\frac{1}{6}\cdot\frac{3}{5}+\frac{1}{3}\cdot\frac{3}{7}+\frac{1}{2}\cdot\frac{4}{9}}$$ 2

$= \frac{90}{293}$ 1

CBSE SAMPLE PAPER - II

CLASS XII MATHEMATICS

BLUE PRINT - II

S. No.	Topics	VSA	SA	LA	Total
1. (a)	Relations and Functions	1(1)	4(1)	-	
(b)	Inverse Trigonometric Functions	1(1)	4(1)	-	10(4)
2. (a)	Matrices	2(2)	4(1)	-	
(b)	Determinants	1(1)	-	6(1)	13(5)
3. (a)	Continuity and differentiability	-	8(2)	-	
(b)	Applications of derivatives	-	4(1)	6(1)	
(c)	Integration	2(2)	4(1)	6(1)	
(d)	Applications of Integrals	-	-	6(1)	
(e)	Differential Equations	-	8(2)	-	44(11)
4. (a)	Vectors	2(2)	4(1)	-	
(b)	3-dimensional Geometry	1(1)	4(1)	6(1)	17(6)
5.	Linear - Programming	-	-	6(1)	6(1)
6.	Probability	-	4(1)	6(1)	10(2)
	Total	10(10)	48(12)	42(7)	100(29)

SAMPLE PAPER - II

MATHEMATICS

CLASS - XII

Time : 3 Hours **Max. Marks : 100**

General Instructions

1. All questions are compulsory.
2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.
3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.
4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.
5. Use of calculators in not permitted. You may ask for logarithmic tables, if required.

SECTION A

1. Write the number of all one-one functions from the set $A = \{a, b, c\}$ to itself.

2. Find x if $\tan^{-1} 4 + \cot^{-1} x = \frac{\pi}{2}$

3. What is the value of $|3\ I_3|$, where I_3 is the identity matrix of order 3?

4. For what value of k, the matrix $\begin{bmatrix} 2\text{-}k & 3 \\ -5 & 1 \end{bmatrix}$ is not invertible?

5. If A is a matrix of order 2x3 and B is a matrix of order 3x5, what is the order of matrix $(AB)'$?

6. Write a value of $\int \frac{dx}{\sqrt{4-x^2}}$.

7. Find f(x) satisfying the following :

   $\int e^x (\sec^2 x + \tan x)dx = e^x\ f(x)+c$

8. In a triangle ABC, the sides AB and BC are represented by vectors $2\hat{i}-\hat{j}+2\hat{k}$, $\hat{i}+3\hat{j}+5\hat{k}$ respectively. Find the vector representing CA.

9. Find the value of λ for which the vector $\vec{a} = 3\hat{i}+\hat{j}-2\hat{k}$ and $\vec{b} = \hat{i}+\lambda\hat{j}-3\hat{k}$ are perpendicular to each other.

10. Find the value of λ such that the line $\frac{x-2}{9} = \frac{y-1}{\lambda} = \frac{z+3}{-6}$ is perpendicular to the plane $3x-y-2z=7$

SECTION B

11. Show that the function $f: R \rightarrow R$ defined by $f(x) = 2x^3-7$, for $x \in R$ is bijective.

OR

Let $f,g : R \rightarrow R$ be defined as $f(x) = |x|$ and $g(x) = [x]$ where [x] denotes the greatest integer less than or equal to x. Find fog $\left(\frac{5}{2}\right)$ and gof $\left(-\sqrt{2}\right)$.

12. Prove that $\tan^{-1}1+\tan^{-1}2+\tan^{-1}3 = \pi$

13. If $A = \begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix}$. show that $A^2-5A-14I = O$. Hence find A^{-1}.

14. Show that $f(x) = |x-3|, \forall x \in R$, is continuous but not differentiable at x=3.

OR

If $\tan\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$, then prove that $\frac{dy}{dx} = \frac{y}{x}$

15. Verify Rolle's Theorem for the function f, given by $f(x) = e^x (\sin x - \cos x)$ on $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

16. Using differentials, find the approximate value of $\sqrt{25 \cdot 2}$

OR

Two equal sides of an isosceles triangle with fixed base 'a' are decreasing at the rate of 9 cm/second. How fast is the area of the triangle decreasing when the two sides are equal to 'a'.

17. Evaluate $\int_{-1}^{\frac{1}{2}} |x \cos(\pi x)| \, dx$.

18. Solve the following differential equation :

$$ye^{x/y}dx = (xe^{x/y} + y)dy$$

19. Solve the following differential equation :

$(1+y+x^2y)dx + (x+x^3)dy = 0$, where y=0 when x=1

20. If $\vec{a}, \vec{b}$ and $\vec{c}$ are three unit vectors such that $\vec{a} \cdot \vec{b} = \vec{a} \cdot \vec{c} = 0$ and angle between $\vec{b}$ and $\vec{c}$ is $\frac{\pi}{6}$, prove that $\vec{a} = \pm 2 (\vec{b} \times \vec{c})$.

21. Show that the four points (0, -1, -1), (4, 5, 1), (3, 9, 4) and (-4, 4, 4) are coplanar. Also, find the equation of the plane containing them.

22. A coin is biased so that the head is 3 times as likely to occur as tail. If the coin is tossed three times, find the probability distribution of number of tails.

OR

How many times must a man toss a fair coin, so that the probability of having at least one head is more than 80%?

SECTION C

23. Using properties of determinants, show that

$$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+b)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix} = 2abc\,(a+b+c)^3$$

24. The sum of the perimeter of a circle and a square is k, where k is some constant. Prove that the sum of their areas is least when the side of square is double the radius of the circle.

OR

A helicopter is flying along the curve $y = x^2+2$. A soldier is placed at the point (3, 2). Find the nearest distance between the soldier and the helicopter.

25. Evaluate : $\int \frac{1}{\sin x\,(5 - 4\cos x)}dx$

OR

Evaluate : $\int \sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}\,dx$

26. Using integration, find the area of the region

$\left\{(x, y) : |x-1| \leq y \leq \sqrt{5-x^2}\right\}$

27. Show that the lines $\frac{x+3}{-3} = \frac{y-1}{1} = \frac{z-5}{5}$ and $\frac{x+1}{-1} = \frac{y-2}{2} = \frac{z-5}{5}$ are coplanar. Also find the equation of the plane.

28. From a pack of 52 cards, a card is lost. From the remaining 51 cards, two cards are drawn at random (without replacement) and are found to be both diamonds. What is the probability that the lost card was a card of heart?

29. A diet for a sick person must contain at least 4000 units of vitamins, 50 units of minerals and 1400 calories. Two foods X and Y are available at a cost of Rs 4 and Rs 3 per unit respectively. One unit of food X contains 200 units of vitamins, 1 unit of minerals and 40 calories, whereas 1 unit of food Y contains 100 units of vitamins, 2 units of minerals and 40 calories. Find what combination of foods X and Y should be used to have least cost, satisfying the requirements. Make it an LPP and solve it graphically.

MARKING SCHEME

MATHEMATICS CLASS - XII

SAMPLE PAPER II

SECTION A

1. 6
2. 4
3. 27
4. 17
5. 5x2
6. $\sin^{-1}\left(\frac{x}{2}\right)$
7. tanx
8. $-(3\hat{i}+2\hat{j}+7\hat{k})$
9. $\lambda = -9$
10. $\lambda = -3$ (1 mark for correct answer for Qs. 1 to 10)

SECTION B

11. Let x, y be any two elements of R (domain)

then $f(x) = f(y) \Rightarrow 2x^3-7 = 2y^3-7$

$\Rightarrow x^3=y^3 \Rightarrow x=y$ 1

so, f is an injective function

Let y be any element of R (co-domain)

$\therefore f(x) = y \Rightarrow 2x^3-7=y$

$$\Rightarrow x^3 = \frac{y+7}{2} \Rightarrow x = \left(\frac{y+7}{2}\right)^{\frac{1}{3}}$$

Now for all $y \in R$ (co-domain), there exists $x = \left(\frac{y+7}{2}\right)^{\frac{1}{3}} \in R(\text{domain})$ 1

such that $f(x) = f\left\{\left(\frac{y+7}{2}\right)^{\frac{1}{3}}\right\} = 2\left\{\left(\frac{y+7}{2}\right)^{\frac{1}{3}}\right\}^3 - 7$

$$= 2\cdot\frac{y+7}{2}-7=y$$ 1

so, f is surjective

Hence, f is a bijective function 1

OR

$$fog\left(\frac{5}{2}\right)=f\left[g\left(\frac{5}{2}\right)\right]=f(2)=|2|=2$$ 2

$$gof\left(-\sqrt{2}\right)=g\left[f\left(-\sqrt{2}\right)\right]=g\left[\left|-\sqrt{2}\right|\right]=g\left[\sqrt{2}\right]=1$$ 2

12. L.H.S. $= \tan^{-1}1+\tan^{-1}2+\tan^{-1}3$

$$= \frac{\pi}{4}+\frac{\pi}{2}-\cot^{-1}2+\frac{\pi}{2}-\cot^{-1}3$$ ½

$$= \frac{5\pi}{4}-\tan^{-1}\left(\frac{1}{2}\right)-\tan^{-1}\left(\frac{1}{3}\right)$$ ½

$$= \frac{5\pi}{4}-\left(\tan^{-1}\frac{1}{2}+\tan^{-1}\frac{1}{3}\right)$$ ½

$$= \frac{5\pi}{4}-\tan^{-1}\left(\frac{\frac{1}{2}+\frac{1}{3}}{1-\frac{1}{2}\cdot\frac{1}{3}}\right)$$ 1

$$= \frac{5\pi}{4}-\tan^{-1}(1)$$ ½

$$= \frac{5\pi}{4}-\frac{\pi}{4}$$ ½

$$= \pi = \text{RHS}$$ ½

13. $A=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix} \Rightarrow A^2=\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}$ 1

$$A^2-5A-14I=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}-5\begin{bmatrix}3 & -5\\ -4 & 2\end{bmatrix}-14\begin{bmatrix}1 & 0\\ 0 & 1\end{bmatrix}$$ ½

$$=\begin{bmatrix}29 & -25\\ -20 & 24\end{bmatrix}+\begin{bmatrix}-15 & 25\\ 20 & -10\end{bmatrix}+\begin{bmatrix}-14 & 0\\ 0 & -14\end{bmatrix}$$

$$\begin{bmatrix} 29-15-14 & -25+25-0 \\ -20+20+0 & 24-10-14 \end{bmatrix} = \begin{bmatrix} 0 & 0 \\ 0 & 0 \end{bmatrix} = O$$ 1

Premultiplying $A^2-5A-14I = O$ by A^{-1}, we get

$A^{-1}. A^2 - 5A^{-1} A - 14A^{-1}I = O$

or, $A-5I - 14A^{-1} = O$ ½

$$\text{or } A^{-1} = \frac{1}{14}(A-5I) = \frac{1}{14}\left\{\begin{bmatrix} 3 & -5 \\ -4 & 2 \end{bmatrix} + \begin{bmatrix} -5 & 0 \\ 0 & -5 \end{bmatrix}\right\}$$

$$= \frac{1}{14}\begin{bmatrix} -2 & -5 \\ -4 & -3 \end{bmatrix}$$ 1

14. $f(x) = |(x-3)| \Rightarrow f(x) = \begin{cases} x-3 & \text{if } x \geq 3 \\ -(x-3) & \text{if } x < 3 \end{cases}$ ½

$\lim_{x \to 3^-} f(x) = \lim_{x \to 3^-} -(x-3) = 0$ ½

$\lim_{x \to 3^+} f(x) = \lim_{x \to 3^+} (x-3) = 0$ ½

and $f(3) = 3-3 = 0$

$\therefore \lim_{x \to 3^-} f(x) = \lim_{x \to 3^+} f(x) = f(3)$

$\therefore$ f(x) is continuous at x = 3 ½

For differentiability

$$Lf'(3) = \lim_{x \to 3^-} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3^-} \frac{-(x-3) - 0}{x-3} = -1$$ ½

$$Rf'(3) = \lim_{x \to 3^+} \frac{f(x) - f(3)}{x-3} = \lim_{x \to 3^+} \frac{(x-3) - 0}{x-3} = 1$$ ½

$\therefore Lf'(3) \neq Rf'(3)$

so, f(x) is not differentiable at x=3 1

OR

$$\tan\left(\frac{x^2-y^2}{x^2+y^2}\right) = a$$

$$\Rightarrow \frac{x^2-y^2}{x^2+y^2} = \tan^{-1} a \qquad \text{----------(1)}$$ ½

Differentiating (1) w.r.t. x, we get

$$\frac{\left(x^2+y^2\right)\left(2x-2y\frac{dy}{dx}\right) - \left(x^2-y^2\right)\left(2x+2y\frac{dy}{dx}\right)}{\left(x^2+y^2\right)^2} = 0$$

$$\text{or, } 2x\left(x^2+y^2\right)-2y\left(x^2+y^2\right)\frac{dy}{dx} - 2x(x^2-y^2) - 2y(x^2-y^2)\frac{dy}{dx} = 0$$ 2

$$\text{or, } \frac{dy}{dx}\left[-2x^2y - \cancel{2y^3} - 2x^2y + \cancel{2y^3}\right] = \cancel{-2x^3} - 2xy^2 + \cancel{2x^3} - 2xy^2$$ 1

$$\Rightarrow \frac{dy}{dx}\left[-4x^2y\right] = -4xy^2$$

$$\Rightarrow \frac{dy}{dx} = \frac{-4xy^2}{-4x^2y} = \frac{y}{x}$$ ½

15. We know that e^x, sin x and cos x functions are continuous and differentiable everywhere. Product, sum and difference of two continuous functions is again a continuous function, so

f is also continuous in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$ 1

Also, f(x) is differentiable in $\left[\frac{\pi}{4}, \frac{5\pi}{4}\right]$

$$\text{Now, } f\left(\frac{\pi}{4}\right) = e^{\frac{\pi}{4}}\left(\sin\frac{\pi}{4} - \cos\frac{\pi}{4}\right) = 0$$

$$f\left(\frac{5\pi}{4}\right) = e^{\frac{5\pi}{4}}\left(\sin\frac{5\pi}{4} - \cos\frac{5\pi}{4}\right) = 0$$

$$\Rightarrow f\left(\frac{\pi}{4}\right) = f\left(\frac{5\pi}{4}\right)$$ 1

$\therefore$ Rolle's theorem is applicable

$f'(x) = e^x(\sin x - \cos x) + e^x(\cos x + \sin x) = 2e^x \sin x$

$\therefore$ $f'(x) = 0$ gives $2e^x \sin x = 0$

or $\sin x = 0 \Rightarrow x = 0, \pi$ 1

Now $\pi \in \left(\frac{\pi}{4}, \frac{5\pi}{4}\right)$

$\therefore$ The theorem is verified with $x = \pi$ 1

16. Let $x = 25$, $x+\Delta x = 25\cdot 2$ so $\Delta x = 0\cdot 2$ 1

Let $y = \sqrt{x} \Rightarrow \frac{dy}{dx} = \frac{1}{2\sqrt{x}} = \frac{1}{2\sqrt{25}} = \frac{1}{10}$ at $x = 25$ 1

$$\Delta y \approx \frac{dy}{dx}\cdot \Delta x$$

or $\Delta y \approx \frac{dy}{dx}\cdot \Delta x = \frac{1}{10} \times 0.2 = 0.02$ 1

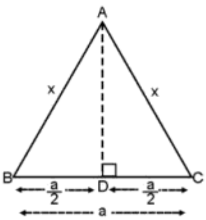
$\therefore \sqrt{25.2} \approx y+\Delta y = 5+0.02 = 5.02$ 1

OR

Let A be the area of Δ ABC in which AB = AC = x and BC = a

$\therefore A = \frac{1}{2} BC \times AD$

$= \frac{1}{2} a \sqrt{x^2 - \frac{a^2}{4}} = \frac{a}{4}\sqrt{4x^2-a^2}$ 1



$\frac{dA}{dt} = \frac{a}{4}\cdot\frac{1}{2\sqrt{4x^2-a^2}}\cdot 8x \cdot \frac{dx}{dt}$ 1

$= \frac{ax \times 9}{\sqrt{4x^2-a^2}}$ 1

$\therefore \left(\frac{dA}{dt}\right)_{\text{at } x=a} = \frac{9a\cdot a}{\sqrt{3a^2}} = 3\sqrt{3}\ a\ \text{cm}^2/\text{second}$ 1

17. $I = \int_{-1}^{1/2} |x\cos(\pi x)|dx$

Three cases arise :

Case I : $-1 < x < \frac{-1}{2}$

$\Rightarrow -\pi < \pi x < -\frac{\pi}{2}$

$\Rightarrow \cos\pi x < 0 \Rightarrow x\cos\pi x > 0$ ½

Case II : $-\frac{1}{2} < x < 0$

$-\frac{\pi}{2} < \pi x < 0$

$\Rightarrow \cos(\pi x) > 0$

$\Rightarrow x\cos(\pi x) < 0$ ½

case III : $0 < x < \frac{1}{2}$

$\Rightarrow 0 < \pi x < \frac{\pi}{2}$

$\Rightarrow \cos \pi x > 0$

$\Rightarrow x\cos \pi x > 0$ ½

$\therefore I = \int_{-1}^{-1/2} x\cos\pi x\, dx + \int_{-1/2}^{0} -x\cos\pi x dx + \int_{0}^{1/2} x\cos\pi x\, dx$ 1

$= \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{-1}^{-\frac{1}{2}} - \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{-\frac{1}{2}}^{0} + \left[\frac{x\sin\pi x}{\pi} + \frac{\cos\pi x}{\pi^2}\right]_{0}^{\frac{1}{2}}$ 1

$= \left[\left(\frac{1}{2\pi} + 0\right) - \left(0 - \frac{1}{\pi^2}\right)\right] - \left[-\left(\frac{1}{2\pi} + 0\right) + \left(0 + \frac{1}{\pi^2}\right)\right] + \left[-\left(0 + \frac{1}{\pi^2}\right) + \left(\frac{1}{2\pi} + 0\right)\right]$

$\frac{1}{2\pi} + \cancel{\frac{1}{\pi^2}} + \frac{1}{2\pi} - \cancel{\frac{1}{\pi^2}} + \frac{1}{2\pi} - \frac{1}{\pi^2}$

$= \frac{3}{2\pi} - \frac{1}{\pi^2}$ ½

18. $y e^{\frac{x}{y}} dx = \left(x e^{\frac{x}{y}} + y\right) dy$

$\Rightarrow \frac{dx}{dy} = \frac{x e^{\frac{x}{y}} + y}{y \cdot e^{\frac{x}{y}}}$ ½

Let $x = vy \Rightarrow \frac{dx}{dy} = v + y \cdot \frac{dv}{dy}$ ½

$$\therefore\ v + y\,\frac{dv}{dy} = \frac{vy\cdot e^{v} + y}{y\cdot e^{v}}$$ 1

$$\Rightarrow y\,\frac{dv}{dy} = \frac{vye^{v} + y}{y\cdot e^{v}} - v = \frac{\cancel{vye^{v}} + y - \cancel{vye^{v}}}{y\cdot e^{v}} = \frac{1}{e^{v}}$$ ½

$$\Rightarrow\ e^{v}dv = \frac{dy}{y}$$ ½

Integrating we get $e^{v} = \log y + \log c = \log cy$ ½

Substituting $v = \frac{x}{y}$, we get ½

$$e^{\frac{x}{y}} = \log cy$$

19. $(1+y+x^2y)dx + (x+x^3)dy = 0$

$$\Rightarrow\ x(1+x^2)dy = -[1+y(1+x^2)]dx$$

$$\Rightarrow\ \frac{dy}{dx} = \frac{-1-y(1+x^2)}{x(1+x^2)} = \frac{-1}{x}\cdot y - \frac{1}{x(1+x^2)}$$ 1

$$\text{or } \frac{dy}{dx} + \frac{1}{x}\cdot y = -\frac{1}{x(1+x^2)}$$

$$\therefore\ \text{I.F.} = e^{\int \frac{1}{x}dx} = e^{\log x} = x$$ 1

$\therefore$ The solution is

$$y.x = -\int \frac{1}{x(1+x^2)}\cdot x\,dx = -\int \frac{dx}{1+x^2}$$

$$= -\tan^{-1}x + c$$ 1

when x=1, y=0

$$\therefore\ 0 = -\tan^{-1}(1) + c \qquad \Rightarrow c = \pi/4$$ ½

$$\therefore\ xy = -\tan^{-1}x + \frac{\pi}{4}$$ ½

20. $\vec{a}, \vec{b} = 0$ and $\vec{a}\cdot\vec{c} = 0$

$\Rightarrow\ \vec{a} \perp \vec{b}$ and $\vec{a} \perp \vec{c}$ ½

$\therefore$ $\vec{a}$ is $\perp$ to the plane of $\vec{b}$ and $\vec{c}$

$\Rightarrow$ $\vec{a}$ is parallel to $\vec{b}\times\vec{c}$ 1

Let $\vec{a} = k\,(\vec{b}\times\vec{c})$, where k is a scalar ½

$$\therefore\ |\vec{a}| = |k|\ |\vec{b}\times\vec{c}|$$

$$= |k|\ |\vec{b}|\ |\vec{c}|\ \sin\frac{\pi}{6}$$ 1

$$\therefore\ 1 = |k|\frac{1}{2}\ \Rightarrow\ |k| = 2$$ ½

$\therefore$ $k = \pm 2$ ½

$\therefore$ $\vec{a} = \pm 2(\vec{b}\times\vec{c})$

21. Equation of plane passing through (0, -1, -1) is

$a(x-0) + b(y+1) + c(z+1) = 0$ ---(i) ½

(i) passes through (4, 5, 1) and (3, 9, 4)

$\Rightarrow$ $4a+6b+2c = 0$ or $2a+3b+c=0$ ----(ii) 1

and $3a + 10b + 5c = 0$ ---(iii)

from (ii) and (iii), we get

$$\frac{a}{15-10} = \frac{-b}{10-3} = \frac{c}{20-9}\ \Rightarrow\ \frac{a}{5} = \frac{-b}{7} = \frac{c}{11} = k \text{ (say)}$$ 1

$\therefore$ $a = 5k,\ b = -7k,\ c = 11k$ ---(iv)

Putting these values of a, b, c in (i), we get

$5kx - 7k(y+1) + 11k(z+1) = 0$

or $5x-7y+11z+4 = 0$ ---(v) ½

Putting the point (-4, 4, 4) in (v), we get

$-20-28+44+4=0$ which is satisfied ½

$\therefore$ The given points are co-planar and equation

of plane is $5x-7y+11z+4 = 0$ ½

22. According to the given question

$$P(H) = \frac{3}{4},\ P(T) = \frac{1}{4}$$ ½

Let X be the random variate, which can take values 0, 1, 2, 3

$P(X=0) = P(\text{No Tails}) = P(HHH) = \frac{3}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$ ½

$P(X=1) = P(1 \text{ Tail}) = P(HHT) + P(HTH) + P(THH)$

$= \frac{3}{4}\times\frac{3}{4}\times\frac{1}{4} + \frac{3}{4}\times\frac{1}{4}\times\frac{3}{4} + \frac{1}{4}\times\frac{3}{4}\times\frac{3}{4} = \frac{27}{64}$ 1

$P(X=2) = P(2 \text{ tails}) = P(HTT) + P(THT) + P(TTH)$

$\frac{3}{4}\times\frac{1}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{3}{4}\times\frac{1}{4} + \frac{1}{4}\times\frac{1}{4}\times\frac{3}{4} = \frac{9}{64}$ 1

$P(X=3) = P(3 \text{ tails}) = P(TTT)$

$\frac{1}{4}\times\frac{1}{4}\times\frac{1}{4} = \frac{1}{64}$ ½

Reqd. Probability Distribution is

X	0	1	2	3
P(X)	27/64	27/64	9/64	1/64

½

OR

For a fair coin, $p(H) = \frac{1}{2}$ and $p(T) = \frac{1}{2}$ where H and T denote Head and Tail respectively. ½

Let the coin be tossed n times

$\therefore$ Required probability $= 1 - p(\text{all Tails})$

$= 1 - \frac{1}{2^n}$ ---(i) 1½

It has to be >80%

Total probability = 1 1

$\therefore$ (i) has to be $> \frac{4}{5}$

$\therefore 1 - \frac{1}{2^n} > \frac{4}{5} \Rightarrow n = 3$

$\therefore$ The fair coin must be tossed 3 times for the desired situation. 1

SECTION C

23.
$$\Delta = \begin{vmatrix} (b+c)^2 & ab & ca \\ ab & (a+c)^2 & bc \\ ac & bc & (a+b)^2 \end{vmatrix}$$

Operating $R_1 \to a\,R_1,\ R_2 \to b\,R_2,\ R_3 \to c\,R_3,$ to get

$$\Delta = \frac{1}{abc}\begin{vmatrix} a(b+c)^2 & a^2b & a^2c \\ ab^2 & b(a+c)^2 & b^2c \\ ac^2 & bc^2 & c(a+b)^2 \end{vmatrix} = \frac{abc}{abc}\begin{vmatrix} (b+c)^2 & a^2 & a^2 \\ b^2 & (a+c)^2 & b^2 \\ c^2 & c^2 & (a+b)^2 \end{vmatrix}$$ 1+½

Operating $c_2 \to c_2 - c_1,\ c_3 \to c_3 - c_1,$ to get

$$\Delta = \begin{vmatrix} (b+c)^2 & a^2-(b+c)^2 & a^2-(b+c)^2 \\ b^2 & (a+c)^2-b^2 & 0 \\ c^2 & 0 & (a+b)^2-c^2 \end{vmatrix} = (a+b+c)^2\begin{vmatrix} (b+c)^2 & a-b-c & a-b-c \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix}$$ 1+½

Operating $R_1 \to R_1 - (R_2 + R_3)$ to get

$$\Delta = (a+b+c)^2\begin{vmatrix} 2bc & -2c & -2b \\ b^2 & a+c-b & 0 \\ c^2 & 0 & a+b-c \end{vmatrix} \quad c_2 \to c_2 + \frac{1}{b}c_1,\ c_3 \to c_3 + \frac{1}{c}c_1$$

$$= (a+b+c)^2\begin{vmatrix} 2bc & 0 & 0 \\ b^2 & a+c & \frac{b^2}{c} \\ c^2 & \frac{c^2}{b} & a+b \end{vmatrix}$$ 1+1

$$(a+b+c)^2\left[2bc(a^2+ac+ab+bc-bc)\right] = (a+b+c)^2(2bc)\,a(a+b+c)$$
$$= (a+b+c)^3 \cdot 2abc$$ 1

24. Let the radius of circle be r and side of square be x

$\therefore\ 2\pi r + 4x = k$ ---(A) 1

Let A be the sum of the areas of circle and square

$\therefore\ A = \pi r^2 + x^2$

$$= \pi\left[\frac{k\text{-}4x}{2\pi}\right]^2 + x^2 \qquad \text{[using (A)]}$$ 1

$$= \pi\left[\frac{k^2+16x^2-8kx}{4\pi^2}\right] + x^2$$

$$= \frac{k^2+16x^2-8kx}{4\pi} + x^2$$ ½

$$\therefore \frac{dA}{dx} = \frac{1}{4\pi}[0+32x\text{-}8k] + 2x$$

$$= \frac{1}{4\pi}[32x\text{-}8k+8\pi x]$$ ½

For optimisation $\frac{dA}{dx} = 0 \Rightarrow (32x+8\pi x) = 8k$

$$\Rightarrow x = \frac{k}{4+\pi} \qquad \text{--(i)}$$ 1

$$\therefore \frac{d^2A}{dx^2} = \frac{1}{4\pi}[32+8\pi] > 0 \Rightarrow \text{Minima}$$ ½

Putting the value of x in (A) to get

$$2\pi r + 4\cdot\frac{k}{4+\pi} = k$$

$$\Rightarrow 2\pi r = k - \frac{4k}{4+\pi} = \frac{\pi k}{4+\pi}$$ 1

$$2r = \frac{k}{4+\pi} \qquad \text{--(ii)}$$

From (i) and (ii), $x = 2r$ ½

OR

Let P(x,y) be the position of the Helicopter and the position of soldier at A(3, 2)

$$\therefore AP = \sqrt{(x\text{-}3)^2 + (y\text{-}2)^2} = \sqrt{(x\text{-}3)^2 + (x^2)^2} \qquad \left[\begin{array}{l}\therefore y=x^2+2 \text{ is the} \\ = \text{n of curve}\end{array}\right]$$ 2

Let $AP^2 = z = (x\text{-}3)^2 + x^4$

$\Rightarrow \dfrac{dz}{dx} = 2(x-3)+4x^3$ 1

For optimisation $\dfrac{dz}{dx} = 0 \Rightarrow 2x^3 + x-3 = 0$

or $(x-1)(2x^2+2x+3) = 0 \Rightarrow x=1$ $\left[\begin{array}{l}\text{other factor} \\ \text{gives no real values}\end{array}\right]$ 1

$\dfrac{d^2z}{dx^2} = 6x^2 + 1 > 0 \Rightarrow$ Minima

when $x=1$, $y=x^2+2=3$

$\therefore$ The required point is (1, 3) 1

And distance AP $= \sqrt{(1-3)^2 + (3-2)^2} = \sqrt{5}$ 1

25. $\displaystyle\int \frac{1}{\sin x\,(5-4\cos x)}\,dx = \int \frac{\sin x}{\sin^2 x\,(5-4\cos x)}\,dx$ ½

$= \displaystyle\int \frac{\sin x}{(1-\cos^2 x)(5-4\cos x)}\,dx$

$= -\displaystyle\int \frac{dt}{(1-t^2)(5-4t)}$, where $\cos x=t$, $dt=-\sin x\,dx$

$= -\displaystyle\int \frac{dt}{(1-t)(1+t)(5-4t)}$ 1

Let $\dfrac{1}{(1-t)(1+t)(5-4t)} = \dfrac{A}{1-t} + \dfrac{B}{1+t} + \dfrac{C}{5-4t}$ ½

$\Rightarrow 1 = A(1+t)(5-4t) + B(1-t)(5-4t) + c(1-t^2)$ --(i)

Compairing Cofficients on both sides to get

$A = \dfrac{1}{2}$

$B = \dfrac{1}{18}$ 1½

$C = -\dfrac{16}{9}$

$\therefore I = -\left[\dfrac{1}{2}\displaystyle\int\frac{dt}{1-t} + \frac{1}{18}\int\frac{dt}{1+t} - \frac{16}{9}\int\frac{dt}{5-4t}\right]$

$$-\left[-\frac{1}{2}\log|1\text{-t}|+\frac{1}{18}\log|1\text{+t}|+\frac{16}{9\text{x}4}\log|5-4t|\right]+c$$ 1½

$$=\frac{1}{2}\log|1\text{-cosx}|-\frac{1}{18}\log|1+\cos x|-\frac{4}{9}\log|5-4\cos x|+c$$ 1

OR

$$I=\int\sqrt{\frac{1-\sqrt{x}}{1+\sqrt{x}}}dx=\int\frac{\sqrt{1-\sqrt{x}}\cdot\sqrt{1-\sqrt{x}}}{\sqrt{1+\sqrt{x}}\sqrt{1-\sqrt{x}}}dx=\int\frac{1\text{-}\sqrt{x}}{\sqrt{1\text{-x}}}dx$$ 1½

$$=\int\frac{dx}{\sqrt{1\text{-x}}}-\int\frac{\sqrt{x}}{\sqrt{1\text{-x}}}dx \quad =I_1\text{-}I_2$$ ½

$$I_1=\int(1\text{-x})^{-\frac{1}{2}}dx=-2(1\text{-x})^{\frac{1}{2}}+c_1 \text{ or } -2\sqrt{1\text{-x}}+c_1$$ 1

$$I_2=\int\frac{\sqrt{x}}{\sqrt{1\text{-x}}}dx : \text{Let } x=\sin^2\theta,\ dx=2\sin\theta\cos\theta\ d\theta$$ ½

$$=\int\frac{\sin\theta\cdot 2\sin\theta\cos\theta\ d\theta}{\cos\theta}=2\int\sin^2\theta\ d\theta$$ 1

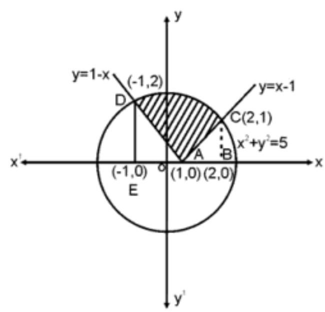
$$=\int(1\text{-cos}2\theta)d\theta=\theta\text{-}\frac{\sin 2\theta}{2}=\theta\text{-sin}\theta\cos\theta+c_2$$

$$=\sin^{-1}\sqrt{x}-\sqrt{x}\sqrt{1\text{-x}}+c_2$$ 1

$$\therefore\ I=-2\sqrt{(1\text{-x})}-\sin^{-1}\sqrt{x}+\sqrt{x}\sqrt{1\text{-x}}+C$$

$$=\sqrt{1\text{-x}}\left[\sqrt{x}-2\right]-\sin^{-1}\sqrt{x}+C$$ ½

26. Equations of curves are



$$x^2+y^2=5 \text{ and } y=\begin{Bmatrix}1\text{-x}, & x<1\\ x\text{-1}, & x>1\end{Bmatrix}$$ ½

correct figure 1

Points of intersection are C(2, 1) ½

D(-1, 2)

Required Area = Area of region (EABCDE) - Area of (ADEA)- Area of (ABCA) ½

$$= \int_{-1}^{2} \sqrt{5-x^2}\,dx - \int_{-1}^{1} (1-x)\,dx - \int_{1}^{2} (x-1)\,dx -$$ 1

$$= \left[\frac{x}{2}\sqrt{5-x^2} + \frac{5}{2}\sin^{-1}\frac{x}{\sqrt{5}}\right]_{-1}^{2} - \left[x - \frac{x^2}{2}\right]_{-1}^{1} - \left[\frac{x^2}{2} - x\right]_{1}^{2}$$ 1

$$= \left[\left\{1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}}\right\} - \left\{-\frac{1}{2}\times 2 + \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right\}\right] - \left[\left(1-\frac{1}{2}\right) - \left(-1-\frac{1}{2}\right)\right]$$

$$-\left[(2-2) - \left(\frac{1}{2} - 1\right)\right]$$ 1

$$= 1 + \frac{5}{2}\sin^{-1}\frac{2}{\sqrt{5}} + 1 - \frac{5}{2}\sin^{-1}\left(\frac{-1}{\sqrt{5}}\right) - 2 - \frac{1}{2}$$

$$= -\frac{1}{2} + \frac{5}{2}\left[\text{Sin}^{-1}\frac{2}{\sqrt{5}} - \sin^{-1}\left(\frac{-1}{\sqrt{5}}\right)\right]$$ ½

27. Lines $\frac{x-x_1}{l_1} = \frac{y-y_1}{m_1} = \frac{z-z_1}{n_1}$ and $\frac{x-x_2}{l_2} = \frac{y-y_2}{m_2} = \frac{z-z_2}{n_2}$ are 1½

coplanar if $\begin{vmatrix} x_2-x_1 & y_2-y_1 & z_2-z_1 \\ l_1 & m_1 & n_1 \\ l_2 & m_2 & n_2 \end{vmatrix} = 0$

In this case $\begin{vmatrix} -2 & -1 & 0 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = -2(5-10)+1(-15+5)+0 = 10-10 = 0$ 1½

$\therefore$ Lines are coplanar

Equation of plane containing this is

$\begin{vmatrix} x+3 & y-1 & z-5 \\ -3 & 1 & 5 \\ -1 & 2 & 5 \end{vmatrix} = 0$ 1½

$\Rightarrow 5x-10y+5z = 0$

or $x-2y+z = 0$ 1½

28. Let events E_1, E_2, E_3, E_4 and A be defined as follows

E_1 : Missing card is a diamond

E_2 : Missing card is a spade

E_3 : Missing card is a club 1

E_4 : Missing card is a heart

A : Drawing two diamond cards

$P(E_1) = P(E_2) = P(E_3) = P(E_4) = \frac{1}{4}$ ½

$P\left(A/E_1\right) = \frac{12}{51} \times \frac{11}{50}$ ½

$P\left(A/E_2\right) = P\left(A/E_3\right) = P\left(A/E_4\right) = \frac{13}{51} \times \frac{12}{50}$ 1

$$P\left(E_4/A\right) = \frac{P(E_4) \cdot P\left(A/E_4\right)}{\sum_1^4 P(E_i) \cdot P\left(A/E_i\right)}$$ 1

$$\frac{\frac{1}{4} \cdot \frac{13}{51} \times \frac{12}{50}}{\frac{1}{4}\left[\frac{12 \times 11 + 13 \times 12 + 13 \times 12 + 13 \times 12}{51 \cdot 50}\right]}$$ 1

$$= \frac{13 \times \cancel{12}}{3 \times 13 \times \cancel{12} + \cancel{12} \times 11} = \frac{13}{39+11} = \frac{13}{50}$$ 1

29. Let x and y be the units taken of Food A and Food B respectively then LPP is, ½

Minimise $z = 4x+3y$

Subject to constraints

$200x + 100y \geq 4000$ or $2x+y \geq 40$

$x+2y \geq 50$

$40x+40y \geq 1400$ or $x+y \geq 35$

$x \geq 0, y \geq 0$ 1½

Correct Graph 2

The corners of feasible region are

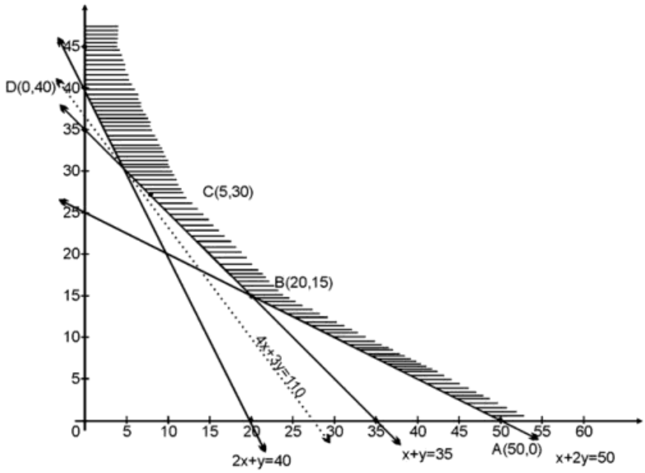
A(50,0), B(20,15), C (5, 30), D(0, 40) 1 1

$Z_A = 200$, $Z_B = 125$, $Z_C = 110$, $Z_D = 120$

$\therefore$ Z is minimum at C

$\therefore$ 5 units of Food A and 30 units of Food B

will give the minimum cost (which is Rs 110) 1



CBSE SAMPLE PAPER - III

CLASS XII MATHEMATICS

BLUE PRINT

S. No.	Topics	VSA	SA	LA	Total
1. (a)	Relations and Functions	-	4(1)	-	
(b)	Inverse Trigonometric Functions	2(2)	4(1)	-	10(4)
2. (a)	Matrics	2(2)	-	6(1)	
(b)	Determinants	1(1)	4(1)	-	13(5)
3. (a)	Continuity and differentiability	1(1)	12(3)	-	
(b)	Applications of derivatives	-	-	6(1)	
(c)	Integration	-	12(3)	-	
(d)	Application of integrals			6(1)	
(e)	Differential Equations	1(1)	-	6(1)	44(11)
4. (a)	Vectors	2(2)	4(1)	-	
(b)	3-dimensional Geometry	1(1)	4(1)	6(1)	17(6)
5.	Linear - Programming	-	-	6(1)	6(1)
6.	Probability	-	4(1)	6(1)	10(2)
	Total	10(10)	48(12)	42(7)	100(29)

SAMPLE PAPER - III

MATHEMATICS

CLASS - XII

Time : 3 Hours **Max. Marks : 100**

General Instructions

1. All questions are compulsory.

2. The question paper consist of 29 questions divided into three sections A, B and C. Section A comprises of 10 questions of one mark each, section B comprises of 12 questions of four marks each and section C comprises of 07 questions of six marks each.

3. All questions in Section A are to be answered in one word, one sentence or as per the exact requirement of the question.

4. There is no overall choice. However, internal choice has been provided in 04 questions of four marks each and 02 questions of six marks each. You have to attempt only one of the alternatives in all such questions.

5. Use of calculators in not permitted. You may ask for logarithmic tables, if required.

SECTION A

1. Write the principal value of $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$.

2. Write the range of the principal branch of $\sec^{-1}(x)$ defined on the domain R-(-1, 1).

3. Find x if $\begin{vmatrix} 3 & 4 \\ -5 & 2 \end{vmatrix} = \begin{vmatrix} 2x & 4 \\ -5 & 3 \end{vmatrix}$.

4. If A is a square matrix of order 3 such that $|\text{adj } A| = 64$. Find $|A'|$

5. If A is a square matrix satisfying A^2=I, then what is the inverse of A?

6. If $f(x) = \sin x°$, find $\frac{dy}{dx}$

7. What is the degree of the following differential equation?

$$y\frac{d^2y}{dx^2}+\left(\frac{dy}{dx}\right)^3 = x\left(\frac{d^3y}{dx^3}\right)^2$$

8. If $\vec{a}$ and $\vec{b}$ represent the two adjacent sides of a parallelogram, then write the area of parallelogram in terms of $\vec{a}$ and $\vec{b}$.

9. Find the angle between two vectors $\vec{a}$ and $\vec{b}$ if $|\vec{a}|=3$, $|\vec{b}| = 4$ and $|\vec{a}\times\vec{b}| = 6$

10. Find the direction cosines of a line, passing through origin and lying in the first octant, making equal angles with the three coordinate axes.

SECTION B

11. Show that the relation R in the set $A = \{x ; x \in Z, 0 \le x \le 12\}$ given by $R = \{(a, b) : |a-b| \text{ is divisible by } 4\}$ is an equivalence relation. Find the set of all elements related to 1.

12. Solve for x : $2\tan^{-1}(\sin x) = \tan^{-1}(2\sec x)$, $0<x<\frac{\pi}{2}$

OR

Show that : $\tan\frac{1}{2}\left[\sin^{-1}\frac{2x}{1+x^2} + \cos^{-1}\frac{1-y^2}{1+y^2}\right] = \frac{x+y}{1-xy}$, $|x|<1$, $y>0$ $xy<1$

13. If none of a, b and c is zero, using properties of determinants.

prove that : $\begin{vmatrix} -bc & b^2+bc & c^2+bc \\ a^2+ac & -ac & c^2+ac \\ a^2+ab & b^2+ab & -ab \end{vmatrix} = (bc+ca+ab)^3$

14. If $\sqrt{1-x^2}+\sqrt{1-y^2}=a(x-y)$, prove that $\frac{dy}{dx}=\sqrt{\frac{1-y^2}{1-x^2}}$

15. If $y=\left(x+\sqrt{x^2-1}\right)^m$, then show that $(x^2+1)\frac{d^2y}{dx^2}+x\frac{dy}{dx}-m^2y=0$

16. Find all the points of dicontinuity of the function $f(x)=[x^2]$ on $[1, 2)$, where [.] denotes the greatest integer function.

OR

Differentiate $\sin^{-1}\left(2x\sqrt{1-x^2}\right)$ w.r.t. $\cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$

17. Evaluate : $\int \frac{1}{\cos(x-a)\cos(x-b)}dx$

OR

Evaluate : $\int x(\log x)^2 \cdot dx$

18. Evaluate : $\int \frac{x}{x^3-1}dx$

19. Using properties of definite integrals, evaluate.

$$\int_0^{\pi} \frac{x\,dx}{4-\cos^2 x}$$

20. The dot products of a vector with the vectors $\hat{i}-3\hat{k}$, $\hat{i}-2\hat{k}$ and $\hat{i}+\hat{j}+4\hat{k}$ are 0, 5 and 8 respectively. Find the vector.

21. Find the equation of plane passing through the point (1, 2, 1) and perpendicular to the line joining the points (1, 4, 2) and (2, 3, 5). Also, find the perpendicular distance of the plane from the origin.

OR

Find the equation of the perpendicular drawn from the point P(2, 4, -1) to the line $\frac{x+5}{1}=\frac{y+3}{4}=\frac{z-6}{-9}$.

22. A biased die is twice as likely to show an even number as an odd number. The die is rolled three times. If occurance of an even number is considered a success, then write the probability distribution of number of successes. Also find the mean number of successes.

SECTION C

23. Using matrices, solve the following system of equations :

$$\frac{1}{x}-\frac{1}{y}+\frac{1}{z}=4;\ \frac{2}{x}+\frac{1}{y}-\frac{3}{z}=0;\ \frac{1}{x}+\frac{1}{y}+\frac{1}{z}=2.\ x\neq 0,\ y\neq 0,\ z\neq 0$$

24. Show that the volume of the greatest cylinder which can be inscribed in a cone of height h and semivertical angle α, is $\frac{4}{27}\pi h^3 \tan^2\alpha$

OR

Show that the normal at any point θ to the curve $x = a\cos\theta + a\theta\sin\theta$ and $y = a\sin\theta - a\theta\cos\theta$ is at a constant distance from the origin.

25. Find the area of the region: $\{(x,y) : 0 \leq y \leq x^2,\ 0 \leq y \leq x+2;\ 0 \leq x \leq 3\}$

26. Find the particular solution of the differential equation

$$(xdy-ydx)y\sin\left(\frac{y}{x}\right) = (ydx+xdy)x\cos\frac{y}{x},\text{ given that } y=\pi \text{ when } x=3.$$

27. Find the equation of the plane passing through the point (1, 1, 1) and containing the line

$\vec{r} = (-3\hat{i}+\hat{j}+5\hat{k}) + \lambda(3\hat{i}-\hat{j}+5\hat{k})$. Also, show that the plane contains the line

$\vec{r} = (-\hat{i}+2\hat{j}+5\hat{k}) + \lambda(\hat{i}-2\hat{j}-5\hat{k})$

28. A company sells two different type of products A and B. The two products are produced in a common production process which has a total capacity of 500 man hours. It takes 5 hours to produce a unit of A and 3 hours to produce a unit of B. The demand in the market shows that the maximum number of units of A that can be sold is 70 and that of B is 125. Profit on each unit of A is Rs. 20 and on B is Rs. 15. How many units of A and B should be produced to maximise the profit. Form an L.P.P. and solve it graphically.

29. Two bags A and B contain 4 white and 3 black balls and 2 white and 2 black balls respectively. From bag A, two balls are drawn at random and then transferred to bag B. A ball is then drawn from bag B and is found to be a black ball. What is the probability that the transferred balls were 1 white and 1 black?

OR

In an examination, 10 questions of true - false type are asked. A student tosses a fair coin to determine his answer to each question. If the coin falls heads, he answers 'true' and if it falls tails, he answers 'false'.

Show that the probability that he answers at most 7 questions correctly is $\frac{121}{128}$.

MARKING SCHEME

MATHEMATICS CLASS - XII

SAMPLE PAPER III

SECTION A

1. $\frac{5\pi}{6}$

2. $\left[0, \frac{\pi}{2}\right) \cup \left(\frac{\pi}{2}, \pi\right]$

3. x=1

4. $|A'| = \pm 8$

5. $A^{-1}=A$

6. $\frac{\pi}{180} \cos x°$

7. 2

8. $|\vec{a} \times \vec{b}|$

9. $\frac{\pi}{6}$

10. $< \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}}, \frac{1}{\sqrt{3}} >$ (1 mark for correct answer for Qs. 1 to 10)

SECTION B

11. (i) $\forall$ a $\in$ A, $|a-a|=0$ is divisible by 4. $\therefore$ R is reflexive --(i) 1

(ii) a, b $\in$ A, (a, b) $\in$ R $\Rightarrow |a-b|$ is divisible by 4.

$\Rightarrow |b-a|$ is divisible by 4 $\therefore$ R is symmetric --(ii) 1

(iii) a, b, c $\in$ A, (a, b) $\in$ R and (b, c) $\in$ R

$\Rightarrow |a-b|$ is divisible by 4 and $|b-c|$ is divisible by 4
$\therefore$ (a-b) and (b-c) are divisible by 4 and so
(a-b) + (b-c) = (a-c) is divisible by 4. Hence 1
$|a-c|$ is divisible by 4 $\Rightarrow$ (a, c) $\in$ R. Hence R is transitive

Hence R is an equivalence relation from (i), (ii) and (iii). ½

Set of all elements of A, related to 1 is {1, 5, 9} ½

12. Given equation can be written as

$$\tan^{-1}\left(\frac{2\sin x}{1-\sin^2 x}\right) = \tan^{-1}\left(\frac{2}{\cos x}\right),\ 0<x<\frac{\pi}{2}$$ 1½

$$\Rightarrow \frac{2\sin x}{\cos^2 x} = \frac{2}{\cos x} \Rightarrow \tan x = 1$$ 1½

$$\Rightarrow x=\frac{\pi}{4}$$ 1

OR

$$\text{LHS} = \tan\frac{1}{2}\left(2\tan^{-1}x+2\tan^{-1}y\right)$$ 1½

$$= \tan\left(\tan^{-1}x+\tan^{-1}y\right) = \tan\tan^{-1}\left(\frac{x+y}{1-xy}\right)$$ 1½

$$= \frac{x+y}{1-xy}$$ 1

13. Given determinant can be writeen as

$$\Delta = \frac{1}{abc}\begin{vmatrix} -abc & ab(b+c) & ac(b+c) \\ ab(a+c) & -abc & bc(a+c) \\ ac(a+b) & bc(b+a) & -abc \end{vmatrix}$$ 1

$$\Delta = \frac{abc}{abc}\begin{vmatrix} -bc & ab+ac & ab+ac \\ ab+bc & -ac & ab+bc \\ ac+bc & bc+ac & -ab \end{vmatrix} \begin{matrix} R_1 \to R_1+R_2+R_3 \\ = (ab+bc+ac) \end{matrix} \begin{vmatrix} 1 & 1 & 1 \\ ab+bc & -ac & ab+bc \\ ac+bc & bc+ac & -ab \end{vmatrix}$$ 1

$$\begin{matrix} C_2 \to C_2-C_1 \\ C_3 \to C_3-C_1 \end{matrix}\ \Delta = (ab+bc+ac)\begin{vmatrix} 1 & 0 & 0 \\ ab+bc & -(ab+bc+ac) & ab+bc \\ ac+bc & 0 & -(ab+bc+ac) \end{vmatrix}$$ 1

$$= (ab+bc+ac)^3$$ 1

14. Putting $x = \cos\alpha$ and $y = \cos\beta$ to get

$$\sin\alpha + \sin\beta = a(\cos\alpha - \cos\beta) \Rightarrow \frac{2\sin\frac{\alpha+\beta}{2}\cos\left(\frac{\alpha-\beta}{2}\right)}{-2\sin\frac{\alpha+\beta}{2}\sin\frac{\alpha-\beta}{2}} = a$$ 1

$$\Rightarrow \cot\left(\frac{\alpha-\beta}{2}\right) = -a, \Rightarrow \alpha-\beta = 2\cot^{-1}(-a) \text{ or } \cos^{-1}x - \cos^{-1}y = 2\cot^{-1}(-a)$$ 1

Differentiating to get $-\frac{1}{\sqrt{1-x^2}} + \frac{1}{\sqrt{1-y^2}}\frac{dy}{dx} = 0$ 1

$$\Rightarrow \frac{dy}{dx} = \sqrt{\frac{1-y^2}{1-x^2}}$$

15. Getting

$$\frac{dy}{dx} = m\cdot\left(x+\sqrt{x^2+1}\right)^{m-1}\left(1+\frac{x}{\sqrt{x^2+1}}\right) = \frac{m\left(x+\sqrt{x^2+1}\right)^m}{\sqrt{x^2+1}} = \frac{m}{\sqrt{x^2+1}}\cdot y$$ 1

$$\Rightarrow \sqrt{x^2+1}\cdot\frac{dy}{dx} = my \qquad \text{--(i)}$$ ½

$$\therefore \sqrt{x^2+1}\cdot\frac{d^2y}{dx^2} + \frac{x}{\sqrt{x^2+1}}\cdot\frac{dy}{dx} = m\cdot\frac{dy}{dx}$$ 1

$$\Rightarrow (x^2+1)\frac{d^2y}{dx^2} + x\cdot\frac{dy}{dx} = m\sqrt{x^2+1}\frac{dy}{dx} = m\cdot my = m^2y \qquad \text{(using i)}$$ 1

or $(x^2+1)\frac{d^2y}{dx^2} + x\frac{dy}{dx} - m^2y = 0$ ½

16. $f(x) = [x^2],\ 1 \le x < 2 \Rightarrow f(x) = \begin{cases} 1, & 1 \le x < \sqrt{2} \\ 2, & \sqrt{2} \le x < \sqrt{3} \\ 3, & \sqrt{3} \le x < 2 \end{cases}$ 1

At $x = \sqrt{2}$, LHL =1, RHL = 2 $\therefore$ $x = \sqrt{2}$ is a discontinuity of f(x) 1½

At $x = \sqrt{3}$, LHL =2, RHL = 3 $\therefore$ $x = \sqrt{3}$ is also a discontinuity of(fx) 1

i.e. $\sqrt{2}, \sqrt{3}$ are two discontinuities in [1, 2) ½

OR

Let $y = \sin^{-1}\left(2x\sqrt{1-x^2}\right)$ and $z = \cos^{-1}\left(\frac{1-x^2}{1+x^2}\right)$ ½

Put $x = \sin\theta$ to get

$y = \sin^{-1}(\sin 2\theta) = 2\theta = 2\sin^{-1}x$ and $z = 2\tan^{-1}x$ 1+½

$\frac{dy}{dx} = \frac{2}{\sqrt{1-x^2}}$ and $\frac{dz}{dx} = \frac{2}{1+x^2}$ 1

$\Rightarrow \frac{dy}{dz} = \frac{1+x^2}{\sqrt{1-x^2}}$ 1

17. $I = \int \frac{1}{\cos(x-a)\cos(x-b)}dx = \frac{1}{\sin(b-a)}\int \frac{\sin\left[(x-a)-(x-b)\right]}{\cos(x-a)\cos(x-b)}dx$ 1

$= \frac{1}{\sin(b-a)}\int\left[\frac{\sin(x-a)\cos(x-b)}{\cos(x-a)\cos(x-b)} - \frac{\cos(x-a)\sin(x-b)}{\cos(x-a)\cos(x-b)}\right]dx$ 1

$= \frac{1}{\sin(b-a)}\int\left[\tan(x-a)-\tan(x-b)\right]dx$ 1

$= \frac{1}{\sin(b-a)}\cdot\left[\log\left|\sec(x-a)\right| - \log\left|\sec(x-b)\right|\right] + c$ 1

OR

$I = \int(\log x)^2\cdot x\,dx = (\log x)^2\cdot\frac{x^2}{2} - \int 2\cdot\frac{\log x}{x}\cdot\frac{x^2}{2}dx$ 1½

$= \frac{x^2}{2}\cdot(\log x)^2 - \log x\cdot\frac{x^2}{2} + \int\frac{1}{x}\cdot\frac{x^2}{2}dx$ 1½

$= \frac{x^2}{2}(\log x)^2 - \frac{x^2}{2}\log x + \frac{x^2}{4} + c$ or $\frac{x^2}{2}\cdot\left[(\log x)^2 - \log x + \frac{1}{2}\right] + c$ 1

18. $\frac{x}{x^3-1} = \frac{x}{(x-1)(x^2+x+1)} = \frac{A}{x-1} + \frac{Bx+C}{x^2+x+1} \Rightarrow x = A(x^2+x+1) + (Bx+C)(x-1)$ ½

$\Rightarrow$ A+B = 0, A-B+C = 1 and A-C = 0 $\Rightarrow A = \frac{1}{3},\ b = -\frac{1}{3},\ c = \frac{1}{3}$ 1

$\therefore I = \int\frac{x}{x^3-1}dx = \frac{1}{3}\int\frac{1}{x-1}dx - \frac{1}{3}\int\frac{x-1}{x^2+x+1}dx$ 1

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\int \frac{2x+1}{x^2+x+1}dx + \frac{1}{2}\int \frac{1}{x^2+x+1}dx$$ 1

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\log|x^2+x+1| + \frac{1}{2}\int \frac{1}{\left(x+\frac{1}{2}\right)+\left(\frac{\sqrt{3}}{2}\right)^2}dx$$

$$= \frac{1}{3}\log|x-1| - \frac{1}{6}\log|x^2+x+1| + \frac{1}{\sqrt{3}}\tan^{-1}\frac{2x+1}{\sqrt{3}} + c$$ ½

19. $$I = \int_0^{\pi} \frac{x\,dx}{4-\cos^2 x} = \int_0^{\pi} \frac{(\pi-x)dx}{4-\cos^2(\pi-x)} = \int_0^{\pi} \frac{(\pi-x)dx}{4-\cos^2 x}$$ 1

$$\therefore\ 2I = \pi\int_0^{\pi} \frac{1}{4-\cos^2 x}\,dx = 2\pi\int_0^{\pi/2} \frac{\sec^2 x}{4\tan^2 x+3}$$ 1

$$I = \frac{\pi}{4}\int_0^{\infty} \frac{dt}{t^2+3/4},\ \text{where } \tan x = t \Rightarrow I = \frac{\pi}{4}\cdot\frac{2}{\sqrt{3}}\tan^{-1}\frac{2t}{\sqrt{3}}\Big]_0^{\infty}$$ 1

$$I = \frac{\pi}{2\sqrt{3}}\cdot\frac{\pi}{2} = \frac{\pi^2}{4\sqrt{3}}$$ 1

20. Let the required vector be $\vec{a} = x\hat{i}+y\hat{j}+z\hat{k}$

$\therefore\ \vec{a}\cdot(\hat{i}-3\hat{k}) = 0 \Rightarrow x-3z = 0$ --(i) 1

$\vec{a}\cdot(\hat{i}-2k) = 5 \Rightarrow x-2z = 5$ --(ii) ½

$\vec{a}\cdot(\hat{i}+\hat{j}+4\hat{k}) = 8 \Rightarrow x+y+4z = 8$ --(iii) ½

solving (i) and (ii) to get x=15, z=5 1

Putting in (iii) to get y = -27 ½

$\vec{a} = 15\hat{i}-27\hat{j}+5\hat{k}$ ½

21. If equation of plans is $\vec{r}\,.\,\vec{n} = \vec{a}\,.\,\vec{n}$, then

$\vec{a} = \hat{i}+2\hat{j}+\hat{k}$ and $\vec{n}=(2-1)\hat{i}+(3-4)\hat{j}+(5-2)\hat{k}$

$= \hat{i}-\hat{j}+3\hat{k}$ 1

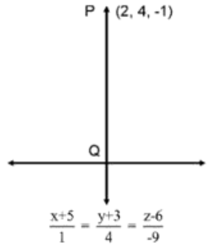
$\therefore$ equation of plane is $\vec{r}\cdot(\hat{i}-\hat{j}+3\hat{k}) = (\hat{i}+2\hat{j}+\hat{k})\cdot(\hat{i}-\hat{j}+3\hat{k})=2$ 1+1

or $x-y+3z-2 = 0$

Distance from origin $= \dfrac{2}{\sqrt{1+1+9}} = \dfrac{2}{\sqrt{11}}$ or $\dfrac{2\sqrt{11}}{11}$ units 1

OR

Any point on the given line is, $(\lambda - 5,\ 4\lambda\text{-}3,\ \text{-}9\lambda\text{+}6)$ for some value of λ, this point is Q, such that PQ is $\perp$ to the line 1



$\Rightarrow$ $(\lambda\text{-}7)1+(4\lambda\text{-}7)4+(\text{-}9\lambda\text{+}7)(\text{-}9) = 0 \Rightarrow \lambda\text{=}1$ 1½

$\therefore$ Q is (-4, 1, -3) and equation of line PQ is

$\dfrac{x-2}{6} = \dfrac{y-4}{3} = \dfrac{z+1}{2}$ 1

22. Getting P(odd number) $= \dfrac{1}{3}$, P(even number) $= \dfrac{2}{3}$ ½

(1 or 3 or 5) (2 or 4 or 6)

Let X be the random variable "getting an even number"

$\therefore$ X	0	1	2	3	½
P(X)	$\frac{1}{27}$	$\frac{6}{27}$	$\frac{12}{27}$	$\frac{8}{27}$	1½
$X \cdot P(X)$	0	$\frac{6}{27}$	$\frac{24}{27}$	$\frac{24}{27}$	½

Mean $= \sum XP(X) = \dfrac{54}{27} = 2$ 1

SECTION C

23. Given equation can be written as

$$\begin{pmatrix} 1 & -1 & 1 \\ 2 & 1 & -3 \\ 1 & 1 & 1 \end{pmatrix} \begin{pmatrix} \frac{1}{x} \\ \frac{1}{y} \\ \frac{1}{z} \end{pmatrix} = \begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} \text{ or } A \cdot X = B$$ ½

$|A| = 1(4)+1(5)+1(1) = 10 \neq 0 \ \therefore\ X = A^{-1} \cdot B$ 1

cofactors are :

$A_{11}=4,\quad A_{12}=-5,\quad A_{13}=1$

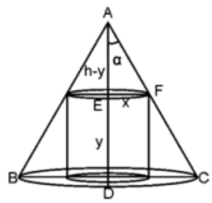
$A_{21}=2,\quad A_{22}=0,\quad A_{23}=-2$
2

$A_{31}=2,\quad A_{32}=5,\quad A_{33}=3$

$\therefore\ A^{-1} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & 2 & 3 \end{pmatrix}$ 1

$\begin{pmatrix} 1/x \\ 1/y \\ 1/z \end{pmatrix} = \frac{1}{10}\begin{pmatrix} 4 & 2 & 2 \\ -5 & 0 & 5 \\ 1 & -2 & 3 \end{pmatrix}\begin{pmatrix} 4 \\ 0 \\ 2 \end{pmatrix} = \begin{pmatrix} 2 \\ -1 \\ 1 \end{pmatrix}$ 1

$\Rightarrow\ x=\frac{1}{2},\ y=-1,\ z=1$ ½



24. Let the radius of inscribed cylinder be x and its height be y

$\therefore$ Volume (v) = $\pi x^2 y$ ½

$V = \pi(h-y)^2 \tan^2\alpha \cdot y$

$V = \pi \tan^2\alpha\left[h^2y-2hy^2+y^3\right]$ 1

$\frac{dv}{dy} = \pi\tan^2\alpha\left[h^2-4hy+3y^2\right]$ 1

$\frac{dv}{dy} = 0\ \Rightarrow\ 3y^2-4hy+h^2=0$ or $(y-h)(3y-h)=0\ \Rightarrow\ y=h,\ y=h/3$ 1½

since y=h is not possible $\therefore\ y=h/3$ is the only point

$\frac{d^2v}{dy^2} = 6y-4h = 6\left(h/3\right)-4h = -2h<0\ \therefore\ y=\frac{h}{3}$ is a maxima 1

$V = \frac{4}{27}\pi h^3 \tan^2\alpha$ 1

OR

$\frac{dx}{d\theta} = -a\sin\theta + a\sin\theta + a\theta\cos\theta = a\theta\cos\theta$ 1

$$\frac{dy}{d\theta} = a\cos\theta - a\cos\theta + a\theta\sin\theta = a\theta\sin\theta$$ 1

$$\Rightarrow \frac{dy}{dx} = \tan\theta \;\therefore\; \text{slope of normal} = -\cot\theta$$ 1

$\therefore$ Equation of normal is

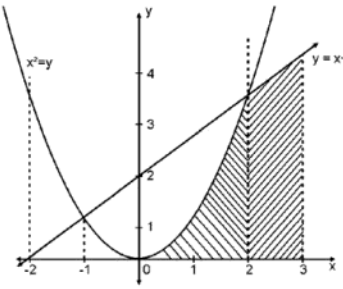
$$y - a(\sin\theta - \theta\cos\theta) = -\frac{\cos\theta}{\sin\theta}\left[x - a(\cos\theta + \theta\sin\theta)\right]$$ 1½

Simplifying to get $x\cos\theta + y\sin\theta - a = 0$

Length of perpendicular from orgin $= \dfrac{|a|}{\sqrt{\sin^2\theta + \cos^2\theta}} = |a|$ (constant) 1½

25. For correct figure and getting points of intersection as x=-1, x=2 1½

Required area $= \int_0^2 x^2 dx + \int_2^3 (x+2)dx$ 2



$$\left[\frac{x^3}{3}\right]_0^2 + \left[\frac{(x+2)^2}{2}\right]_2^3$$ 1½

$$\frac{8}{3} + \frac{25}{2} - 8 = \frac{43}{6} \text{ sq.U}$$ 1

26. Given differential equation can be written as

$$\left(xy\frac{dy}{dx} - y^2\right)\sin\left(\frac{y}{x}\right) = \left(xy + x^2\frac{dy}{dx}\right)\cos\left(y/x\right) \quad \text{--(i)}$$ ½

Putting $\frac{y}{x} = v$ or $y = vx$ gives $\frac{dy}{dx} = v + x\frac{dv}{dx}$ 1

$$\therefore \text{(i) becomes } v\sin v\left(v + x\frac{dv}{dx}\right) - v^2\sin v = v\cos v + \left(v + x\frac{dv}{dx}\right)\cos v$$ 1

$$\Rightarrow (vx\sin v - x\cos v)\frac{dv}{dx} = 2v\cos v$$

$$\Rightarrow -\int\frac{v\sin v - \cos v}{v\cos v}dv = -\int\frac{2}{x}dx \Rightarrow \log|v\cos v| = -2\log x + \log c$$ 1+1

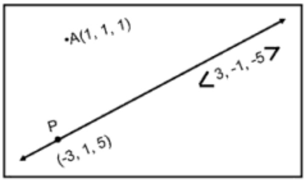
$$\Rightarrow x^2 \cdot v \cdot \cos v = c \quad \Rightarrow xy\cos y/x = c$$ ½

$x=3,\ y=\pi$ gives $c=\dfrac{3\pi}{2}$ ½

$\Rightarrow$ solution is $2xy \cos y/x = 3\pi$ ½

27. Let the given point be A (1, 1, 1) and the point on the line is P(-3, 1, 5)

$\therefore\ \overrightarrow{AP} = -4\hat{i} + 4\hat{k}$ 1



$\therefore$ The vector $\perp$ to the plane is

$(-4\hat{i}+4\hat{k})\times(3\hat{i}-\hat{j}-5\hat{k}) = 4\hat{i}-8\hat{j}+4\hat{k}$ or $\hat{i}-2\hat{j}+\hat{k}$ 1½

$\therefore$ Equation of plane is

$\vec{r}\cdot(\hat{i}-2\hat{j}+\hat{k}) = (\hat{i}+\hat{j}+\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k}) = 0$ --(i) 1½

or $x-2y+z=0$

Now, since $(\hat{i}-2\hat{j}-5\hat{k})\cdot(\hat{i}-2\hat{j}+\hat{k}) = 1+4-5 = 0$

$\therefore$ The line $\vec{r} = (-\hat{i}+2\hat{j}+5\hat{k})+\lambda(\hat{i}-2\hat{j}-5\hat{k})$ is parallel to the plane 1

Also, the point (-1, 2, 5) satisfies the equation of plane

as $(-1-4+5)=0 \Rightarrow$ point lines on plane 1

hence the plane contains the line.

28. Let x be the number of units of A and y of B, which are produced

$\therefore$ LPP is Maximise $z = 20x+15y$ 1

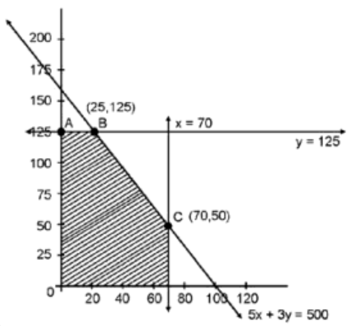
Subject to $5x+3y \le 500$

$x \le 70$

$y \le 125$

$x \ge 0,\ y \ge 0$

1½



2

Getting vertices of feasible region as :

A(0, 125), B(25, 125), C(70, 50), D(70, 0) ½

Maximum Profit = Rs. 2375 at B

$\therefore$ Number of Units of A = 25 1

Number of Units of B = 125

29. Let the events are defined as :

E_1 : 2 white balls are transferred from A to B

E_2 : 2 black balls are transferred

E_3 : 1 white and 1 black ball is transffered 1

A : 1 black ball is drawn from B

$P(E_1) = \frac{4c_2}{7c_2} = \frac{4.3}{7.6} = \frac{2}{7},\ P(E_2) = \frac{3c_2}{7c_2} = \frac{3.2}{7.6} = \frac{1}{7},\ P(E_3) = \frac{4c_1 \cdot 3c_1}{7c_2} = \frac{4}{7}$ 1½

$P(A/E_1) = \frac{2}{6} = \frac{1}{3},\ P(A/E_2) = \frac{4}{6} = \frac{2}{3},\ P(A/E_3) = \frac{3}{6} = \frac{1}{2}$ 1½

$P(E_3/A) = \frac{P(E_3)\cdot P(A/E_3)}{P(E_1)\ P(A/E_1) + P(E_2)\ P(A/E_2) + P(E_3)\ P(A/E_3)}$ ½

$= \frac{\frac{4}{7}\times\frac{1}{2}}{\frac{2}{7}\cdot\frac{1}{2}+\frac{1}{7}\cdot\frac{2}{3}+\frac{4}{7}\cdot\frac{1}{2}}$ ½

$= \frac{3}{5}$ 1

OR

P(answer is true) $= \frac{1}{2}$

P(answer is false) $= \frac{1}{2}$ 1

P(at most 7 correct) = 1-{P(8) + P(9) + P(10)} 2

(where P(8) etc means probability of 8 correct answers)

$= 1-\left\{ {}^{10}C_8 \left(\frac{1}{2}\right)^8 \cdot \left(\frac{1}{2}\right)^2 + {}^{10}C_9 \left(\frac{1}{2}\right)^9 \left(\frac{1}{2}\right) + {}^{10}C_{10} \left(\frac{1}{2}\right)^{10} \right\}$ 1

$= 1- \left\{ {}^{10}C_2 + {}^{10}C_1 + {}^{10}C_0 \right\} \left(\frac{1}{2}\right)^{10}$ 1

$= 1- \{45+10+1\}\ \frac{1}{1024}$

$= 1-\frac{56}{1024} = 1-\frac{7}{128} = \frac{121}{128}$ 1