ANNA UNIVERSITY CHENNAI :: CHENNAI – 600 025

B.E / B.TECH. DEGREE EXAMINATIONS – I YEAR ANNUAL PATTERN

# MODEL QUESTION PAPER

# MA 1X01 - ENGINEERING MATHEMATICS - I

**(Common to all Branches of Engineering and Technology)**
**Regulation 2004**

**Time : 3 Hrs** **Maximum: 100 Marks**

**Answer all Questions**

## PART – A (10 x 2 = 20 Marks)

1. Find the sum and product of the eigen values of the matrix $\begin{bmatrix} 3 & -1 & 1 \\ -1 & 5 & -1 \\ 1 & -1 & 3 \end{bmatrix}$

2. If x = r cosθ, y = r sinθ, find $\frac{\partial(r,\theta)}{\partial(\mathrm{x},y)}$

3. Solve $(D^3+D^2+4D+4)y = 0$.

4. The differential equation for a circuit in which self-inductance L and capacitance C neutralize each other is L $\frac{d^2i}{dt^2}+\frac{i}{C}=0.$ Find the current i as a function of t.

5. Find, by double integration, the area of circle $x^2+y^2 = a^2$.

6. Prove that curl grad $\phi = \bar{o}$.

7. State the sufficient conditions for a function f(z) to be analytic.

8. State Cauchy's integral theorem.

9. Find the Laplace transform of unit step function at t = a.

10. Find $L^{-1}$ $[\frac{s+3}{s^2+4s+13}]$.

## PART – B (5 x 16 = 80 marks)

11.(a).(i). Verify Cayley-Hamilton theorem for the matrix A = $\begin{pmatrix} 7 & 2 & -2 \\ -6 & -1 & 2 \\ 6 & 2 & -1 \end{pmatrix}$.

Hence find its inverse. (8)

(ii). Find the radius of curvature at any point 't' on the curve

x = a (cost + t sint), y = a(sint–t cost) (8)

**(OR)**

(b).(i). Diagonalise the matrix $\begin{bmatrix} 8 & -6 & 2 \\ -6 & 7 & -4 \\ 2 & -4 & 3 \end{bmatrix}$ by orthogonal transformation. (8).

(ii). A rectangular box open at the top is to have volume of 32 c.c. Find the dimensions of the box requiring least material for its construction, by Lagrange's multiplier method. (8).

12(a). (i). Solve $(3x+2)^2 \dfrac{d^2y}{dx^2} + 3(3x+2) \dfrac{dy}{dx} - 36y = 3x^2+4x+1$ (8)

(ii). For the electric circuit gover ned by $(LD^2+RD+\frac{1}{C})\, q = E$ where

$D = \dfrac{d}{dt}$ if L = 1 henry, R = 100 Ohms, C = $10^{-4}$ farad and E = 100 volts,

$q = \dfrac{dq}{dt} = 0$ when t = 0, find the charge q and the current i. (8)

**(OR)**

(b).(i). Solve $\dfrac{dx}{dt} + 2x + 3y = 0$, $3x + \dfrac{dy}{dt} + 2y = 2e^{2t}$ (8)

(ii). The differential equation satisfied by a beam uniformly loaded

(w kg/ metre) with one end fixed and the second end subjected

to tensile force P is given by $EI\dfrac{d^2y}{dx^2} = Py - \dfrac{1}{2}wx^2$ . Show that

the elastic curve for the beam with conditions y = 0 = $\dfrac{dy}{dx}$ at x = 0 is

given by y = $\dfrac{w}{Pn^2}$ (1-coshnx) + $\dfrac{wx^2}{2P}$ where $n^2 = \dfrac{P}{EI}$ (8)

13. a.(i). Change the order of integration in $\int_0^a \int_{\frac{x^2}{a}}^{2a-x} xy\ dx\, dy$ and hence evaluate the same. (8).

(ii). Prove that $\overline{F}$ = ($y^2$cosx + $z^3$)$\bar{i}$ +(2ysinx–4) $\bar{j}$ +3x$z^2$$\bar{k}$ is irrotational and find its scalar potential. (8)

**(OR)**

b.(i). By changing to polar co-ordinates, evaluate $\int_0^a \int_y^a \dfrac{x^2 dx dy}{\sqrt{x^2+y^2}}$ (8)

(ii). Verify Gauss divergence theorem for $\overline{F} = 4xz\bar{i} - y^2\bar{j} + yz\bar{k}$ , taken over the cube bounded by x = 0, x = 1, y = 0, y = 1, z = 0 and z = 1. (8)

14. (a).(i). If f(z) is an analytic function, prove that

$$\left(\frac{\partial^2}{\partial x^2} + \frac{\partial^2}{\partial y^2}\right) |f(z)|^2 = 4|f'(z)|^2.$$ (8).

(ii). Find the Laurent's series expansion of the function

f(z) = $\dfrac{z^2 - 6z - 1}{(z-1)(z-3)(z+2)}$ in the region 3 < |z+2| < 5. (8).

**(OR)**

(b).(i). Find the bilinear map which maps –1, 0, 1 of the z-plane onto –1,-i, 1 of the w-plane. Show that the upper half of the z-plane maps onto the interior of the unit circle | w | = 1. (8).

(ii). Using contour integration, evaluate $\int_0^{\infty} \frac{x^2 dx}{(x^2 + a^2)(x^2 + b^2)}$ (8).

15.(a) (i). Find the Laplace transform of t sint sinh2t and $\frac{1 - \cos at}{t}$ (8)

(ii). Using convolution theorem, find $L^{-1} \frac{1}{(s^2 + a^2)^2}$ (8)

**(OR)**

(b).(i).Find the Laplace transform of the function

$$f(t) = \begin{cases} t, & 0 < t < \pi \\ 2\pi - t, & \pi < t < 2\pi, \quad f(t + 2\pi) = f(t) \end{cases}$$ (8)

(ii).Using Laplace transform technique, solve

$$\frac{d^2 y}{dt^2} + 2\frac{dy}{dt} + 5y = e^{-t} \sin t,$$

$$y = 0, \frac{dy}{dt} = 0 \;\; when\; t = 0$$ (8)

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