Code No: RR221101 **Set No. 1**

II B.Tech II Semester Supplementary Examinations, November/December 2005
PROBABILITY THEORY & STOCHASTIC PROCESS
(Bio-Medical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) Define Probability density function and obtain the relationship between probability and probability density.
   (b) Consider the probability density f (x) = $ae^{-b|x|}$ where x is a random variable Whose allowable values range from x = $-\infty$to∞. Find
      i. the CDF F (x)
      ii. the relationship between a and b. and
      iii. the probability that the out come x lies between 1 and 2.

   [7+9]

2. Two discrete random variables X and Y have joint p.m.f. given by the following table

X ↓	1	2	3	Y ←
1	1/12	1/6	1/12	
2	1/6	1/4	1/12	
3	1/12	1/12	0	

   Compute the probability of each of the following events
   (a) X ≤ 1½
   (b) XY is even
   (c) Y is even given that X is even.

   [5+5+6]

3. (a) For a function Y=$(X - m_x)/\sigma_x$, prove that mean is zero & variance is 1
   (b) For the joint distribution of (X,Y) given by
   $f_{xy}(x, y) = \frac{1}{4a^2}\left[(1 + xy)\left(x^2 - y^2\right]\right., |x| <= a, |y| <= a, a > 0$
   $= 0$, otherwise
   Show that the Characteristic function of X+Y is equal to the product of the characteristic function of X & Y.

   [8+8]

4. (a) State and prove properties of cross correlation function.

(b) Consider the Random process x(t)= A $\cos(\varpi_0 t + \theta)$ where A and ϖ_0 are real constants and θ is a random variable uniformly distributed on the interval $(0, \pi/2)$ find the average power P_{xx} in x(t).

[8+8]

5. Find the input auto correlation function, output autocorrelation and o/p spectral density of RC low pass filter, where the filter is subjected to a white noise of spectral density $N_0/2$. [16]

6. Write short notes on

   (a) Flicker noise

   (b) Partition noise

   (c) Johnson's noise

[5+5+6]

7. (a) Derive the equation for narrow band noise and illustrate all its properties

   (b) Show their noise figure F of a n/w is given by F= $\frac{Go(f)}{K^2 Gin(f)}$ where Go(f), Gin(f), and K are respectively open circuited voltage, spectral density and the voltage gain of n/w.

[10+6]

8. (a) Consider an AWGN channel with S/N = 10^4. Find the maximum rate for reliable information transmission when, B = 1 KHz, 10 KHz and 100 KHz.

   (b) The Binary Erasure Channel (BEC) has two source symbols 0 and 1, and three destination symbols 0, 1 and E, where E denotes a detected but uncorrectable error. The forward transition probabilities are,

$$P(0/0) = 1 - \alpha \qquad P(E/0) = \alpha \qquad P(1/0) = 0$$

$$P(0/1) = 0 \qquad P(E/1) = \alpha \qquad P(1/1) = 1 - \alpha$$

   I (x, y) is maximum when source symbols are equiprobable. Find C_s (channel capacity) in terms of α.

[6+10]

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Code No: RR221101 **Set No. 2**

II B.Tech II Semester Supplementary Examinations, November/December 2005
PROBABILITY THEORY & STOCHASTIC PROCESS
(Bio-Medical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) State and prove Bayes theorem of probability.
   (b) In a single throw of two dice, what is the probability of obtaining a sum of at least 10?

[8+8]

2. Two discrete random variables X and Y have joint p.m.f. given by the following table

X ↓	1	2	3	Y ←
1	1/12	1/6	1/12	
2	1/6	1/4	1/12	
3	1/12	1/12	0	

Compute the probability of each of the following events

   (a) $X \leq 1\frac{1}{2}$
   (b) XY is even
   (c) Y is even given that X is even.

[5+5+6]

3. (a) For a function Y=$(X - m_x)/\sigma_x$, prove that mean is zero & variance is 1
   (b) For the joint distribution of (X,Y) given by
   $f_{xy}(x, y) = \frac{1}{4a^2}\left[(1 + xy)\left(x^2 - y^2\right]\right., |x| <= a, |y| <= a, a > 0$
   $= 0$, otherwise
   Show that the Characteristic function of X+Y is equal to the product of the characteristic function of X & Y.

[8+8]

4. Consider a Random binary waveform that consists of a sequence of pulses with the following properties

   (a) Each pulse is of duration T_0
   (b) Pulses are Equally likely to be ± 1
   (c) All pulses are statistically independent
   (d) The pulses are not synchronized, that is, the starting time T of the first pulse is Equally likely to be anywhere between 0 and Tb

Find the Auto correlation and power spectral density function of x(t). [8+8]

5. (a) Find the PSD of a random process $z(t) = X(t) + y(t)$ where x(t) and y(t) are zero mean, individual random process.
   (b) A wss random process x(t) is applied to the input of an LTI system whose impulse response is $5t.e^{-2t}$ The mean of x(t) is 3. Find the output of the system.

[8+8]

6. (a) What are the causes of thermal noise?
   (b) What are the causes of shot noise?

[8+8]

7. In TV receivers, the antenna is often mounted on a tall mask and a long lossy cable is used to connect the antenna and receiver. To overcome the effect of noisy cable, a preamplifier is mounted on the antenna. The parameters of the different stages are

Preamplifier gain	= 20 dB
Preamplifier Noise figure	= 6 dB
Lossy cable noisy figure	= 3 dB
Cable Loss	= -20 dB
Receiver front end gain	= 60 dB
Receiver Noise figure	= 16 dB

Determine the overall noise figure of the system. [16]

8. (a) Discuss the necessity for "Source coding".
   (b) A source has an alphabet {a1, a2, a3, a4, a5, and a6} with corresponding probabilities {0.1, 0.2, 0.3, 0.05, 0.15, and 0.2}. Find the entropy of its source. Compare the entropy with the entropy of a uniformly distributed source with same alphabet.

[8+8]

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Code No: RR221101 **Set No. 3**

II B.Tech II Semester Supplementary Examinations, November/December 2005
PROBABILITY THEORY & STOCHASTIC PROCESS
(Bio-Medical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) If A and B are any events, not necessarily mutually exclusive events, derive an expression for probability of A Union B. When A and B are mutually exclusive, what happens to the above expression derived?
   (b) Define the term Independent events. State the conditions for independence of
      i. any two events A and B.
      ii. any three events A, B and C.
   (c) A coin is tossed. If it turns up heads, two balls will be drawn from box A, otherwise, two balls will be drawn from box B. Box A contains three black and five white balls. Box B contains seven black and one white balls. In both cases, selections are to be made with replacement. What is the probability that Box A is used, given that both balls drawn are black?

[5+6+5]

2. The Rayleigh density function is given by

$$f(x) = x\,e^{-x^2/2} \qquad x \geq 0$$
$$= 0\, x < 0$$

   (a) Prove that f (x) satisfies the properties of the p.d.f.
      i. f(x)$\geq$ 0 for all x and
      ii. $\int_{\infty}^{\infty} f(x)\,dx = 1$
   (b) Find the distribution function F (x)
   (c) Find P(0.5 < x $\leq$ 2)
   (d) Find P(0.5 $\leq$ x < 2).

[2+2+4+4+4]

3. (a) Given the following table

X	1	2	3	4	5	6	7
P(x)	0.05	0.1	0.3	0	0.3	0.15	0.1

   Find
      i. E[X]

ii. $E[X^2]$
iii. V[X]
iv. V[2x $\pm$ 3]

(b) Prove that cov(ax,by) = ab cov(x,y)

[8+8]

4. (a) Explain Ergodic random process
(b) State and prove properties of Auto correlation function

[8+8]

5. White noise n(t) with G(f) $=\eta/2$ is passed through a low pass RC network with a 3dB frequency f_c.
(a) Find the autocorrelation R(τ) of the out put noise of the network.
(b) Sketch P(τ) $=R(\tau)/R(0)$
(c) Find $\varpi_c(\tau)$such that $P(\tau)$ $\leq$0.1.

[8+4+4]

6. (a) What are the causes of thermal noise?
(b) What are the causes of shot noise?

[8+8]

7. (a) Show that the effective noise temperature of n networks in cascade is given by, $T_e = T_{e1} + T_{e2}/g_1 + T_{e3}/g_1g_2 +$ .... $+ T_{en}/g_1g_2g_{n-1}$
(b) A low noise receiver for satellite ground station consists of the following stages
Antenna with T_i = 125K
Waveguide with a loss of 0.5dB
Power amplifier with $g_a = 30dB, T_e = 6K, B_N$ = 20 MHz
TWT amplifier with $g_a = 16dB, F = 6dB, B_N$ = 20 MHz
Calculate the effective noise temperature of the system.

[8+8]

8. (a) A code is composed of dots and dashes. Assume that a dash is three times as long as the dot and has one-third the probability of occurrence.
Find,
i. The information in a dot and that in a dash, and
ii. The entropy in the dot - dash code.
(b) Suppose 100 voltage levels are employed to transmit 100 equally likely messages. Assume the system to be a Gaussian channel with λ = 3.5 and bandwidth B = 104 Hz. Find S/N.

[8+8]

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Code No: RR221101 **Set No. 4**

II B.Tech II Semester Supplementary Examinations, November/December 2005
PROBABILITY THEORY & STOCHASTIC PROCESS
(Bio-Medical Engineering)

Time: 3 hours **Max Marks: 80**

Answer any FIVE Questions
All Questions carry equal marks
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1. (a) State and prove Bayes theorem of probability.
   (b) In a single throw of two dice, what is the probability of obtaining a sum of at least 10?

[8+8]

2. Two discrete random variables X and Y have joint p.m.f. given by the following table

X ↓	1	2	3	Y ←
1	1/12	1/6	1/12	
2	1/6	1/4	1/12	
3	1/12	1/12	0	

Compute the probability of each of the following events

   (a) $X \leq 1\frac{1}{2}$
   (b) XY is even
   (c) Y is even given that X is even.

[5+5+6]

3. (a) Prove that the second moment of binomial distribution is given by $E(X^2) = (np)^2 + npq$.
   (b) From the nth moment of exponential distribution, determine its variance to be $1/\alpha^2$, where α is a constant.

[8+8]

4. (a) If the auto correlation function of a wss process is $R(\tau) = k\ .e^{-k(\tau)}$, show that its spectral density is given by $S(\omega) = \frac{2}{1+\left(\frac{\omega}{k}\right)^2}$
   (b) Find the PSD of a random process x(t) if E[x(t)] = 1 and $R_{xx}(\tau) = 1 + e^{-\alpha|\tau|}$

[8+8]

5. (a) Find the PSD of a random process $z(t) = X(t) + y(t)$ where x(t) and y(t) are zero mean, individual random process.
   (b) A wss random process x(t) is applied to the input of an LTI system whose impulse response is $5t.e^{-2t}$ The mean of x(t) is 3. Find the output of the system.

[8+8]

6. (a) Explain how partition noise is present in electron devices?
   (b) Explain the usefulness of knowing the noise power spectral density of a network.

[8+8]

7. (a) Bring out the difference between narrowband and broadband noises
   (b) Describe the quadrature representation of narrowband noise.

[8+8]

8. (a) Consider an AWGN channel with S/N = 10^4. Find the maximum rate for reliable information transmission when, B = 1 KHz, 10 KHz and 100 KHz.
   (b) The Binary Erasure Channel (BEC) has two source symbols 0 and 1, and three destination symbols 0, 1 and E, where E denotes a detected but uncorrectable error. The forward transition probabilities are,

$$P(0/0) = 1 - \alpha \qquad P(E/0) = \alpha \qquad P(1/0) = 0$$

$$P(0/1) = 0 \qquad P(E/1) = \alpha \qquad P(1/1) = 1 - \alpha$$

   I (x, y) is maximum when source symbols are equiprobable. Find C_s (channel capacity) in terms of α.

[6+10]

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