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Aakash

IIT-JEE

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Time : 3 hrs. Max. Marks: 210

SOLUTIONS

for

IIT-JEE 2012

PAPER - 1 (Code - 0)

INSTRUCTIONS

1. The question paper consists of **3 parts** (Physics, Chemistry and Mathematics). Each part consists of **three sections.**
2. **Section I** contains **10 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**
3. **Section II** contains **5 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**
4. **Section III** contains **5 questions**. The answer to each question is a **single digit integer**, ranging from 0 to 9 (both inclusive).

PART–I : PHYSICS

SECTION - I
(Single Correct Answer Type)

This section contains **10 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

1. Three very large plates of same area are kept parallel and close to each other. They are considered as ideal black surfaces and have very high thermal conductivity. The first and third plates are maintained at temperatures $2T$ and $3T$ respectively. The temperature of the middle (*i.e.* second) plate under steady state condition is

(A) $\left(\frac{65}{2}\right)^{\frac{1}{4}}T$ (B) $\left(\frac{97}{4}\right)^{\frac{1}{4}}T$ (C) $\left(\frac{97}{2}\right)^{\frac{1}{4}}T$ (D) $(97)^{\frac{1}{4}}T$

Answer (C)

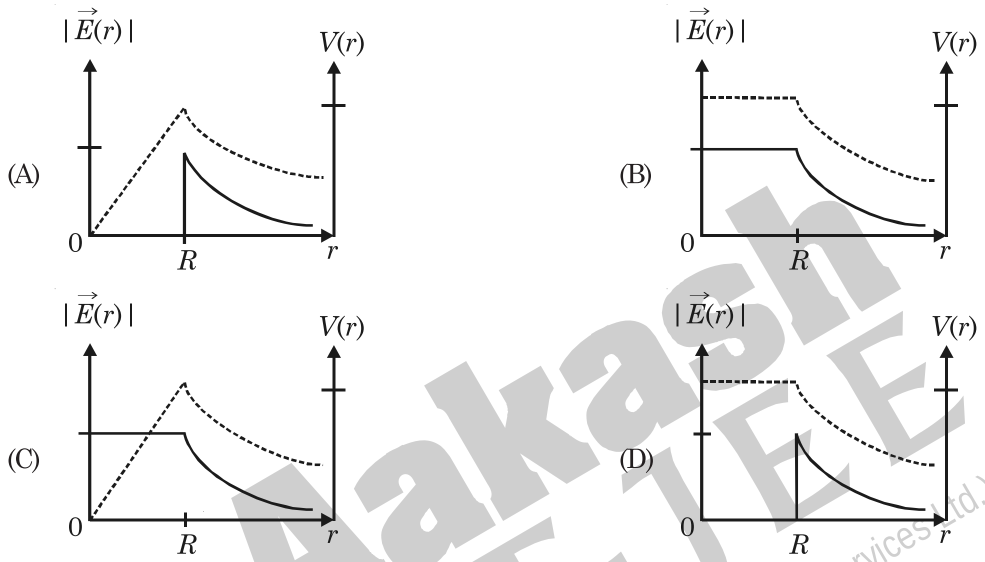
Hints : At equilibrium,

Heat absorbed = Heat radiated

$\Rightarrow \quad A(3T)^4 + A(2T)^4 = 2AT_0^{\,4}$

$\Rightarrow \quad T_0 = \left(\dfrac{97}{2}\right)^{1/4} T$

2. Consider a thin spherical shell of radius R with its centre at the origin, carrying uniform positive surface charge density. The variation of the magnitude of the electric field $|\vec{E}(r)|$ and the electric potential $V(r)$ with the distance r from the centre, is best represented by whcih graph?



Answer (D)

Hints : Inside, $E(r) = 0$, outside $E(r) \propto \dfrac{1}{r^2}$

Inside $V(r)$ = constant, outside $V(r) \propto \dfrac{1}{r}$

3. Young's double slit experiment is carried out by using green, red and blue light, one color at a time. The fringe widths recorded are β_G, β_R and β_B, respectively. Then,

(A) $\beta_G > \beta_B > \beta_R$ (B) $\beta_B > \beta_G > \beta_R$ (C) $\beta_R > \beta_B > \beta_G$ (D) $\beta_R > \beta_G > \beta_B$

Answer (D)

Hints : As $\lambda_R > \lambda_G > \lambda_B$

$\Rightarrow \quad \beta_R > \beta_G > \beta_B$ as $\beta \propto \lambda$

4. Two large vertical and parallel metal plates having a separation of 1 cm are connected to a DC voltage source of potential difference X. A proton is released at rest midway between the two plates. It is found to move at 45° to the vertical JUST after release. Then X is nearly

(A) 1×10^{-5} V (B) 1×10^{-7} V (C) 1×10^{-9} V (D) 1×10^{-10} V

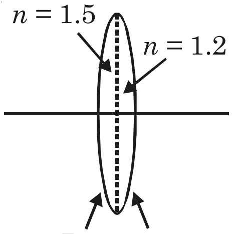
Answer (C)

Hints : Here, $qE = mg$

$$1.6 \times 10^{-19} \times \frac{X}{(1 \times 10^{-2})} = 1.6 \times 10^{-27} \times 10$$

$\Rightarrow \quad X = 10^{-9}$ V

5. A bi-convex lens is formed with two thin plano convex lenses as shown in the figure. Refractive index n of the first lens is 1.5 and that of the second lens is 1.2. Both the curved surfaces are of the same radius of curvature R = 14 cm. For this bi-convex lens, for an object distance of 40 cm, the image distance will be



(A) –280.0 cm (B) 40.0 cm (C) 21.5 cm (D) 13.3 cm

Answer (B)

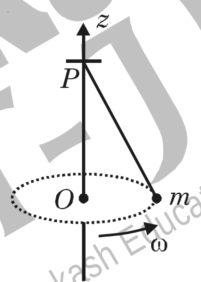
Hints : $\frac{1}{f} = (1.5-1)\left(\frac{1}{14}-\frac{1}{\infty}\right)+(1.2-1)\left(\frac{-1}{\infty}-\frac{1}{-14}\right)$

$\Rightarrow$ f = 20 cm

$\Rightarrow$ As object distance = 40 cm

$\Rightarrow$ Image distance = 40 cm

6. A small mass m is attached to a massless string whose other end is fixed at P as shown in the figure. The mass is undergoing circular motion in the x-y plane with centre at O and constant angular speed ω. If the angular momentum of the system, calculated about O and P are denoted by $\vec{L}_O$ and $\vec{L}_P$ respectively, then



(A) $\vec{L}_O$ and $\vec{L}_P$ do not vary with time

(B) $\vec{L}_O$ varies with time while $\vec{L}_P$ remains constant

(C) $\vec{L}_O$ remains constant while $\vec{L}_P$ varies with time

(D) $\vec{L}_O$ and $\vec{L}_P$ both vary with time

Answer (C)

Hints : The torque about O is zero, but about P is non-zero.

As $\vec{\tau} = \frac{d\vec{L}}{dt}$ thus $\vec{L}_P$ must be a variable.

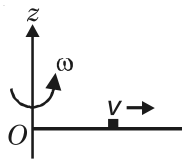
7. A mixture of 2 moles of helium gas (atomic mass = 4 amu) and 1 mole of argon gas (atomic mass = 40 amu) is kept at 300 K in a container. The ratio of the rms speeds $\left(\frac{v_{rms}\text{(helium)}}{v_{rms}\text{(argon)}}\right)$ is

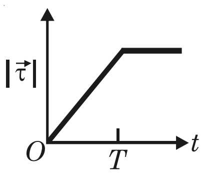
(A) 0.32 (B) 0.45 (C) 2.24 (D) 3.16

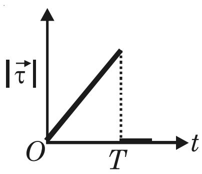
Answer (D)

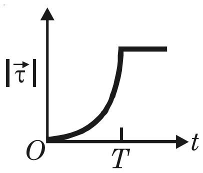
Hints : $\frac{v_{He}}{v_{Ar}} = \sqrt{\frac{M_{Ar}}{M_{He}}} = \sqrt{10} = 3.16$

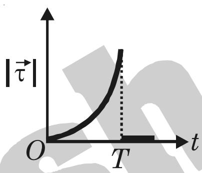
8. A thin uniform rod, pivoted at O, is rotating in the horizontal plane with constant angular speed ω, as shown in the figure. At time $t = 0$, a small insect starts from O and moves with constant speed v with respect to the rod towards the other end. It reaches the end of the rod at $t = T$ and stops. The angular speed of the system remains ω throughout. The magnitude of the torque ($|\vec{\tau}|$) on the system about O, as a function of time is best represented by which plot?



(A) 

(B) 

(C) 

(D) 

Answer (B)

Hints : $\tau = \dfrac{d}{dt}(I\omega)$

$$= \frac{d}{dt}\left(\frac{ML^2}{3} + mx^2\right)\omega$$

$$= 2mx\frac{dx}{dt}\omega$$

Now, $x = vt$

$\Rightarrow \quad \tau \propto t$

Finally torque becomes zero.

9. In the determination of Young's modulus $\left(Y = \dfrac{4MLg}{\pi l d^2}\right)$ by using Searle's method, a wire of length $L = 2$ m and diameter $d = 0.5$ mm is used. For a load $M = 2.5$ kg, an extension $l = 0.25$ mm in the length of the wire is observed. Quantities d and l are measured using a screw gauge and a micrometer, respectively. They have the same pitch of 0.5 mm. The number of divisions on their circular scale is 100. The contributions to the maximum probable error of the Y measurement

(A) Due to the errors in the measurements of d and l are the same

(B) Due to the error in the measurement of d is twice that due to the error in the measurement of l

(C) Due to the error in the measurement of l is twice that due to the error in the measurement of d

(D) Due to the error in the measurement of d is four times that due to the error in the measurement of l

Answer (A)

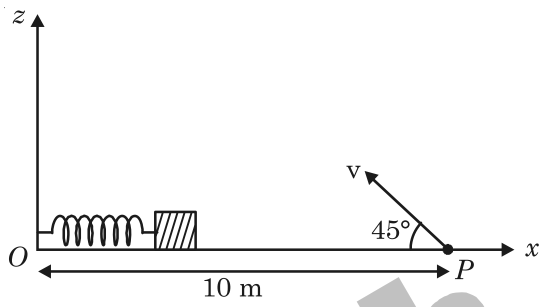
Hints : $\dfrac{\Delta y}{y} = \dfrac{\Delta l}{l} + \dfrac{2\Delta d}{d}$

$$\Delta y = y\left(\frac{\Delta l}{l} + \frac{2\Delta d}{d}\right)$$

$$= \frac{y\Delta l}{l} + \frac{2y\Delta d}{d}$$

The two quantities are equal.

10. A small block is connected to one end of a massless spring of un-stretched length 4.9 m. The other end of the spring (see the figure) is fixed. The system lies on a horizontal frictionless surface. The block is stretched by 0.2 m and released from rest at $t = 0$. It then executes simple harmonic motion with angular frequency $\omega = \frac{\pi}{3}$ rad/s. Simultaneously at $t = 0$, a small pebble is projected with speed v from point P at an angle of 45° as shown in the figure. Point P is at a horizontal distance of 10 m from O. If the pebble hits the block at $t = 1$ s, the value of v is (take $g = 10$ m/s^2)



(A) $\sqrt{50}$ m/s (B) $\sqrt{51}$ m/s (C) $\sqrt{52}$ m/s (D) $\sqrt{53}$ m/s

Answer (A)

Hints : For the block, $T = \frac{2\pi}{\omega} = 6$ seconds.

For block, the position in 1 second will be

$x = 0.2\cos\omega t$

At $t = 1$ s, $x = 0.1$ m

$\Rightarrow$ At $t = 1$ s, block will be at a distance 4.9 m + 0.1 m = 5 m from wall.

$\therefore$ Range of pebble is also 5 m

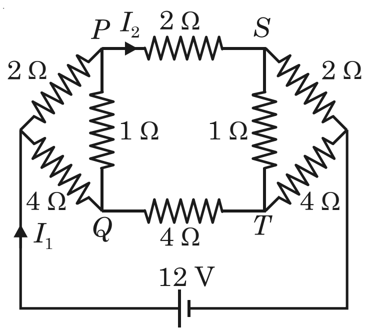
Now, for pebble, vcos45° × (1 sec) = 5 m

$\Rightarrow$ $v = 5\sqrt{2} = \sqrt{50}$ m/s

SECTION - II
(Multiple Correct Answer(s) Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct.**

11. For the resistance network shown in the figure, choose the correct option(s).



(A) The current through PQ is zero

(B) $I_1 = 3$ A

(C) The potential at S is less than that at Q

(D) $I_2 = 2$ A

Answer (A, B, C, D)

Hints : The network is an extension of balanced Wheatstone bridge, thus current PQ and ST is zero.

$$\Rightarrow \quad R_{eq} = \left(\frac{1}{6}+\frac{1}{12}\right)^{-1} = 4\ \Omega$$

$$\Rightarrow \quad I_1 = 3\text{A}$$

Also, $I_2 = \dfrac{12}{6} = 2$ A

Now, $V_S + 2I_2 + 2I_2 - 4I_3 = V_Q$

$$V_S - V_Q = 4I_3 - 2I_2 - 2I_2$$
$$= 4 \times 1 - 2 \times 2 - 2 \times 2$$
$$= -4\text{ V}$$

12. A person blows into open-end of a long pipe. As a result, a high-pressure pulse of air travels down the pipe. When this pulse reaches the other end of the pipe

(A) A high-pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(B) A low-pressure pulse starts traveling up the pipe, if the other end of the pipe is open

(C) A low-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

(D) A high-pressure pulse starts traveling up the pipe, if the other end of the pipe is closed

Answer (B, D)

Hints : At open end, a compression is reflected as a rarefaction

At closed end, a compression is reflected as a compression

13. Consider the motion of a positive point charge in a region where there are simultaneous uniform electric and magnetic fields $\vec{E} = E_0\hat{j}$ and $\vec{B} = B_0\hat{j}$. At time $t = 0$, this charge has velocity $\vec{v}$ in the x-y plane, making an angle θ with the x-axis. Which of the following option(s) is/are correct for time $t > 0$?

(A) If $\theta = 0°$, the charge moves in a circular path in the x-z plane

(B) If $\theta = 0°$, the charge undergoes helical motion with constant pitch along the y-axis

(C) If $\theta = 10°$, the charge undergoes helical motion with its pitch increasing with time, along the y-axis

(D) If $\theta = 90°$, the charge undergoes linear but accelerated motion along the y-axis

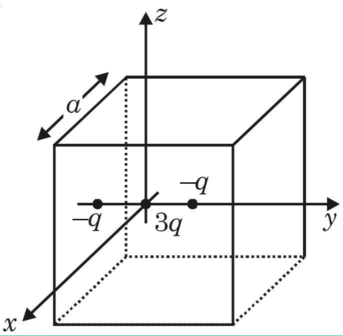
Answer (C, D)

Hints : For $\theta = 10°$, path is helical.

Since there is an electric field, pitch is increasing.

For $\theta = 90°$, path is straight line as $\vec{B}$ will not exert any force.

14. A cubical region of side a has its centre at the origin. It encloses three fixed point charges, $-q$ at $\left(0, -\frac{a}{4}, 0\right)$, $+3q$ at (0, 0, 0) and $-q$ at $\left(0, +\frac{a}{4}, 0\right)$. Choose the correct option(s).



(A) The net electric flux crossing the plane $x = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = -\frac{a}{2}$

(B) The net electric flux crossing the plane $y = +\frac{a}{2}$ is more than the net electric flux crossing the plane $y = -\frac{a}{2}$

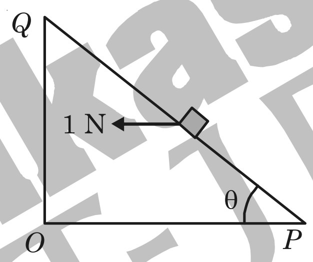
(C) The net electric flux crossing the entire region is $\frac{q}{\varepsilon_0}$

(D) The net electric flux crossing the plane $z = +\frac{a}{2}$ is equal to the net electric flux crossing the plane $x = +\frac{a}{2}$

Answer (A, C, D)

Hints : Due to symmetry.

15. A small block of mass of 0.1 kg lies on a fixed inclined plane PQ which makes an angle θ with the horizontal. A horizontal force of 1 N acts on the block through its centre of mass as shown in the figure. The block remains stationary if (take g =10 m/s^2)



(A) θ = 45°

(B) θ > 45° and a frictional force acts on the block towards P.

(C) θ > 45° and a frictional force acts on the block towards Q.

(D) θ < 45° and a frictional force acts on the block towards Q.

Answer (A, C)

Hints : If 1 cos θ = 1 sin θ

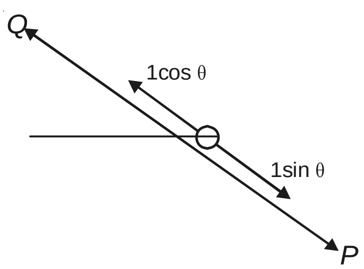
$\Rightarrow \quad Q = 45°, F_{net} = 0$

If 1 cos θ > 1 sin θ

i.e., θ < 45, friction must be towards P.

If 1 cos θ < 1 sin θ

θ > 45°, friction must be towards Q.

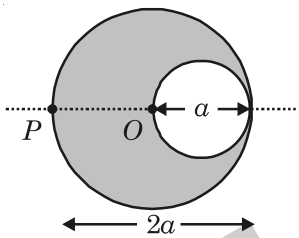


SECTION - III

(Integer Answer Type)

This section contains **5 questions.** The answer to each of the questions is a **single digit integer,** ranging from 0 to 9 (*both inclusive*).

16. A cylindrical cavity of diameter a exists inside a cylinder of diameter $2a$ as shown in the figure. Both the cylinder and the cavity are infinitely long. A uniform current density J flows along the length. If the magnitude of the magnetic field at the point P is given by $\frac{N}{12}\mu_0 aJ$, then the value of N is



Answer (5)

Hints : $B = B_1 - B_2$

Here, B_1 is $\frac{\mu_0 Ja}{2}$ (field due to complete cylinder)

$$B_2 = \frac{\mu_0 J\pi\left(\frac{a}{2}\right)^2}{2\pi\left(\frac{3a}{2}\right)} = \frac{\mu_0 Ja}{12}$$

$$\Rightarrow \quad B = \frac{5\mu_0 aJ}{12}$$

17. A proton is fired from very far away towards a nucleus with charge $Q = 120\,e$, where e is the electronic charge. It makes a closest approach of 10 fm to the nucleus. The de Broglie wavelength (in units of fm) of the proton at its start is: (take the proton mass, $m_p = \left(\frac{5}{3}\right)\times 10^{-27}$ kg; h/e = 4.2×10^{-15} J.s/C; $\frac{1}{4\pi\varepsilon_0} = 9\times 10^9$. m/F; 1 fm $= 10^{-15}$ m)

Answer (7)

Hints : $\frac{120e\times e}{4\pi\varepsilon_0(10\times 10^{-15})} = \frac{p^2}{2m} = \frac{h^2}{\lambda^2\times 2m}$

$$\Rightarrow \quad \lambda^2 = \frac{h^2}{2m}\times\frac{4\pi\varepsilon_0\times 10^{-14}}{120e^2}$$

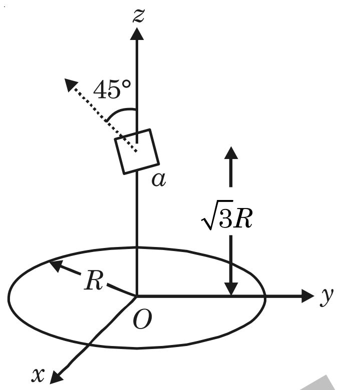
$$\Rightarrow \quad \lambda = \frac{h}{e}\times\sqrt{\frac{4\pi\varepsilon_0\times 10^{-4}}{2\times 120\times m}}$$

$$= 4.2\times 10^{-15}\times\sqrt{\frac{10^{-14}\times 3}{9\times 10^9\times 2\times 120\times 5\times 10^{-27}}}$$

$= 7 \times 10^{-15}$ m

$= 7$ *fm*

18. A circular wire loop of radius R is placed in the x-y plane centered at the origin O. A square loop of side $a(a << R)$ having two turns is placed with its centre at $z = \sqrt{3}R$ along the axis of the circular wire loop, as shown in figure. The plane of the square loop makes an angle of 45° with respect to the z-axis. If the mutual inductance between the loops is given by $\frac{\mu_0 a^2}{2^{p/2} R}$, then the value of p is



Answer (7)

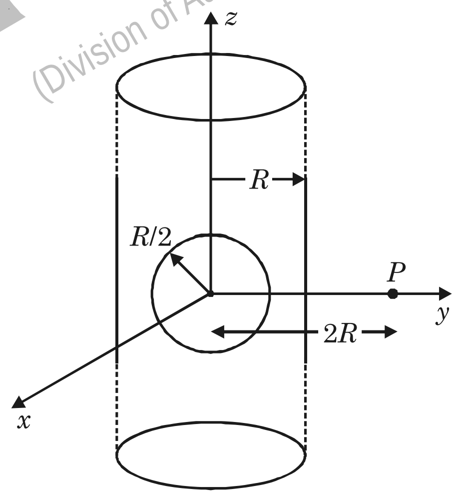
Hints : $M = \frac{\phi}{I}$

$$M = \frac{\mu_0 2 \times I \times \pi R^2}{4\pi(R^2 + (\sqrt{3}R)^2)^{3/2}} \times \frac{1}{I} \times a^2 \cos 45° \times 2$$

$$= \frac{\mu_0 R^2}{2(8R^3)} \times a^2 \times \frac{1}{\sqrt{2}} \times 2$$

$$= \frac{\mu_0 a^2}{R} \times \frac{1}{8\sqrt{2}} = \frac{\mu_0 a^2}{R 2^{7/2}}$$

19. An infinitely long solid cylinder of radius R has a uniform volume charge density ρ. It has a spherical cavity of radius $R/2$ with its centre on the axis of the cylinder, as shown in the figure. The magnitude of the electric field at the point P, which is at a distance $2R$ from the axis of the cylinder, is given by the expression $\frac{23\rho R}{16k\varepsilon_0}$. The value of k is



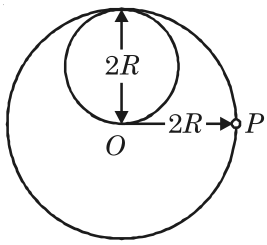
Answer (6)

Hints : $E_P = E_{\text{cylinder}} - E_{\text{sphere}}$

$$= \frac{\rho R^2}{2\varepsilon_0(2R)} - \frac{\rho(R^3/8)}{3\varepsilon_0(2R)^2}$$

$$= \frac{\rho R}{4\varepsilon_0} - \frac{\rho R}{96\varepsilon_0} = \frac{23\rho R}{96\varepsilon_0} \Rightarrow k = 6$$

20. A lamina is made by removing a small disc of diameter $2R$ from a bigger disc of uniform mass density and radius $2R$, as shown in the figure. The moment of inertia of this lamina about axes passing through O and P is I_O and I_P, respectively. Both these axes are perpendicular to the plane of the lamina. The ratio $\frac{I_P}{I_O}$ to the nearest integer is



Answer (3)

Hints : Let the mass density be σ.

$$I_O = \frac{\sigma\pi(2R)^2(2R)^2}{2} - \frac{3}{2}\sigma\pi(R)^2(R)^2$$

$$= \frac{13}{2}\sigma\pi R^4$$

$$I_P = \frac{3}{2}\sigma\pi(2R)^2(2R)^2 - \left[\frac{\sigma\pi R^2(R)^2}{2} + \sigma\pi R^2 \times (\sqrt{5}R)^2\right]$$

$$= \frac{37}{2}\sigma\pi R^4$$

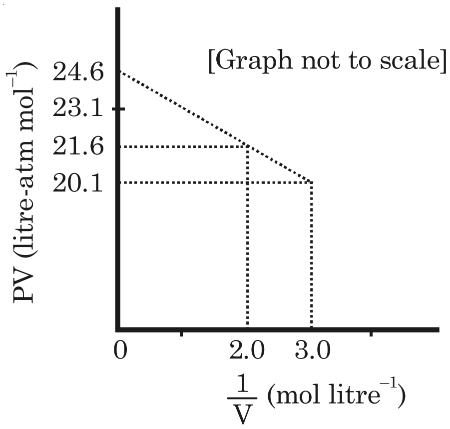
$$\text{Ratio } \frac{I_P}{I_O} = \frac{37}{13} \approx 3$$

PART–II : CHEMISTRY

SECTION - I
(Single Correct Answer Type)

This section contains **10 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

21. For one mole of a van der Waal's gas when b = 0 and T = 300 K, the PV vs. 1/V plot is shown below. The value of the van der Waal's constant 'a' (atm. litre^2 mol^{-2}) is



(A) 1.0 (B) 4.5 (C) 1.5 (D) 3.0

Answer (C)

Hints :

$$\left(P+\frac{a}{V^2}\right)V = RT$$

$$PV + \frac{a}{V} = RT$$

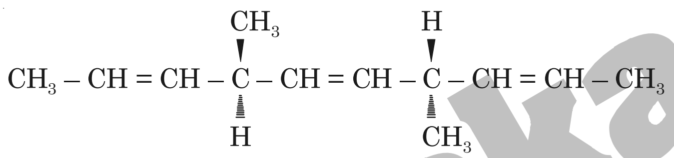
$$PV = RT - \frac{a}{V}$$

So, Plot of (PV) vs $\left(\frac{1}{V}\right)$ has slope – a.

$\therefore$ slope $= \frac{20.1-24.6}{3} = -1.5$

$\therefore$ a = 1.5 atm L^2 mol^{-2}

22. The number of optically active products obtained from the **complete** ozonolysis of the given compound is



(A) 0 (B) 1 (C) 2 (D) 4

Answer (A)

Hints :

Ozonolysis produces CH_3CHO and $CH_3–CH(CHO)_2$

23. The colour of light absorbed by an aqueous solution of $CuSO_4$ is

(A) Orange-red (B) Blue-green (C) Yellow (D) Violet

Answer (A)

Hints :

Complementary colour of blue is (Orange-red)

24. Which ordering of compounds is according to the decreasing order of the oxidation state of nitrogen?

(A) HNO_3, NO, NH_4Cl, N_2
(B) HNO_3, NO, N_2, NH_4Cl
(C) HNO_3, NH_4Cl, NO, N_2
(D) NO, HNO_3, NH_4Cl, N_2

Answer (B)

Hints :

$$H\overset{(+5)}{N}O_3,\ \overset{(+2)}{N}O,\ \overset{(0)}{N_2},\ \overset{(-3)}{N}H_4Cl$$

25. As per IUPAC nomenclature, the name of the complex $[Co(H_2O)_4(NH_3)_2]Cl_3$ is

(A) Tetraaquadiaminecobalt (III) chloride
(B) Tetraaquadiamminecobalt (III) chloride
(C) Diaminetetraaquacobalt(III) chloride
(D) Diamminetetraaquacobalt (III) chloride

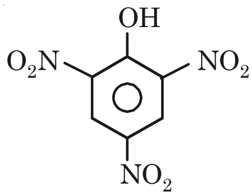
Answer (D)

26. The carboxyl functional group (–COOH) is present in

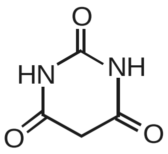
(A) Picric acid (B) Barbituric acid (C) Ascorbic acid (D) Aspirin

Answer (D)

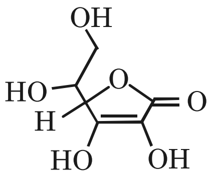
Hints :



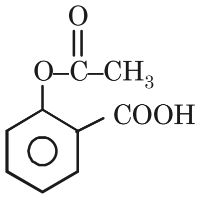
(Picric acid)



(Barbituric acid)



(Ascorbic acid)



(Aspirin)

27. The kinetic energy of an electron in the second Bohr orbit of a hydrogen atom is [a_0 is Bohr radius]

(A) $\frac{h^2}{4\pi^2 m a_0^2}$ (B) $\frac{h^2}{16\pi^2 m a_0^2}$ (C) $\frac{h^2}{32\pi^2 m a_0^2}$ (D) $\frac{h^2}{64\pi^2 m a_0^2}$

Answer (C)

Hints :

Centripital force = Coulombic force of attraction

$$\frac{mV^2}{r} = \frac{KZe^2}{r^2}$$

$$V^2 = \frac{KZe^2}{mr}$$

$$V = \frac{nh}{2\pi mr}$$

$$V^2 = \frac{n^2h^2}{4\pi^2m^2r^2}$$

$$\therefore \frac{KZe^2}{mr} = \frac{n^2h^2}{4\pi^2m^2r^2} \Rightarrow r = \left(\frac{h^2}{4\pi^2mKe^2}\right)\frac{n^2}{Z} = \frac{a_0n^2}{Z}$$

$$V_n = \frac{hZ}{2\pi m a_0 n}$$

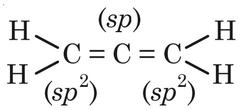
$$K.E. = \frac{1}{2}mv_n^2 = \frac{h^2Z^2}{8\pi^2ma_0^2n^2} \text{ ; K.E. (for x = 2 and Z = 1)} = \frac{h^2}{32\pi^2ma_0^2}$$

28. In allene (C_3H_4), the type(s) of hybridisation of the carbon atoms is (are)

(A) *sp* and sp^3 (B) *sp* and sp^2 (C) Only sp^2 (D) sp^2 and sp^3

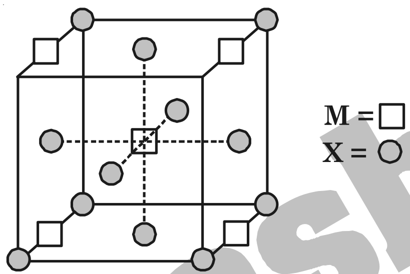
Answer (B)

Hints :



(Allene)

29. A compound M_pX_q has cubic close packing (ccp) arrangement of X. Its unit cell structure is shown below. The empirical formula of the compound is



(A) MX (B) MX_2 (C) M_2X (D) M_5X_{14}

Answer (B)

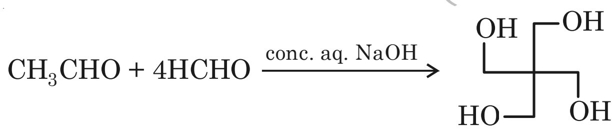
Hints :

Number of X atoms/ ions per unit cell = $8 \times \frac{1}{8} + 6 \times \frac{1}{2} = 4$

Number of M atoms/ions per unit cell = $1 + 4 \times \frac{1}{4} = 2$

∴ Empirical formula of the compound is MX_2

30. The number of aldol reaction(s) that occurs in the given transformation is



(A) 1 (B) 2 (C) 3 (D) 4

Answer (C)

Hints :

$$CH_3-CHO + H-CHO \xrightarrow[\text{Aldol}]{\bar{O}H} HOCH_2-CH_2-CHO \xrightarrow[\text{Aldol}]{+HCHO/O\bar{H}} HOCH_2-\underset{}{CH}(CH_2OH)-CHO$$

$$\xrightarrow[\text{Aldol}]{+HCHO/O\bar{H}} HOCH_2-C(CH_2OH)_2-CHO \xrightarrow[\text{Cannizzaro}]{+HCHO/O\bar{H}} HOCH_2-C(CH_2OH)_2-CH_2OH$$

∴ Number of aldol reactions = 3

SECTION - II
(Multiple Correct Answer(s) Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct.**

31. Choose the correct reason(s) for the stability of the lyophobic colloidal particles.

(A) Preferential adsorption of ions on their surface from the solution

(B) Preferential absorption of solvent on their surface from the solution

(C) Attraction between different particles having opposite charges on their surface

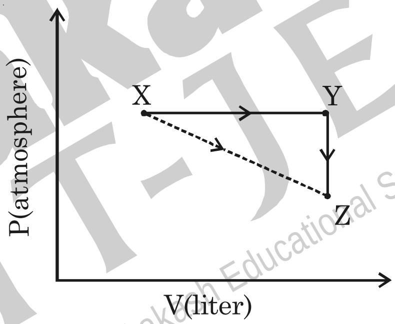
(D) Potential difference between the fixed layer and the diffused layer of opposite charges around the colloidal particles

Answer (A, D)

Hints :

Lyophobic colloids carry either +ve or –ve charge. In order to acquire stability the colloidal particles attract oppositely charged ions forming a diffused layer. A potential is developed between fixed layer and diffused layer called zeta potential. Higher the value of zeta potential higher will be the stability of lyophobic colloid.

32. For an ideal gas, consider only P-V work in going from an initial state X to the final state Z. The final state Z can be reached by either of the two paths shown in the figure. Which of the following choice(s) is (are) correct? [take ΔS as change in entropy and w as work done]



(A) $\Delta S_{x\to z} = \Delta S_{x\to y} + \Delta S_{y\to z}$

(B) $w_{x\to z} = w_{x\to y} + w_{y\to z}$

(C) $w_{x\to y\to z} = w_{x\to y}$

(D) $\Delta S_{x\to y\to z} = \Delta S_{x\to y}$

Answer (A, C)

Hints :

Entropy is a state function. In this diagram initial state is X and final state is Z.

Therefore, $\Delta S_{X\to Z} = \Delta S_{X\to Y} + \Delta S_{Y\to Z}$

Work is a path function. Therefore $W_{X\to Y\to Z} = W_{X\to Y}$

33. Which of the following hydrogen halides react(s) with $AgNO_3$ (aq) to give a precipitate that dissolves in $Na_2S_2O_3$(aq)?

(A) HCl

(B) HF

(C) HBr

(D) HI

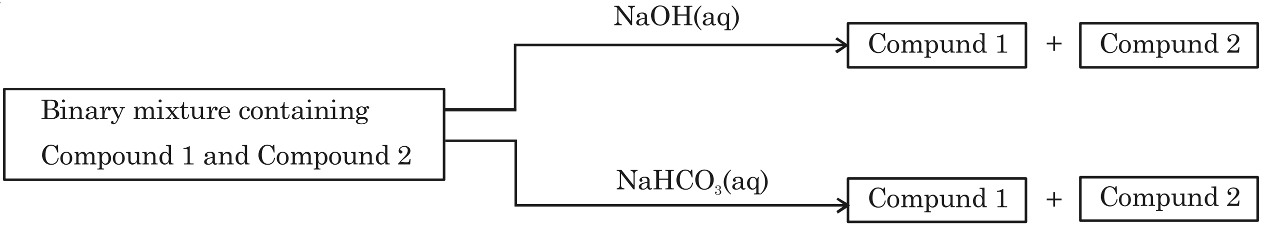
Answer (A, C, D)

Hints :

$$Na_2S_2O_3 + AgX \longrightarrow Ag_2S_2O_3 \xrightarrow{2Na_2S_2O_3} Na_2[Ag(S_2O_3)_3]$$

[X = Cl and Br]

34. Identify the binary mixture(s) that can be separated into individual compounds, by differential extraction, as shown in the given scheme.



(A) C_6H_5OH and C_6H_5COOH

(B) C_6H_5COOH and $C_6H_5CH_2OH$

(C) $C_6H_5CH_2OH$ and C_6H_5OH

(D) $C_6H_5CH_2OH$ and $C_6H_5CH_2COOH$

Answer (B, D)

Hints :

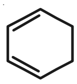
$C_6H_5COOH + NaOH \longrightarrow C_6H_5COONa + H_2O$; $C_6H_5COOH + NaHCO_3 \rightarrow C_6H_5COONa + CO_2 + H_2O$

$C_6H_5CH_2 - OH + NaOH \longrightarrow$ No reaction ; $C_6H_5CH_2OH + NaHCO_3 \rightarrow$ No reaction

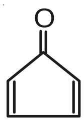
$C_6H_5CH_2COOH + NaOH \longrightarrow C_6H_5CH_2 - COONa + H_2O$; $C_6H_5CH_2COOH + NaHCO_3 \rightarrow C_6H_5CH_2COONa + CO_2 + H_2O$

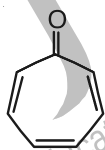
$C_6H_5CH_2OH + NaOH \longrightarrow$ No reaction ; $C_6H_5CH_2OH + NaHCO_3 \rightarrow$ No reaction

35. Which of the following molecules, in pure form, is (are) unstable at room temperature?

(A) 

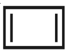
(B) 

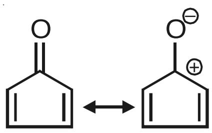
(C) 

(D) 

Answer (B, C)

Hints :

The compound  is antiaromatic and hence unstable at room temperature. The other compound



is also unstable at room temperature due to partial positive charge at carbonyl C-atom

SECTION - III
(Integer Answer Type)

This section contains **5 questions.** The answer to each of the questions is a **single digit integer,** ranging from 0 to 9 (*both inclusive*).

36. The periodic table consists of 18 groups. An isotope of copper, on bombardment with protons, undergoes a nuclear reaction yielding element **X** as shown below. To which group, element **X** belongs in the periodic table?

$^{63}_{29}Cu + ^{1}_{1}H \longrightarrow 6^{1}_{0}n + \alpha + 2^{1}_{1}H + X$

Answer (8)

Hints :

$^{63}_{29}Cu + ^{1}_{1}H \longrightarrow 6^{1}_{0}n + \alpha(^{4}_{2}He) + 2^{1}_{1}H + X(^{52}_{26}Fe)$

Fe belongs to group-8.

37. An organic compound undergoes first-order decomposition. The time taken for its decomposition to $\frac{1}{8}$ and $\frac{1}{10}$ of its initial concentration are $t_{\frac{1}{8}}$ and $t_{\frac{1}{10}}$ respectively. What is the value of $\frac{\left[t_{\frac{1}{8}}\right]}{\left[t_{\frac{1}{10}}\right]}\times 10$? (take $\log_{10}2 = 0.3$)

Answer (9)

Hints :

$$t_{1/8} = \frac{2.303}{k}\log_{10}\frac{1}{1/8} \qquad ...(i)$$

$$t_{1/10} = \frac{2.303}{k}\log_{10}\frac{1}{1/10} \qquad ...(ii)$$

$$\frac{[t_{1/8}]}{[t_{1/10}]}\times 10 = \frac{\log 8}{\log 10}\times 10 = 3\times 0.3\times 10 = 9$$

38. 29.2% (w/w) HCl stock solution has a density of 1.25 g mL^{-1}. The molecular weight of HCl is 36.5 g mol^{-1}. The volume (mL) of stock solution required to prepare a 200 mL solution of 0.4 M HCl is

Answer (8)

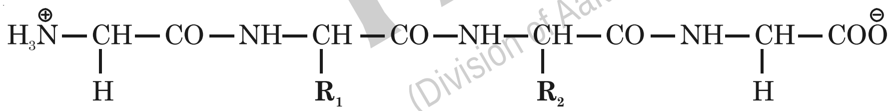
Hints :

$$\text{Molarity of HCl} = \frac{1.25\times 29.2\times 1000}{100\times 1\times 36.5} = 10(M)$$

V × 10 = 200 × 0.4

V = 8 mL

39. The substituents $\mathbf{R_1}$ and $\mathbf{R_2}$ for nine peptides are listed in the table given below. How many of these peptides are positively charged at pH = 7.0?



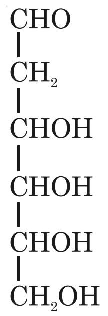
Peptide	R_1	R_2
I	H	H
II	H	CH_3
III	CH_2COOH	H
IV	CH_2CONH_2	$(CH_2)_4NH_2$
V	CH_2CONH_2	CH_2CONH_2
VI	$(CH_2)_4NH_2$	$(CH_2)_4NH_2$
VII	CH_2COOH	CH_2CONH_2
VIII	CH_2OH	$(CH_2)_4NH_2$
IX	$(CH_2)_4NH_2$	CH_3

Answer (4)

Hints :

When any group in R_1 and R_2 is basic group then amino acid is positively charged at pH = 7.0. So, answers are peptide IV, VI, VIII and IX.

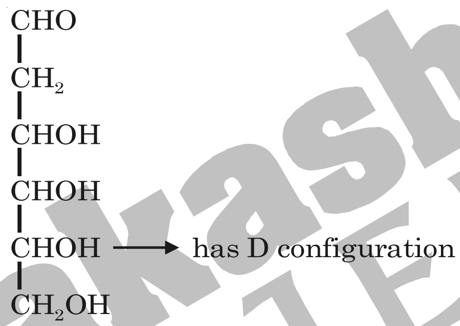
40. When the following aldohexose exists in its **D**-configuration, the total number of stereoisomers in its pyranose form is

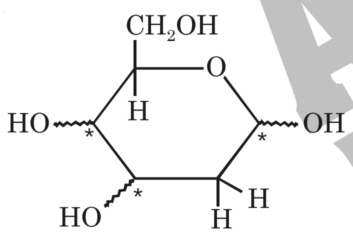


Answer (8)

Hints :

Given





C^* are chiral carbon atoms. Hence total stereoisomers are 8.

PART–III : MATHEMATICS

SECTION - I
(Single Correct Answer Type)

This section contains **10 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

41. Let $f(x) = \begin{cases} x^2\left|\cos\dfrac{\pi}{x}\right|, & x \neq 0 \\ 0, & x = 0 \end{cases}$, $x \in \mathbb{R}$ then f is

(A) Differentiable both at $x = 0$ and at $x = 2$

(B) Differentiable at $x = 0$ but not differentiable at $x = 2$

(C) Not differentiable at $x = 0$ but differentiable at $x = 2$

(D) Differentiable neither at $x = 0$ nor at $x = 2$

Answer (B)

Hints :

$$f(x)=\begin{cases} x^2\left|\cos\dfrac{\pi}{x}\right| & , \quad x\neq 0 \\ 0 & , \quad x=0 \end{cases} \quad x\in\mathbb{R}$$

Let $p(x) = x^2$ and $g(x)=\left|\cos\dfrac{\pi}{x}\right|$

Now $g(x)$ is a continuous non-differentiable at $x = 0$, and 2 but since $p(0) = 0$, so, $p(x).g(x)$ is non-differentiable at $x = 2$.

42. The function $f : [0, 3] \to [1, 29]$, defined by $f(x) = 2x^3 - 15x^2 + 36x + 1$, is

(A) One-one and onto

(B) Onto but not one-one

(C) One-one but not onto

(D) Neither one-one nor onto

Answer (B)

Hints :

$f(x)=2x^3-15x^2+36x+1$

$f'(x) = 6x^2 - 30x + 36$

$= 6(x^2 - 5x + 6)$

$= 6(x - 2)(x - 3)$

Clearly the derivative changes sign in [0, 3] so, f is NOT one-one.

Now the function is increasing in [0, 2] and decreasing in [2, 3]

Also, $f(0) = 1$

$f(2) = 29$

$f(3) = 8$

Hence the range is [1, 29] and so, the function is onto.

43. The ellipse $E_1 : \dfrac{x^2}{9}+\dfrac{y^2}{4}=1$ is inscribed in a rectangle R whose sides are parallel to the coordinate axes. Another ellipse E_2 passing through the point (0, 4) circumscribes the rectangle R. The eccentricity of the ellipse E_2 is

(A) $\dfrac{\sqrt{2}}{2}$ (B) $\dfrac{\sqrt{3}}{2}$ (C) $\dfrac{1}{2}$ (D) $\dfrac{3}{4}$

Answer (C)

Hints :

Let the equation of E_2 be

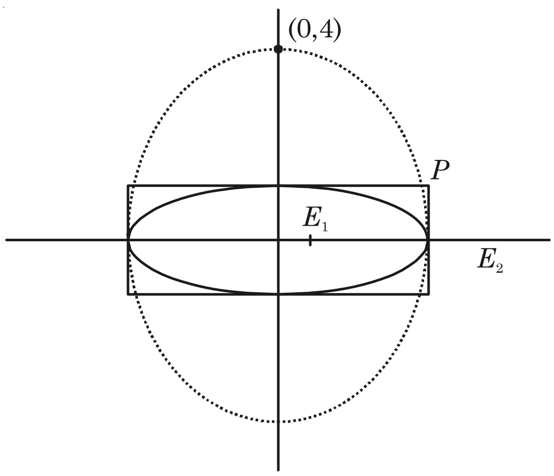
$$\frac{x^2}{a^2}+\frac{y^2}{b^2}=1$$

Clearly the point P has coordinates (3, 2)

So, (3, 2) and (0, 4) satisfy E_2

$$\frac{16}{b^2}=1 \Rightarrow b^2=16$$

and $\dfrac{9}{a^2}+\dfrac{4}{b^2}=1$



$$\frac{9}{a^2} = \frac{3}{4} \Rightarrow a^2 = 12$$

$$e = \sqrt{1 - \frac{12}{16}} = \frac{1}{2}$$

44. The point P is the intersection of the straight line joining the points Q (2, 3, 5) and R (1, –1, 4) with the plane $5x - 4y - z = 1$. If S is the foot of the perpendicular drwan from the point T (2, 1, 4) to QR, then the length of the line segment PS is

(A) $\frac{1}{\sqrt{2}}$ (B) $\sqrt{2}$ (C) 2 (D) $2\sqrt{2}$

Answer (A)

Hints :

The equation of the line QR is

$$\frac{x-2}{-1} = \frac{y-3}{-4} = \frac{z-5}{-1} = k$$

so any arbitrary point is $(-k+2,\ -4k+3,\ -k+5)$

Now this will satisfy the equation of the plane

$$5(-k+2) - 4(-4k+3) - (-k+5) = 1$$

$$\Rightarrow \quad k = \frac{2}{3}$$

So, the point is $\left(\frac{4}{3}, \frac{1}{3}, \frac{13}{3}\right)$

Now, let point S be $(-\lambda + 2, -4\lambda + 3, -\lambda + 5)$

So, D.R's of ST is $(-\lambda, -4\lambda + 2, -\lambda + 1)$

$\Rightarrow \quad (-\lambda)(-1) - 4(-4\lambda + 2) - 1(-\lambda + 1) = 0$

$$\Rightarrow \quad \lambda = \frac{1}{2}$$

The point is $\left(\frac{3}{2}, 1, \frac{9}{2}\right)$

So, PS is $\frac{1}{\sqrt{2}}$

45. The locus of the mid-point of the chord of contact of tangents drawn from points lying on the straight line $4x - 5y = 20$ to the circle $x^2 + y^2 = 9$ is

(A) $20(x^2 + y^2) - 36x + 45y = 0$

(B) $20(x^2 + y^2) + 36x - 45y = 0$

(C) $36(x^2 + y^2) - 20x + 45y = 0$

(D) $36(x^2 + y^2) + 20x - 45y = 0$

Answer (A)

Hints :

Let, $P(\alpha, \beta)$ be a point on the line $4x - 5y = 20$.

i.e., $4\alpha - 5\beta = 20$ ... (1)

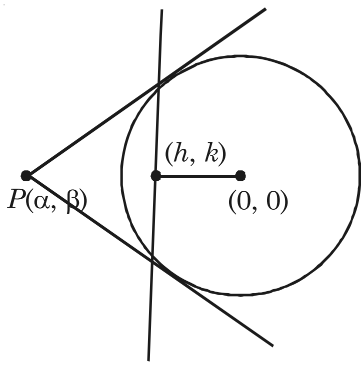
If (h, k) be the mid point of chord of contact, then $T = S_1$, gives

$\Rightarrow xh + yk - 9 = h^2 + k^2 - 9$

$\Rightarrow xh + yk = h^2 + k^2$. ... (2)

Also, equation of chord of contact w.r.t. $P(\alpha, \beta)$ is

$\alpha x + \beta y = 9$ ... (3)



as equations (2) & (3) are identical,

So, $\frac{\alpha}{h} = \frac{\beta}{k} = \frac{9}{h^2 + k^2}$

$\Rightarrow \alpha = \frac{9h}{h^2 + k^2}, \beta = \frac{9k}{h^2 + k^2}$

Using equation (1), we get

$$4\left(\frac{9h}{h^2 + k^2}\right) - 5\left(\frac{9k}{h^2 + k^2}\right) = 20$$

i.e., locus of (h, k) is given by

$20(x^2 + y^2) - 36x + 45y = 0$

46. Let $P = [a_{ij}]$ be a 3×3 matrix and let $Q = [b_{ij}]$, where $b_{ij} = 2^{i+j}a_{ij}$ for $1 \le i, j \le 3$. If the determinant of P is 2, then the determinant of the matrix Q is

(A) 2^{10} (B) 2^{11} (C) 2^{12} (D) 2^{13}

Answer (D)

Hints :

Given, $P = [a_{ij}]_{3\times 3} = \begin{bmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{33} \\ a_{31} & a_{32} & a_{33} \end{bmatrix}$

Now, $Q = 2^{i+j}a_{ij} = [b_{ij}]$

$\Rightarrow Q = \begin{bmatrix} b_{11} & b_{12} & b_{13} \\ b_{21} & b_{22} & b_{23} \\ b_{31} & b_{32} & b_{33} \end{bmatrix}$

$= \begin{bmatrix} 2^2 a_{11} & 2^3 a_{12} & 2^4 a_{13} \\ 2^3 a_{21} & 2^4 a_{22} & 2^5 a_{23} \\ 2^4 a_{31} & 2^5 a_{32} & 2^6 a_{33} \end{bmatrix}$

So, $|Q| = 2^{12} \begin{vmatrix} a_{11} & a_{12} & a_{13} \\ a_{21} & a_{22} & a_{23} \\ a_{31} & a_{32} & a_{33} \end{vmatrix}$

$= 2^{13}$.

(as $|P| = 2$)

47. The integral $\int \frac{\sec^2 x}{(\sec x + \tan x)^{9/2}} dx$ equals (for some arbitrary constant K)

(A) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(B) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} - \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(C) $-\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

(D) $\frac{1}{(\sec x + \tan x)^{11/2}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

Answer (C)

Hints :

Let $\sec x + \tan x = t$

$\Rightarrow \sec x (\sec x + \tan x)\, dx = dt$

$\Rightarrow dx = \frac{2}{1+t^2} dt$

Now, $I = \int \frac{\left(\frac{t + \frac{1}{t}}{2}\right)^2 \frac{2}{1+t^2} dt}{t^{9/2}}$

$= \frac{1}{2}\int \left(t^{-\frac{9}{2}} + t^{-\frac{13}{2}}\right) dt$

$= \frac{1}{2}\left\{\frac{-2}{7} t^{\frac{-7}{2}} - \frac{2}{11} t^{-\frac{11}{2}}\right\} + K$

$= -t^{-\frac{11}{2}}\left\{\frac{1}{7}t^2 + \frac{1}{11}\right\} + K$

$= -\frac{1}{(\sec x + \tan x)^{\frac{11}{2}}}\left\{\frac{1}{11} + \frac{1}{7}(\sec x + \tan x)^2\right\} + K$

48. The total number of ways in which 5 balls of different colours can be distributed among 3 persons so that each person gets at least one ball is

(A) 75 (B) 150 (C) 210 (D) 243

Answer (B)

Hints :

x	y	z	
3	1	1	$\longrightarrow 3 \times {}^5C_3 \times {}^2C_1 \times {}^1C_1 = 60$
2	2	1	$\longrightarrow 3 \times {}^5C_2 \times {}^3C_2 \times {}^1C_1 = 90$

Total = 150

49. If $\lim_{x\to\infty}\left(\frac{x^2+x+1}{x+1}-ax-b\right)=4$, then

(A) $a = 1, b = 4$ (B) $a = 1, b = -4$ (C) $a = 2, b = -3$ (D) $a = 2, b = 3$

Answer (B)

Hints :

Here, $\lim_{x\to\infty}\frac{(x^2+x+1)-ax(x+1)-b(x+1)}{x+1}=4$

$\Rightarrow \lim_{x\to\infty}\frac{x^2(1-a)+x(1-a-b)+1-b}{x+1}=4$

This is possible iff

$1-a=0 \Rightarrow a=1$

and $1-a-b=4 \Rightarrow b=-4$

50. Let z be a complex number such that the imaginary part of z is nonzero and $a = z^2 + z + 1$ is real. Then a **cannot** take the value

(A) -1 (B) $\frac{1}{3}$ (C) $\frac{1}{2}$ (D) $\frac{3}{4}$

Answer (D)

Hints :

As, a is real,

So $a=\bar{a}$ gives

$\Rightarrow z^2+z+1=\bar{z}^2+\bar{z}+1$

$\Rightarrow (z-\bar{z})(z+\bar{z}+1)=0$

As, $z\neq\bar{z}$

So, $z+\bar{z}=-1$

$\Rightarrow x=-\frac{1}{2}$ {where $z=x+iy$}

Now,

$a=z^2+z+1$

$=\left(-\frac{1}{2}+iy\right)^2+\left(-\frac{1}{2}+iy\right)+1$

$=\frac{3}{4}-y^2$

As, $y\neq 0$ so, $\boxed{a\neq\frac{3}{4}}$

SECTION - II
(Multiple Correct Answer(s) Type)

This section contains **5 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct.**

51. Let S be the area of the region enclosed by $y=e^{-x^2}$, $y = 0$, $x = 0$, and $x = 1$. Then

(A) $S\geq\frac{1}{e}$ (B) $S\geq 1-\frac{1}{e}$ (C) $S\leq\frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)$ (D) $S\leq\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{e}}\left(1-\frac{1}{\sqrt{2}}\right)$

Answer (A, B, D)

Hints :

$x \in [0, 1] \Rightarrow x^2 \le x \Rightarrow -x^2 \ge -x \Rightarrow e^{-x^2} \ge e^{-x}$

$S = \int_0^1 e^{-x^2} dx \ge \int_0^1 e^{-x} dx = [-e^{-x}]_0^1 = 1 - \frac{1}{e}$

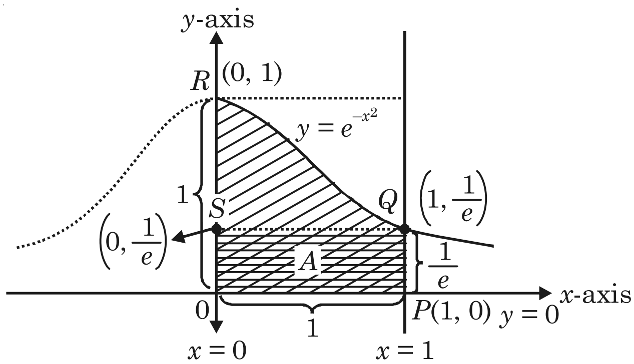
i.e., $S \ge 1 - \frac{1}{e} \Rightarrow$ Option (B) is correct.

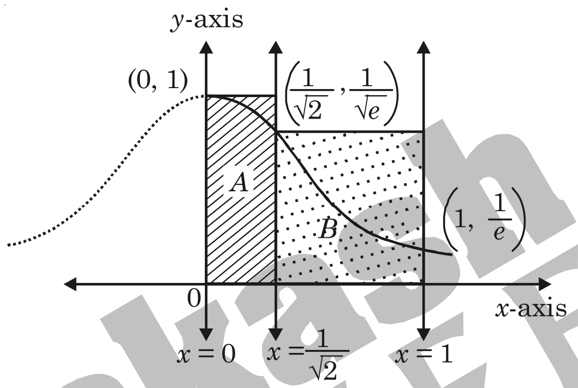
S is the area under the curve $y = e^{-x^2}$

Hence, $S \ge$ area of the rectangle $OPQS$

i.e., $S \ge 1 \times \frac{1}{e} = \frac{1}{e}$

Hence, option (A) is correct.





$S = \int_0^1 e^{-x^2} dx$

$= \int_0^{\frac{1}{\sqrt{2}}} e^{-x^2} dx + \int_{\frac{1}{\sqrt{2}}}^1 e^{-x^2} dx$

$\le$ Area A + Area B

Now, Area $A = \int_0^{\frac{1}{\sqrt{2}}} 1 \cdot dx = \frac{1}{\sqrt{2}}$

Area $B = \int_{\frac{1}{\sqrt{2}}}^1 \frac{1}{\sqrt{e}} dx = \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Hence, $S \le \frac{1}{\sqrt{2}} + \frac{1}{\sqrt{e}}\left(1 - \frac{1}{\sqrt{2}}\right)$

Hence, option (D) is correct

Now, consider the integral $\int_1^e \frac{dx}{x^{3/2}} = 2\left[1 - \frac{1}{\sqrt{e}}\right]$

Substitute $t = \ln x$

$\Rightarrow \int_1^e \frac{dx}{x^{3/2}} = \int_0^1 e^{-t/2} dt$

Now, for $0 < x < 1$

$e^{-x^2} > e^{-x/2}$

$$\Rightarrow \int_0^1 e^{-x^2}dx > 2\left[1-\frac{1}{\sqrt{e}}\right]$$

Now, $2\left[1-\frac{1}{\sqrt{e}}\right]-\frac{1}{4}\left(1+\frac{1}{\sqrt{e}}\right)=\frac{7}{4}-\frac{9}{4\sqrt{e}}$

$$=\frac{7\sqrt{e}-9}{4\sqrt{e}}>0$$

So, option (C) is **not** correct.

52. If $y(x)$ satisfies the differential equation $y' - y \tan x = 2x \sec x$ and $y(0) = 0$, then

(A) $y\left(\frac{\pi}{4}\right)=\frac{\pi^2}{8\sqrt{2}}$ (B) $y'\left(\frac{\pi}{4}\right)=\frac{\pi^2}{18}$ (C) $y\left(\frac{\pi}{3}\right)=\frac{\pi^2}{9}$ (D) $y'\left(\frac{\pi}{3}\right)=\frac{4\pi}{3}+\frac{2\pi^2}{3\sqrt{3}}$

Answer (A, D)

Hints :

$$\frac{dy}{dx}-y\tan x = 2x\sec x$$

$$\cos x\frac{dy}{dx}-(\sin x)y=2x$$

$d(y\cos x) = d(x^2)$

$y\cos x = x^2 + c$

$y(0) = c = 0$

$y = x^2\sec x$

also, $y' = 2x\sec x + x^2\sec x\tan x$

53. Tangents are drawn to the hyperbola $\frac{x^2}{9}-\frac{y^2}{4}=1$, parallel to the straight line $2x - y = 1$. The point of contact of the tangents on the hyperbola are

(A) $\left(\frac{9}{2\sqrt{2}},\frac{1}{\sqrt{2}}\right)$ (B) $\left(-\frac{9}{2\sqrt{2}},-\frac{1}{\sqrt{2}}\right)$ (C) $(3\sqrt{3},-2\sqrt{2})$ (D) $(-3\sqrt{3},2\sqrt{2})$

Answer (A, B)

Hints :

As, points of contact

$$=\left(\frac{\pm a^2m}{\sqrt{a^2m^2-b^2}},\frac{\pm b^2}{\sqrt{a^2m^2-b^2}}\right)$$

Here, $a^2 = 9$, $b^2 = 4$, $m = 2$

so, required points are $\left(\frac{\pm 9}{2\sqrt{2}},\frac{\pm 1}{\sqrt{2}}\right)$

54. Let $\theta, \varphi \in [0, 2\pi]$ be such that

$$2\cos\theta(1-\sin\varphi)=\sin^2\theta\left(\tan\frac{\theta}{2}+\cot\frac{\theta}{2}\right)\cos\varphi-1,$$

$$\tan(2\pi-\theta)>0 \text{ and } -1<\sin\theta<-\frac{\sqrt{3}}{2}.$$

Then φ **cannot** satisfy

(A) $0<\varphi<\frac{\pi}{2}$ (B) $\frac{\pi}{2}<\varphi<\frac{4\pi}{3}$ (C) $\frac{4\pi}{3}<\varphi<\frac{3\pi}{2}$ (D) $\frac{3\pi}{2}<\varphi<2\pi$

Answer (A, C, D)

Hints :

$$2\cos\theta(1-\sin\phi)=\sin^2\theta\left(\frac{1}{\sin\frac{\theta}{2}\cdot\cos\frac{\theta}{2}}\right)\cos\phi-1$$

$\Rightarrow \quad 2\cos\theta(1-\sin\phi)=2\sin\theta\cdot\cos\phi-1$

$\Rightarrow \quad 2\cos\theta + 1 = 2\sin(\theta + \phi)$...(i)

Now, $\tan\theta<0$ and $-1<\sin\theta<\frac{-\sqrt{3}}{2}$ gives

$\theta\in\left(\frac{3\pi}{2},\frac{5\pi}{3}\right)$

Using (i) ;

$\sin(\theta+\phi)=\cos\theta+\frac{1}{2}$

$\Rightarrow \quad \frac{1}{2}<\sin(\theta+\phi)<1$

i.e. $\frac{\pi}{6}<\theta+\phi<\frac{5\pi}{6}$ or, $\frac{13\pi}{6}<\theta+\phi<\frac{17\pi}{6}$

$\Rightarrow \quad \frac{13\pi}{6}-\theta<\phi<\frac{17\pi}{6}-\theta$

$\Rightarrow \quad \frac{2\pi}{3}<\phi<\frac{7\pi}{6}$

So, options (A), (C) and (D) are correct.

55. A ship is fitted with three engines E_1, E_2 and E_3. The engines function independently of each other with respective probabilities $\frac{1}{2}$, $\frac{1}{4}$ and $\frac{1}{4}$. For the ship to be operational at least two of its engines must function. Let X denote the event that the ship is operational and let X_1, X_2 and X_3 denote respectively the events that the engines E_1, E_2 and E_3 are functioning. Which of the following is (are) true?

(A) $P[X_1^c \mid X]=\frac{3}{16}$

(B) P [Exactly two engines of the ship are functioning | X] = $\frac{7}{8}$

(C) $P[X \mid X_2]=\frac{5}{16}$

(D) $P[X \mid X_1]=\frac{7}{16}$

Answer (B, D)

Hints :

(A) $P\left[X_1^c \middle| X\right]$

$$=\frac{P(\bar{X}_1\cap X)}{P(X)}$$

$$=\frac{\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\left(\frac{1}{4}\right)}{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}}=\frac{1}{8}$$

(B) Required probability $= \dfrac{\frac{3}{2\cdot 4\cdot 4}+\frac{3}{2\cdot 4\cdot 4}+\frac{1}{2\cdot 4\cdot 4}}{\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{3}{4}+\frac{1}{2}\cdot\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{1}{4}\cdot\frac{1}{4}}$

$= \dfrac{7}{8}$

(C) $P\left[X|X_2\right]$

$= \dfrac{P(X\cap X_2)}{P(X_2)}$

$= \dfrac{\left(\frac{1}{4}\right)\left[\frac{1}{2}\cdot\frac{1}{4}+\frac{1}{2}\cdot\frac{3}{4}+\left(\frac{1}{2}\right)\left(\frac{1}{4}\right)\right]}{\left(\frac{1}{4}\right)}$

$= \dfrac{\frac{1}{4}\cdot\frac{5}{8}}{\frac{1}{4}} = \dfrac{5}{8}$

(D) $\dfrac{P(X\cap X_1)}{P(X_1)}$

$= \dfrac{\left(\frac{1}{2}\right)\left[\frac{1}{4}\cdot\frac{1}{4}+\frac{3}{4}\cdot\frac{1}{4}+\frac{1}{4}\cdot\frac{3}{4}\right]}{\frac{1}{2}}$

$= \dfrac{7}{16}$

SECTION - III
(Integer Answer Type)

This section contains **5 questions.** The answer to each of the questions is a **single digit integer,** ranging from 0 to 9 (*both inclusive*).

56. The value of $6+\log_{\frac{3}{2}}\left(\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\sqrt{4-\frac{1}{3\sqrt{2}}\ldots}}}\right)$ is

Answer (4)

Hints :

As, $y = \dfrac{1}{3\sqrt{2}}\sqrt{4-y}$

$\Rightarrow\ y = \dfrac{4}{9}$ (as $y > 0$)

so, $6+\log_{\frac{3}{2}}\left(\dfrac{4}{9}\right) = 4$

57. Let S be the focus of the parabola $y^2 = 8x$ and let PQ be the common chord of the circle $x^2 + y^2 - 2x - 4y = 0$ and the given parabola. The area of the triangle PQS is

Answer (4)

Hints :

Clearly $P(2t^2, 4t)$ satisfies the equation of circle,

$x^2 + y^2 - 2x - 4y = 0$

$\Rightarrow 4t^4 + 16t^2 - 4t^2 - 16t = 0$

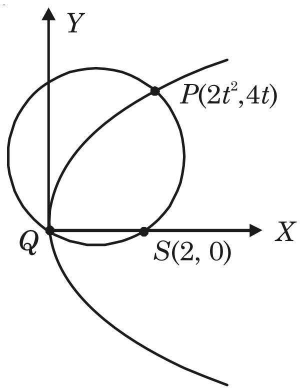
$\Rightarrow t(t^3 + 3t - 4) = 0$

$\Rightarrow t(t - 1)(t^2 + t + 4) = 0$

$\Rightarrow t = 0,1$

i.e., P be (2, 4)

So, area of $\Delta PQS = \frac{1}{2} \times 2 \times 4 = 4$ units



58. If $\vec{a}, \vec{b}$ and $\vec{c}$ are unit vectors satisfying $|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$, then $|2\vec{a}+5\vec{b}+5\vec{c}|$ is

Answer (3)

Hints :

$|\vec{a}-\vec{b}|^2 + |\vec{b}-\vec{c}|^2 + |\vec{c}-\vec{a}|^2 = 9$

$\Rightarrow 2(|\vec{a}|^2 + |\vec{b}|^2 + |\vec{c}|^2 - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{c} - \vec{c}.\vec{a}) = 9$ ...(i)

$\Rightarrow 2(3 - \vec{a}\cdot\vec{b} - \vec{b}\cdot\vec{c} - \vec{c}\cdot\vec{a}) = 9$

$\Rightarrow \vec{a}\cdot\vec{b} + \vec{b}\cdot\vec{c} + \vec{c}\cdot\vec{a} = -\frac{3}{2} \Rightarrow$ angle between each pair is 120°.

Now, $|2\vec{a}+5\vec{b}+5\vec{c}|^2 = 4|\vec{a}|^2 + 25|\vec{b}|^2 + 25|\vec{c}|^2 + 20\vec{a}\cdot\vec{b} + 20\vec{a}\cdot\vec{c} + 50\vec{a}\cdot\vec{c}$

$= 4 + 25 + 25 - \frac{20}{2} - \frac{20}{2} - \frac{50}{2}$

$= 54 - 10 - 10 - 25$

$= 9$

$\therefore |2\vec{a}+5\vec{b}+5\vec{c}| = 3$

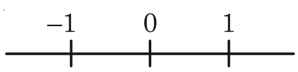
59. Let $f : \mathbb{R} \to \mathbb{R}$ be defined as $f(x) = |x| + |x^2 - 1|$. The total number of points at which f attains either a local maximum or a local minimum is

Answer (5)

Hints :

$f(x) = |x| + |x^2 - 1|$

$f(x) = |x| + |(x - 1)(x + 1)|$

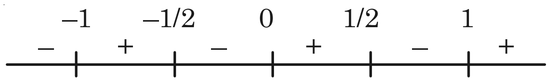


$$f(x) = \begin{cases} x^2 - x - 1 & , \quad x < -1 \\ -x^2 - x + 1 & , \quad -1 \le x < 0 \\ -x^2 + x + 1 & , \quad 0 \le x < 1 \\ x^2 + x - 1 & , \quad x \ge 1 \end{cases}$$

$$f'(x) = \begin{cases} 2x - 1 & , \quad x < -1 \\ -2x - 1 & , \quad -1 \le x < 0 \\ -2x + 1 & , \quad 0 \le x < 1 \\ 2x + 1 & , \quad x \ge 1 \end{cases}$$

$f(x)$ is not differentiable at $x = 0, \pm 1$ and $f'(x) = 0$ at $x = -\frac{1}{2}, \frac{1}{2}$

Also sign scheme of $f'(x)$



$x = -1, 0, 1$ are point of minima

$x = \frac{-1}{2}, \frac{1}{2}$ are point of maxima

total number of point of maxima and minima = 5

60. Let $p(x)$ be a real polynomial of least degree which has a local maximum at $x = 1$ and a local minimum at $x = 3$. If $p(1) = 6$ and $p(3) = 2$, then $p'(0)$ is

Answer (9)

Hints :

$p(x)$ will be of degree 3

$p'(x) = k(x - 1)(x - 3)$, $k > 0$ as 1 is point of maxima and 3 is point of minima

$= k(x^2 - 4x + 3)$

Now $p(x) = k\left(\frac{x^3}{3} - 2x^2 + 3x\right) + c$

given, $p(1) = 6$

$\Rightarrow 4k + 3c = 18$ ...(i)

and $p(3) = 2$

$\Rightarrow c = 2$

so, $k = 3$

$p'(x) = 3(x - 1)(x - 3)$

$p'(0) = 3(-1)(-3) = 9$

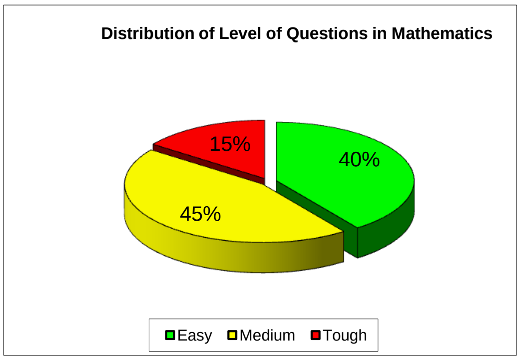
❑ ❑ ❑

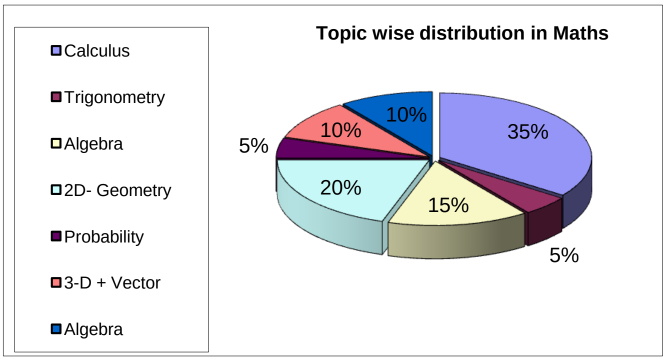
Topic-wise Analysis of MATHEMATICS - IIT-JEE 2012 (Paper-1)

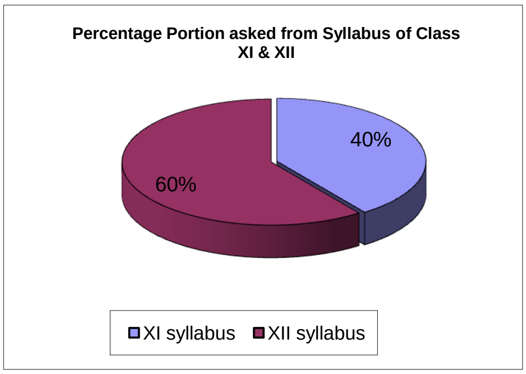
	XII	XI	XI	XI	XII	XII	XII	
	Calculus	Trigonometry	Algebra	2D- Geometry	Probability	3-D + Vector	Algebra	*Total*
Easy	2	0	1	3	0	1	1	*8*
Medium	4	0	2	1	0	1	1	*9*
Tough	1	1	0	0	1	0	0	*3*
Total	*7*	*1*	*3*	*4*	*1*	*2*	*2*	***20***
Markwise	**26**	**4**	**10**	**13**	**4**	**7**	**6**	70

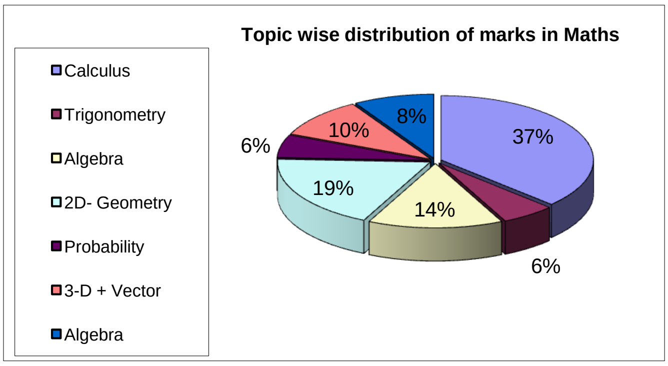
XI Marks	**27**	**XII Marks**	**43**

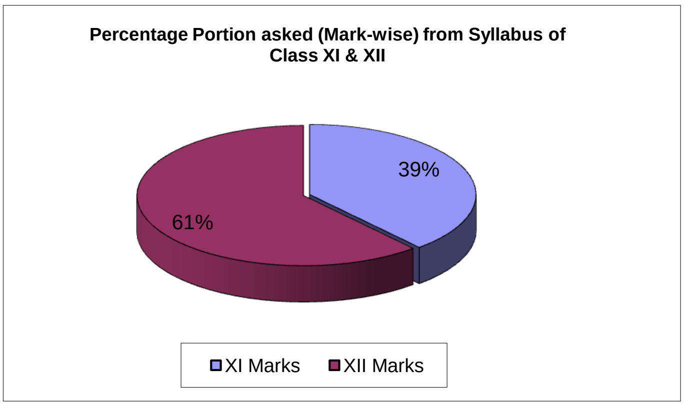
XI syllabus	**8**	**XII syllabus**	**12**











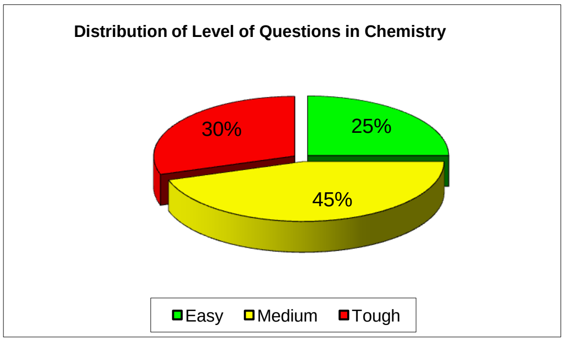
Topic-wise Analysis of CHEMISTRY - IIT-JEE 2012
(Paper-1)

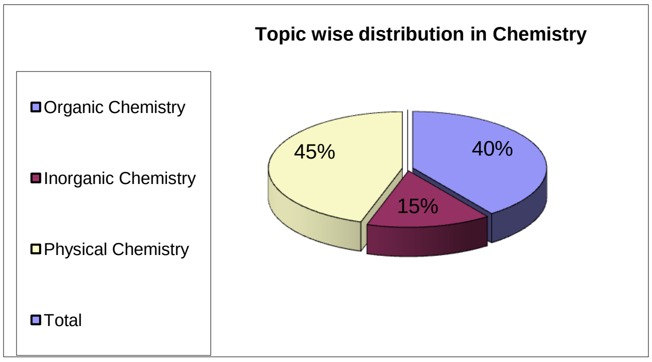
	Organic Chemistry	Inorganic Chemistry	Physical Chemistry	*Total*
Easy	2	1	2	*5*
Medium	3	1	5	*9*
Tough	3	1	2	*6*
Total	*8*	*3*	*9*	***20***

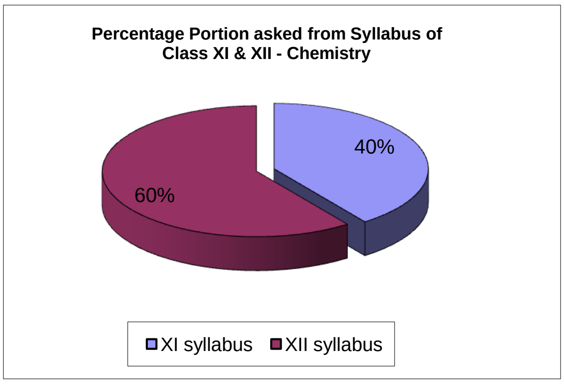
Markwise	**28**	**10**	**32**	**70**

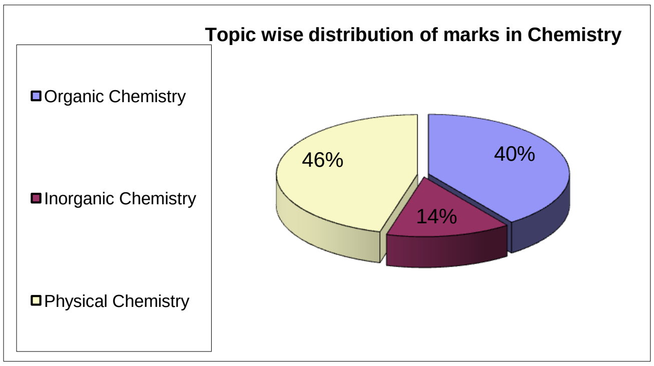
XI Marks	**27**	**XII Marks**	**43**

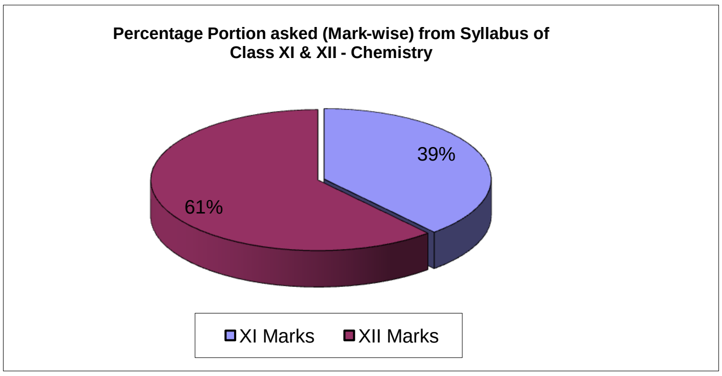
XI syllabus	**8**	**XII syllabus**	**12**











Topic-wise Analysis of PHYSICS - IIT-JEE 2012 (Paper-1)

	XII	XI	XII	XI	XI	XII	XII	XII	
	Electricity	Heat & Thermodynamics	Modern Physics	Mechanics	Oscillation & Waves	Magnetism and EMI	Optics	Experimental physics	*Total*
Easy	3	1	0	0	0	1	1	0	*6*
Medium	1	1	1	3	2	1	1	1	*11*
Tough	1	0	0	1	0	1	0	0	*3*
Total	*5*	*2*	*1*	*4*	*2*	*3*	*2*	**1**	***20***

Markwise	**18**	**6**	**4**	**14**	**7**	**12**	**6**	**3**	70

XI Marks	**27**	**XII Marks**	**43**
XI syllabus	**8**	**XII syllabus**	**12**