DATE : 08/04/2012

# Aakash IIT-JEE

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**Time : 3 hrs.** **Max. Marks: 198**

# SOLUTIONS

*for*

# IIT-JEE 2012

## PAPER - 2 (Code - 0)

### INSTRUCTIONS

1. The question paper consists of **3 parts** (Physics, Chemistry and Mathematics). Each part consists of **three sections.**
2. **Section I** contains **8 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**
3. **Section II** contains **3 paragraphs** each describing theory, experiment, data etc. There are **6 multiple choice questions** relating to three paragraphs with **2 questions on each paragraph**. Each question of a particular paragraph has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**
4. **Section III** contains **6 multiple choice questions**. Each question has four choices (A), (B), (C) and (D) out of which **ONE or MORE are correct.**

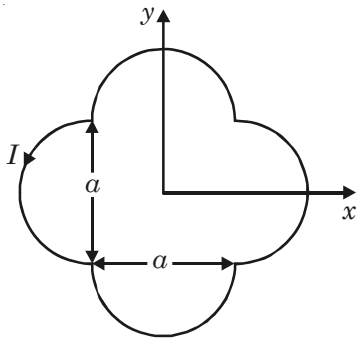
## PART–I : PHYSICS

### SECTION - I

**(Single Correct Answer Type)**

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

1. A loop carrying current $I$ lies in the $x$-$y$ plane as shown in the figure. The unit vector $\hat{k}$ is coming out of the plane of the paper. The magnetic moment of the current loop is




(A) $a^2 I\hat{k}$ (B) $\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$

(C) $-\left(\frac{\pi}{2}+1\right)a^2 I\hat{k}$ (D) $(2\pi+1)a^2 I\hat{k}$

**Answer (B)**

**Hints :**

$$\vec{M} = I\vec{A}$$

$$\vec{A} = a^2 + 4 \times \frac{\pi\left(\frac{a}{2}\right)^2}{2}$$

$$= \left(\frac{\pi}{2} + 1\right) a^2$$

2. A thin uniform cylindrical shell, closed at both ends, is partially filled with water. It is floating vertically in water in half-submerged state. If $\rho_c$ is the relative density of the material of the shell with respect to water, then the correct statement is that the shell is

(A) More than half-filled if $\rho_c$ is less than 0.5 (B) More than half-filled if $\rho_c$ is more than 1.0

(C) Half-filled if $\rho_c$ is more than 0.5 (D) Less than half-filled if $\rho_c$ is less than 0.5

**Answer (A)**

**Hints :**

Let $V_0$ be the volume of walls of cylinder and $V$ be the volume of water in it. Further let $V_1$ be the volume of empty space in the cylinder.

For equilibrium,

$$\rho_c V_0 g + Vg = \left(\frac{V_1 + V_0}{2}\right) g$$

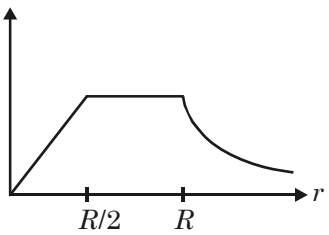
$$V = \frac{V_1}{2} + \frac{V_0}{2}(1 - 2\rho_c)$$

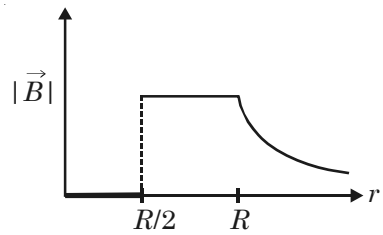
if $\rho_c = 0.5$, $V = \frac{V_1}{2}$

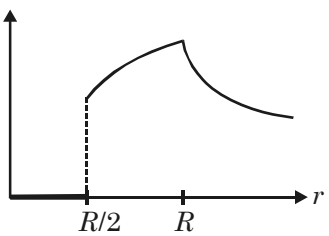
$\rho_c > 0.5$, $V < \frac{V_1}{2}$

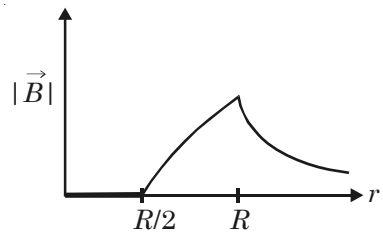
$\rho_c < 0.5$, $V > \frac{V_1}{2}$

3. An infinitely long hollow conducting cylinder with inner radius $R/2$ and outer radius $R$ carries a uniform current density along its length. The magnitude of the magnetic field, $|\vec{B}|$ as a function of the radial distance $r$ from the axis is best represented by

(A) 


(B) 


(C) 

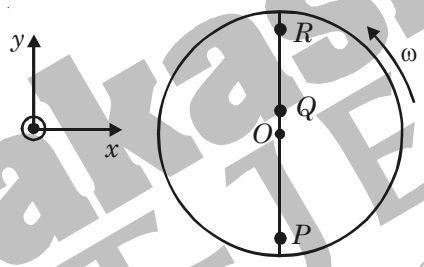

(D) 

**Answer (D)**

**Hints :**

$$B = \begin{cases} 0, r < \dfrac{R}{2} \\ \dfrac{\mu_0 J\left(r^2 - \dfrac{R^2}{4}\right)}{2r}, \dfrac{R}{2} < r < R \\ \dfrac{\mu_0 J\left(\dfrac{3R^2}{4}\right)}{2r}, r > R \end{cases}$$

4. Consider a disc rotating in the horizontal plane with a constant angular speed ω about its centre $O$. The disc has a shaded region on one side of the diameter and an unshaded region on the other side as shown in the figure. When the disc is in the orientation as shown, two pebbles $P$ and $Q$ are simultaneously projected at an angle towards $R$. The velocity of projection is in the $y$-$z$ plane and is same for both pebbles with respect to the disc. Assume that (i) they land back on the disc before the disc has completed $\frac{1}{8}$ rotation, (ii) their range is less than half the disc radius, and (iii) ω remains constant throughout. Then




(A) $P$ lands in the shaded region and $Q$ in the unshaded region

(B) $P$ lands in the unshaded region and $Q$ in the shaded region

(C) Both $P$ and $Q$ land in the unshaded region

(D) Both $P$ and $Q$ land in the shaded region

**Answer (C)**

**Hints :**

Horizontal component (towards right), of the velocity of $P$ at any time $t$ will always be greater than that of any point on the disc below it. Therefore it will land in unshaded region.

Similarly, horizontal component towards left of $Q$ is always less than that of the point below it and it will also land in unshaded region.

5. A student is performing the experiment of Resonance Column. The diameter of the column tube is 4 cm. The frequency of the tuning fork is 512 Hz. The air temperature is 38°C in which the speed of sound is 336 m/s. The zero of the meter scale coincides with the top end of the Resonance Column tube. When the first resonance occurs, the reading of the water level in the column is

(A) 14.0 cm (B) 15.2 cm (C) 16.4 cm (D) 17.6 cm

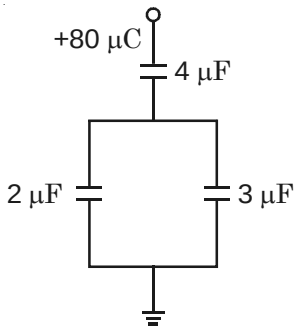
**Answer (B)**

**Hints :**

$$L = \frac{\lambda}{4} - e$$

$$= \frac{\lambda}{4} - 0.3d = 15.2 \text{ cm}$$

6. In the given circuit, a charge of +80 μC is given to the upper plate of the 4 μF capacitor. Then in the steady state, the charge on the upper plate of the 3 μF capacitor is




(A) +32 μC (B) +40 μC (C) +48 μC (D) +80 μC

**Answer (C)**

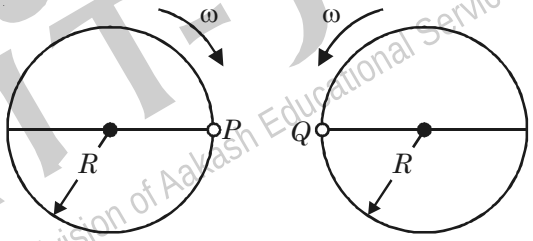
**Hints :**

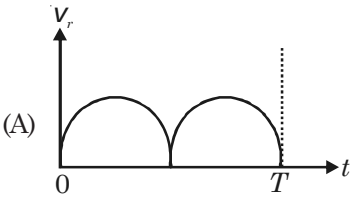
For the 2 μF and 3 μF capacitor, equivalent capacitance is 5 μF

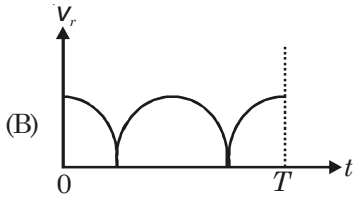
$\Rightarrow \quad V = \frac{Q}{C} = 16$ V

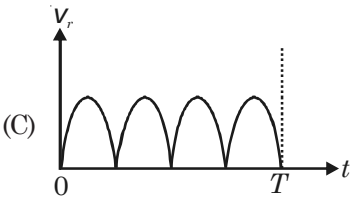
Now, $q = CV = 48$ μC

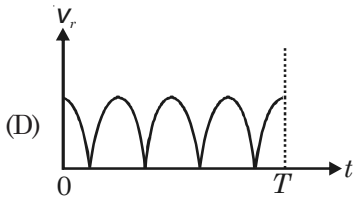
7. Two identical discs of same radius $R$ are rotating about their axes in opposite directions with the same constant angular speed ω. The discs are in the same horizontal plane. At time $t = 0$, the points $P$ and $Q$ are facing each other as shown in the figure. The relative speed between the two points $P$ and $Q$ is $v_r$. In one time period ($T$) of rotation of the discs, $v_r$ as a function of time is best represented by



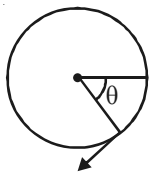




**Answer (A)**

**Hints :** $v_r = 2v|\sin\theta|$



8. Two moles of ideal helium gas are in a rubber balloon at 30°C. The balloon is fully expandable and can be assumed to require no energy in its expansion. The temperature of the gas in the balloon is slowly changed to 35°C. The amount of heat required in raising the temperature is nearly (take $R$ = 8.31 J/mol.K)

(A) 62 J (B) 104 J (C) 124 J (D) 208 J

**Answer (D)**

**Hints :**

$Q = nCp\Delta T$ = 208 J

## SECTION - II
## (Paragraph Type)

This section contains **6 multiple choice questions** relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which **ONE is correct.**

### Paragraph for Questions 9 and 10

The β-decay process, discovered around 1900, is basically the decay of a neutron ($n$). In the laboratory, a proton ($p$) and an electron ($e^-$) and observed as the decay products of the neutron. Therefore, considering the decay of a neutron as a two body decay process, it was predicted theoretically that the kinetic energy of the electron should be a constant. But experimentally, it was observed that the electron kinetic energy has a continuous spectrum. Considering a three-body decay process, i.e. $n \rightarrow p + e^- + \bar{\nu}_e$, around 1930, Pauli explained the observed electron energy spectrum. Assuming the anti-neutrino ($\bar{\nu}_e$) to be massless and possessing negligible energy, and the neutron to be at rest, momentum and energy conservation principles are applied From this calculation, the maximum kinetic energy of the electron is 0.8 × $10^6$ eV. The kinetic energy carried by the proton is only the recoil energy.

9. What is the maximum energy of the anti-neutrino?

(A) Zero

(B) Much less than 0.8 × $10^6$ eV

(C) Nearly 0.8 × $10^6$ eV

(D) Much larger than 0.8 × $10^6$ eV

**Answer (C)**

**Hints :**

Maximum energy of the antineutrino will be nearly 0.8 × $10^6$ eV.

10. If the anti-neutrino had a mass of 3 eV/$c^2$ (where $c$ is the speed of light) instead of zero mass, what should be the range of the kinetic energy, $K$, of the electron?

(A) $0 \le K \le 0.8 \times 10^6$ eV

(B) 3.0 eV $\le K \le 0.8 \times 10^6$ eV

(C) 3.0 eV $\le K < 0.8 \times 10^6$ eV

(D) $0 \le K < 0.8 \times 10^6$ eV

**Answer (D)**

**Hints :**

Minimum kinetic energy of electron can be zero or greater than zero. But maximum kinetic energy will be less than 0.8 × $10^6$ eV.

**Paragraph for Questions 11 and 12**

Most materials have the refractive index, $n > 1$. So, when a light ray from air enters a naturally occurring material, then by Snell's law, $\frac{\sin\theta_1}{\sin\theta_2} = \frac{n_2}{n_1}$, it is understood that the refracted ray bends towards the normal. But it never emerges on the same side of the normal as the incident ray. According to electromagnetism, the refractive index of the medium is given by the relation, $n = \left(\frac{c}{v}\right) = \pm\sqrt{\varepsilon_r\mu_r}$, where $c$ is the speed of electromagnetic waves in vacuum, $v$ its speed in the medium, $\varepsilon_r$ and $\mu_r$ are the relative permittivity and permeability of the medium respectively.

In normal materials, both $\varepsilon_r$ and $\mu_r$ are positive, implying positive $n$ for the medium. When both $\varepsilon_r$ and $\mu_r$ are negative, one most choose the negative root of $n$. Such negative refractive index materials can now be artificially prepared and are called meta-materials. They exhibit significantly different optical behavior, without violating any physical laws. Since $n$ is negative, it results in a change in the direction of propagation of the refracted light. However, similar to normal materials, the frequency of light remains unchanged upon refraction even in meta-materials.

11. Choose the correct statement.

(A) The speed of light in the meta-material is $v = c|n|$

(B) The speed of light in the meta-material is $v = \frac{c}{|n|}$

(C) The speed of light in the meta-material is $v = c$

(D) The wavelength of the light in the meta-material ($\lambda_m$) is given by $\lambda_m = \lambda_{air}|n|$, where $\lambda_{air}$ is the wavelength of the light in air

**Answer (B)**

**Hints :** Speed is given by $\frac{C}{|n|}$

12. For light incident from air on a meta-material, the appropriate ray diagram is

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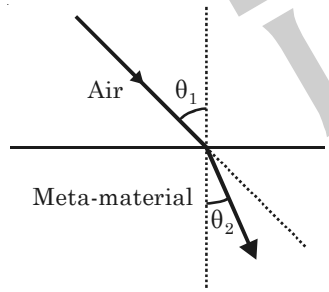
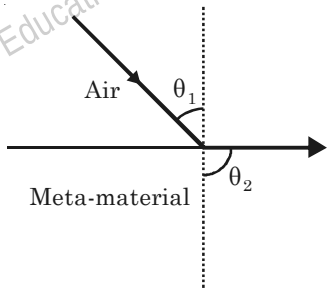
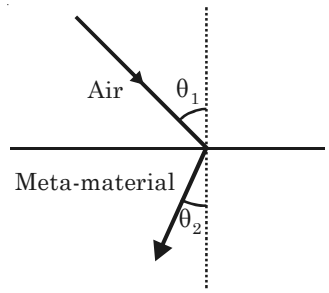
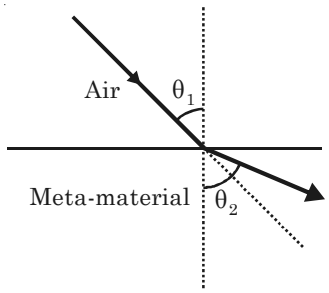



(B)



(C)


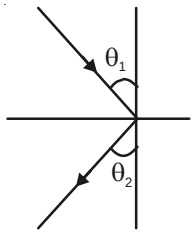

(D)



**Answer (C)**

**Hints :**

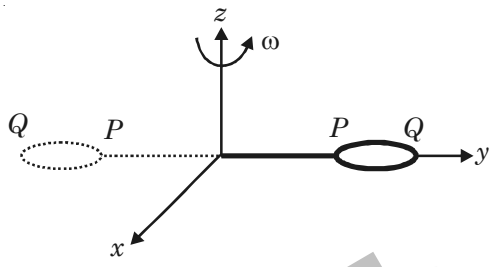
$1\sin\theta_1 = -n\sin\theta_2$

$\sin\theta_2 = -\frac{1}{n}\sin\theta_1$

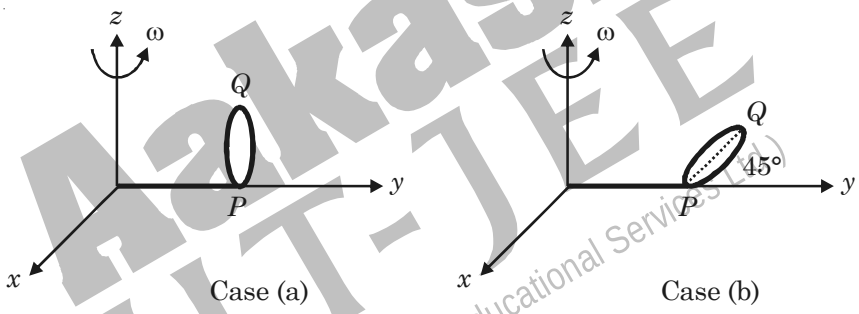


**Paragraph for Questions 13 and 14**

The general motion of a rigid body can be considered to be a combination of (i) a motion of its centre of mass about an axis, and (ii) its motion about an instantaneous axis passing through the centre of mass. These axes need not be stationary. Consider, for example, a thin uniform disc welded (rigidly fixed) horizontally at its rim to a massless stick, as shown in the figure. When the disc-stick system is rotated about the origin on a horizontal frictionless plane with angular speed ω, the motion at any instant can be taken as a combination of (i) a rotation of the centre of mass of the disc about the $z$-axis, and (ii) a rotation of the disc through an instantaneous vertical axis passing through its centre of mass (as is seen from the changed orientation of points $P$ and $Q$). Both these motions have the same angular speed ω in this case.




Now consider two similar systems as shown in the figure: Case (a) the disc with its face vertical and parallel to $x$-$z$ plane; Case (b) the disc with its face making an angle of 45° with $x$-$y$ plane and its horizontal diameter parallel to $x$-axis. In both the cases, the disc is welded at point $P$, and the systems are rotated with constant angular speed ω about the $z$-axis.




13. Which of the following statements regarding the angular speed about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is $\sqrt{2}\omega$ for both the cases

(B) It is ω for case (a); and $\frac{\omega}{\sqrt{2}}$ for case (b)

(C) It is ω for case (a); and $\sqrt{2}\omega$ for case (b)

(D) It is ω for both the cases

**Answer (D)**

**Hints :**

In the frame of reference of the centre of mass the disc it appears to rotate with the same angular velocity ω.

14. Which of the following statement about the instantaneous axis (passing through the centre of mass) is correct?

(A) It is vertical for both the cases (a) and (b)

(B) It is vertical for case (a); and is at 45° to the $x$-$z$ plane and lies in the plane of the disc for case (b)

(C) It is horizontal for case (a); and is at 45° to the $x$-$z$ plane and is normal to the plane of the disc for case (b)

(D) It is vertical for case (a); and is at 45° to the $x$-$z$ plane and is normal to the plane of the disc for case (b)

**Answer (A)**

**Hints :**

The axis of rotation must be parallel to the angular velocity of rotation.

## SECTION - III
## (Multiple Correct Answer(s) Type)

This section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct.**

15. Two solid cylinders $P$ and $Q$ of same mass and same radius start rolling down a fixed inclined plane from the same height at the same time. Cylinder $P$ has most of its mass concentrated near its surface, while $Q$ has most of its mass concentrated near the axis. Which statement(s) is(are) correct?

(A) Both cylinders $P$ and $Q$ reach the ground at the same time

(B) Cylinder $P$ has larger linear acceleration than cylinder $Q$

(C) Both cylinders reach the ground with same translational kinetic energy

(D) Cylinder $Q$ reaches the ground with larger angular speed

**Answer (D)**

**Hints :**

$$a_{cm} = \frac{g\sin\theta}{1+\frac{I_{cm}}{mR^2}}$$

$I_{cmP} > I_{cmQ}$

$\Rightarrow$ $a_{cmQ} > a_{cmP}$. Thus $Q$ reaches the ground with larger angular speed.

16. A current carrying infinitely long wire is kept along the diameter of a circular wire loop, without touching it. The correct statement(s) is(are)

(A) The emf induced in the loop is zero if the current is constant

(B) The emf induced in the loop is finite if the current is constant

(C) The emf induced in the loop is zero if the current decreases at a steady rate

(D) The emf induced in the loop is finite if the current decreases at a steady rate

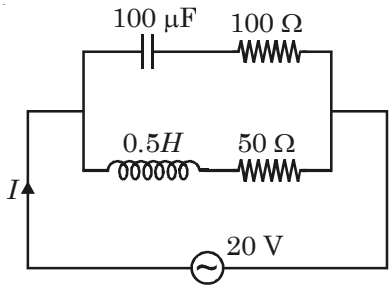
**Answer (A, C)**

**Hints :**

If the current is constant then emf is zero.

If the current is varying even then the net flux remains zero hence emf is zero.

17. In the given circuit, the $AC$ source has $\omega = 100$ rad/s. Considering the inductor and capacitor to be ideal, the correct choice(s) is(are)




(A) The current through the circuit, $I$ is 0.3 A

(B) The current through the circuit, $I$ is $0.3\sqrt{2}$ A

(C) The voltage across 100 Ω resistor = $10\sqrt{2}$ V

(D) The voltage across 50 Ω resistor = 10 V

**Answer (A, C)**

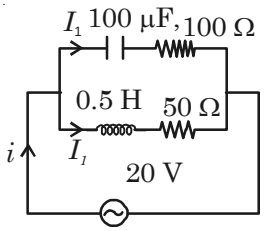
**Hints :**

$X_C = 100\ \Omega$

$X_L = 50\ \Omega$

$I_1 = \dfrac{20}{100\sqrt{2}}$

$I_2 = \dfrac{20}{50\sqrt{2}}$




$I = \sqrt{I_1^2 + I_2^2} = \dfrac{1}{\sqrt{10}} \approx 0.3\ \text{A}$

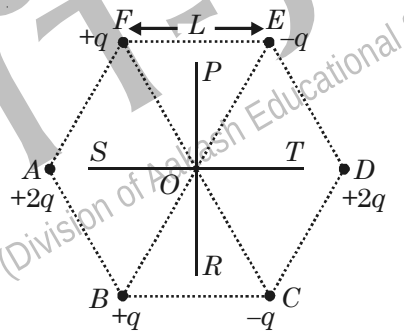
$V^2 + V^2 = (20)^2$

$2V^2 = 20^2$

$V = \dfrac{20}{\sqrt{2}}$

$V = 10\sqrt{2}$ .

18. Six point charges are kept at the vertices of a regular hexagon of side $L$ and centre $O$, as shown in the figure. Given that $K = \dfrac{1}{4\pi\varepsilon_0}\dfrac{q}{L^2}$, which of the following statement(s) is(are) correct?




(A) The electric field at $O$ is $6K$ along $OD$

(B) The potential at $O$ is zero

(C) The potential at all points on the line $PR$ is same

(D) The potential at all points on the line $ST$ is same

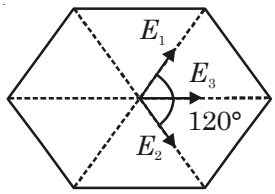
**Answer (A, B, C)**

**Hints :**

$\vec{E} = \vec{E}_1 + \vec{E}_2 + \vec{E}_3$

$E_1 = E_2 = 2 \times \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{q}{L^2}$

$E_3 = 2 \times \dfrac{1}{4\pi\varepsilon_0} \times \dfrac{2q}{L^2}$



Net electric field at $O$.

$E_3 + 2E_1 \cos 60° = 6$ K

Potential at $O$ is zero.

Any point on line $PR$ is equidistant from a pair of equal and opposite charge.

19. Two spherical planets P and Q have the same uniform density ρ, masses $M_P$ and $M_Q$, and surface areas $A$ and $4A$, respectively. A spherical planet $R$ also has uniform density ρ and its mass is $(M_P + M_Q)$. The escape velocities from the planets $P$, $Q$ and $R$, are $V_P$, $V_Q$ and $V_R$, respectively. Then

(A) $V_Q > V_R > V_P$ (B) $V_R > V_Q > V_P$ (C) $V_R/V_P = 3$ (D) $V_P/V_Q = \frac{1}{2}$

**Answer (B, D)**

**Hints :**

As area of $Q$ is 4 times area of $P$ so

radius of $Q$ = 2 times radius of $P$

$R_Q = 2R_P$

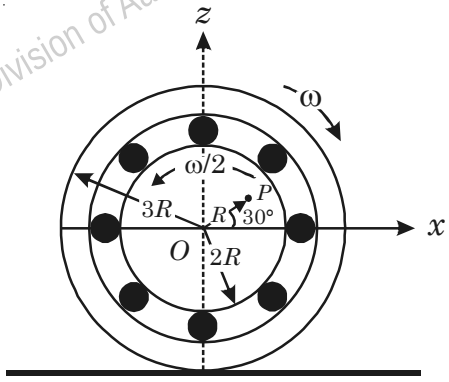
Also $M_Q = 8M_P$

So $M_R = M_Q + M_P = 9M_P$

and $R_R = 9^{1/3}R_P$

Now, $V_e = \sqrt{\frac{GM}{R}}$

20. The figure shows a system consisting of (i) a ring of outer radius $3R$ rolling clockwise without slipping on a horizontal surface with angular speed ω and (ii) an inner disc of radius $2R$ rotating anti-clockwise with angular speed ω/2. The ring and disc are separated by frictionless ball bearings. The system is in the $x$-$z$ plane. The point $P$ on the inner disc is at a distance $R$ from the origin, where $OP$ makes an angle of 30° with the horizontal. Then with respect to the horizontal surface,




(A) the point $O$ has a linear velocity $3R\omega\hat{i}$.

(B) the point $P$ has a linear velocity $\frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$.

(C) the point $P$ has a linear velocity $\frac{13}{4}R\omega\hat{i} - \frac{\sqrt{3}}{4}R\omega\hat{k}$

(D) the point $P$ has a linear velocity $\left(3 - \frac{\sqrt{3}}{4}\right)R\omega\hat{i} + \frac{1}{4}R\omega\hat{k}$

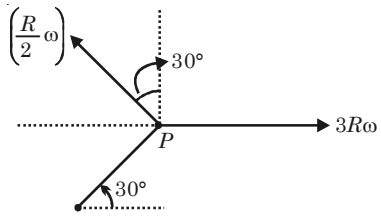
**Answer (A, B)**

**Hints :**

As the system is under pure rooling so

$$V_0 = (3R)\omega\hat{i}$$

For point $P$




$$\vec{V}_P = \frac{11}{4}R\omega\hat{i} + \frac{\sqrt{3}}{4}R\omega\hat{k}$$

# PART–II : CHEMISTRY

## SECTION - I
## (Single Correct Answer Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct**.

21. $NiCl_2\{P(C_2H_5)_2(C_6H_5)\}_2$ exhibits temperature dependent magnetic behaviour (paramagnetic/diamagnetic). The coordination geometries of $Ni^{2+}$ in the paramagnetic and diamagnetic states are respectively

(A) tetrahedral and tetrahedral
(B) square planar and square planar
(C) tetrahedral and square planar
(D) square planar and tetrahedral

**Answer (C)**

**Hints :** The hybridisation of $Ni^{2+}$ in paramagnetic complex is $sp^3$ and hence the complex is tetrahedral. The diamagnetic complex is square planar as the hybridisation of $Ni^{2+}$ is $dsp^2$.

22. The reaction of white phosphorus with aqueous NaOH gives phosphine along with another phosphorus containing compound. The reaction type; the oxidation states of phosphorus in phosphine and the other product are respectively

(A) redox reaction; – 3 and – 5
(B) redox reaction; + 3 and + 5
(C) disproportionation reaction; – 3 and + 5
(D) disproportionation reaction; – 3 and + 3

**Answer (D)**

**Hints :** $P_4$ on disproportionation produces $PH_3$, $NaH_2PO_2$ and $Na_3PO_3$.

23. In the cyanide extraction process of silver from argentite ore, the oxidizing and reducing agents used are

(A) $O_2$ and CO respectively
(B) $O_2$ and Zn dust respectively
(C) $HNO_3$ and Zn dust respectively
(D) $HNO_3$ and CO respectively

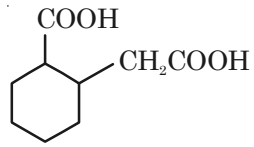
**Answer (B)**

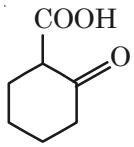
**Hints :** $Ag_2S + 4NaCN \rightleftharpoons 2Na[Ag(CN)_2] + Na_2S$

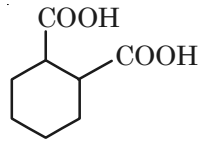
$Na_2S^{-2} + 2O_2 \longrightarrow Na_2^{+6}SO_4$

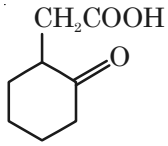
$2Na[Ag^{+1}(CN)_2] + Zn \longrightarrow Na_2[Zn(CN)_4] + 2Ag^0 \downarrow$

24. The compound that undergoes decarboxylation most readily under mild condition is

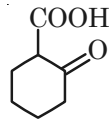
(A) 

(B) 

(C) 

(D) 

**Answer (B)**

**Hints :**  ⇒ β-keto acid undergoes easy decarboxylation just by heating through the formation of six membered transition state.

25. Using the data provided, calculate the multiple bond energy (kJ $mol^{-1}$) of a C ≡ C bond in $C_2H_2$. That energy is (take the bond energy of a C – H bond as 350 kJ $mol^{-1}$)

$$2C(s) + H_2(g) \longrightarrow C_2H_2(g) \qquad \Delta H = 225 \text{ kJ mol}^{-1}$$
$$2C(s) \longrightarrow 2C(g) \qquad \Delta H = 1410 \text{ kJ mol}^{-1}$$
$$H_2(g) \longrightarrow 2H(g) \qquad \Delta H = 330 \text{ kJ mol}^{-1}$$

(A) 1165 (B) 837

(C) 865 (D) 815

**Answer (D)**

**Hints :** ΔH = Bond energy in reactant – Bond energy in product

$225 = (1410 + 330) - (700 + \Delta H_{C\equiv C})$

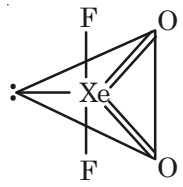
$\Delta H_{C\equiv C} = 1740 - 925 = 815$ kJ/mol

26. The shape of $XeO_2F_2$ molecule is

(A) Trigonal bipyramidal (B) Square planar

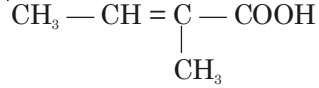
(C) Tetrahedral (D) See-saw

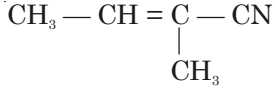
**Answer (D)**

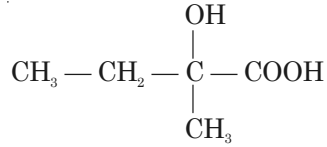
**Hints :**  ⇒ See saw.

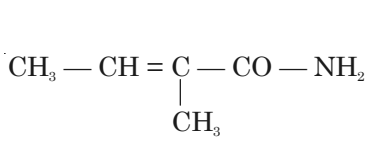
27. The major product H of the given reaction sequence is

$$CH_3 - CH_2 - CO - CH_3 \xrightarrow{\ominus CN} G \xrightarrow[\text{Heat}]{95\%\ H_2SO_4} H$$

(A) 

(B) 

(C) 

(D) 

**Answer (B)**

**Hints :** $CH_3 - CH_2 - \overset{\overset{O}{\|}}{C} - CH_3 \xrightarrow{\ominus CN} CH_3 - CH_2 - \overset{\overset{OH}{|}\ \overset{CN}{|}}{C} - CH_3 \xrightarrow[\Delta]{95\%\ H_2SO_4} CH_3 - CH = \overset{\overset{CN}{|}}{C} - CH_3$

28. For a dilute solution containing 2.5 g of a non-volatile non-electrolyte solute in 100 g of water, the elevation in boiling point at 1 atm pressure is 2°C. Assuming concentration of solute is much lower than the concentration of solvent, the vapour pressure (mm of Hg) of the solution is (take $K_b = 0.76$ K kg mol$^{-1}$)

(A) 724 (B) 740

(C) 736 (D) 718

**Answer (A)**

**Hints :** $\Delta T_b = 1 \times K_b \times m$

$2 = 1 \times 0.76 \times m$

$m = \dfrac{2}{0.76} = 2.63$

$\dfrac{760 - P_S}{760} = 2.63 \times \dfrac{18}{1000}$

$P_S = 724$ mmHg

## SECTION - II
## (Paragraph Type)

This section contains **6 multiple choice questions** relating to three paragraphs with **two questions on each paragraph**. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

### Paragraph for Questions 29 and 30

The electrochemical cell shown below is a concentration cell.

M | $M^{2+}$ (saturated solution of a sparingly soluble salt, $MX_2$) || $M^{2+}$ (0.001 mol dm$^{-3}$) | M

The emf of the cell depends on the difference in concentrations of $M^{2+}$ ions at the two electrodes.

The emf of the cell at 298 K is 0.059 V.

29. The solubility product ($K_{sp}$; mol$^3$ dm$^{-9}$) of $MX_2$ at 298 K based on the information available for the given concentration cell is (take $2.303 \times R \times 298/F = 0.059$ V)

(A) $1 \times 10^{-15}$ (B) $4 \times 10^{-15}$ (C) $1 \times 10^{-12}$ (D) $4 \times 10^{-12}$

**Answer (B)**

**Hints :** $0.059 = +\dfrac{0.059}{2} \log \dfrac{0.001}{(M^{2+})}$

$\log \dfrac{0.001}{[M^{2+}]} = 2$

$\dfrac{0.001}{[M^{2+}]} = 100$

$[M^{2+}] = 10^{-5}$

$K_{sp} = 4 \times (10^{-5})^3 = 4 \times 10^{-15}$

30. The value of $\Delta G$(kJ mol$^{-1}$) for the given cell is (take 1 F = 96500 C mol$^{-1}$)

(A) –5.7 (B) 5.7 (C) 11.4 (D) –11.4

**Answer (D)**

**Hints :** $\Delta G = -nFE$

$= -2 \times 96500 \times 0.059$

$= -11387$ joule mol$^{-1}$

$\approx -11.4$ kJ mol$^{-1}$

**Paragraph for Questions 31 and 32**

Bleaching powder and bleach solution are produced on a large scale and used in several household products. The effectiveness of bleach solution is often measured by iodometry.

31. 25 mL of household bleach solution was mixed with 30 mL of 0.50 M KI and 10 mL of 4 N acetic acid. In the titration of the liberated iodine, 48 mL of 0.25 N $Na_2S_2O_3$ was used to reach the end point. The molarity of the household bleach solution is

(A) 0.48 M (B) 0.96 M (C) 0.24 M (D) 0.024 M

**Answer (C)**

**Hints :** $25 \times N = 48 \times 0.25$

$$N = \frac{48 \times 0.25}{25}$$

$$N = 0.48 \text{ M}$$

$$\Rightarrow \quad M = \frac{0.48}{2} = 0.24 \text{ M}$$

32. Bleaching powder contains a salt of an oxoacid as one of its components. The anhydride of that oxoacid is

(A) $Cl_2O$ (B) $Cl_2O_7$ (C) $ClO_2$ (D) $Cl_2O_6$

**Answer (A)**

**Hints :** Oxoacid is HOCl. The anhydride of oxoacid is $Cl_2O$.

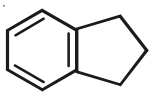
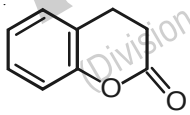
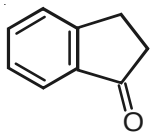
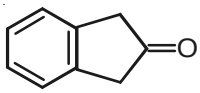
**Paragraph for Questions 33 and 34**

In the following reaction sequence, the compound J is an intermediate.

$$I \xrightarrow[CH_3COONa]{(CH_3CO)_2O} J \xrightarrow[(ii)\ SOCl_2\ (iii)\ anhyd.\ AlCl_3]{(i)\ H_2, Pd/C} K$$

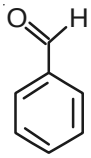
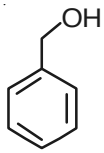
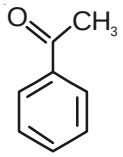
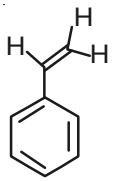
J ($C_9H_8O_2$) gives effervescence on treatment with $NaHCO_3$ and a positive Baeyer's test.

33. The compound K is

(A)  (B)  (C)  (D) 

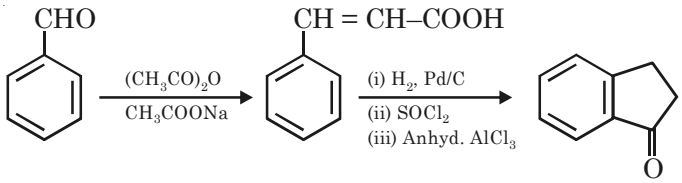
**Answer (C)**

34. The compound I is

(A)  (B)  (C)  (D) 

**Answer (A)**

**Hints :** For Q.No. 33 & 34.



## SECTION - III
## (Multiple Correct Answer(s) Type)

This section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct.**

35. With respect to graphite and diamond, which of the statement(s) given below is (are) correct?

(A) Graphite is harder than diamond

(B) Graphite has higher electrical conductivity than diamond

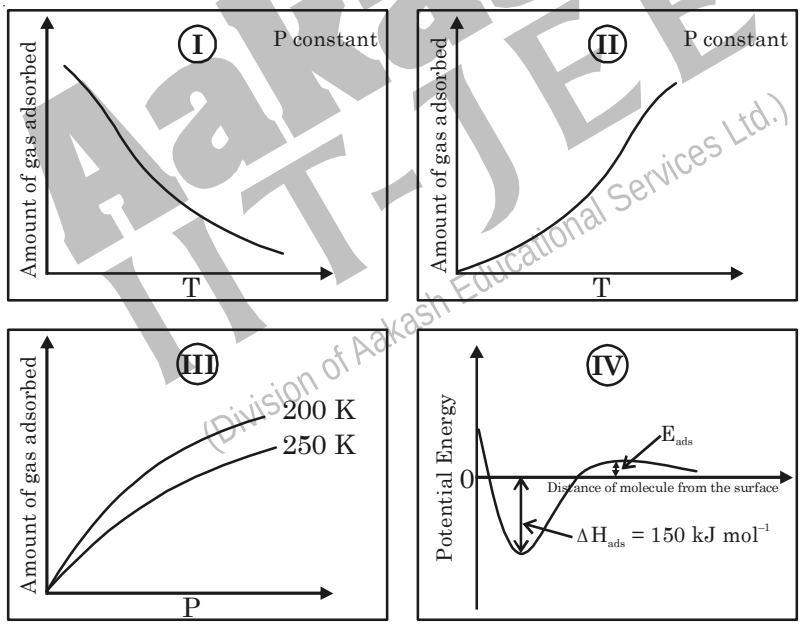
(C) Graphite has higher thermal conductivity than diamond

(D) Graphite has higher C–C bond order than diamond

**Answer (B, C, D)**

**Hints :** Property-wise graphite is softer than diamond. However both electrical and thermal conductivity of graphite is higher than that of diamond. Bond order (C–C) in graphite is 1.5 and higher than C–C bond order in diamond. Hence, answer is (B, C, D).

36. The given graphs/data **I**, **II**, **III** and **IV** represent general trends observed for different physisorption and chemisorption processes under mild conditions of temperature and pressure. Which of the following choice(s) about, **I**, **II**, **III** and **IV** is (are) correct?




(A) **I** is physisorption and **II** is chemisorption

(B) **I** is physisorption and **III** is chemisorption

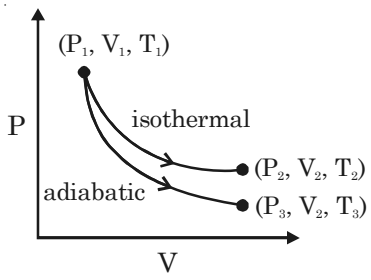
(C) **IV** is chemisorption and **II** is chemisorption

(D) **IV** is chemisorption and **III** is chemisorption

**Answer (A, C)**

**Hints :** Under mild conditions of temperatures extent of physisorption decreases with increasing temperature so graph (I) is correct. As initial temperature provided meets the necessary activation energy needed for chemical bond formation for chemisorption so graph (II) is also correct. Finally graph (IV) is correct for chemisorption as plot is now of potential energy vs distance of molecule from surface. Hence answer is (A, C).

37. The reversible expansion of an ideal gas under adiabatic and isothermal conditions is shown in the figure. Which of the following statement(s) is (are) correct?



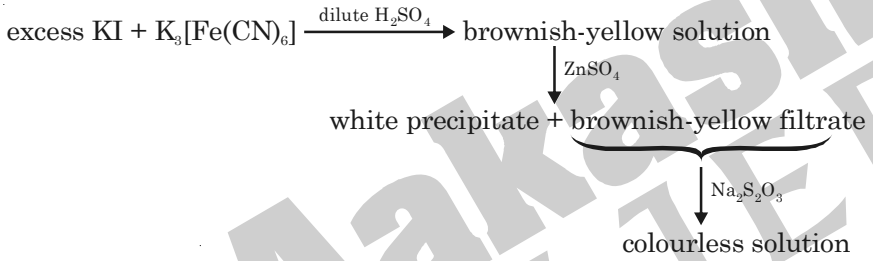

(A) $T_1 = T_2$ (B) $T_3 > T_1$

(C) $w_{isothermal} > w_{adiabatic}$ (D) $\Delta U_{isothermal} > \Delta U_{adiabatic}$

**Answer (A, C, D)**

**Hints :** Obviously here $T_1 > T_3$ from the graph and $T_1 = T_2$ as process curve is isothermal. $\Delta U_{isothermal} = 0$ and that for an adiabatic process is negative so option (D) is correct. Now $W_{isothermal} > W_{adiabatic}$ also hence answer is (A, C, D).

38. For the given aqueous reactions, which of the statement(s) is (are) true?




(A) The first reaction is a redox reaction

(B) White precipitate is $Zn_3[Fe(CN)_6]_2$

(C) Addition of filtrate to starch solution gives blue colour

(D) White precipitate is soluble in NaOH solution

**Answer (A, C, D)**

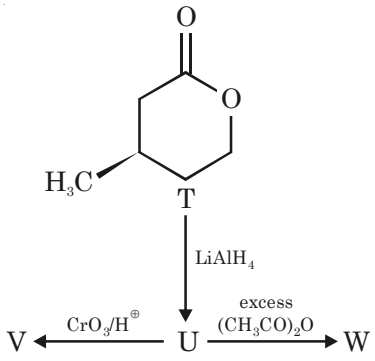
**Hints :** $2[\text{Excess KI}] + 2K_3[Fe(CN)_6] \longrightarrow 2K_4[Fe(CN)_6] + I_2 \xrightarrow{ZnSO_4} \underset{\text{Brown-yellow filtrate}}{I_2} + \underset{\text{(white ppt)}}{K_2Zn_3[Fe(CN)_6]_2}$

$Zn^{+2} + 4NaOH \longrightarrow \underset{\text{Soluble}}{[Zn(OH)_4]^{2-}} + 4Na^+$

$2Na_2S_2O_3 + I_2 \longrightarrow \underset{\text{colourless solution}}{Na_2S_4O_6} + 2NaI$

So answer is (A, C, D).

39. With reference to the scheme given, which of the statement(s) about T, U, V and W is (are) correct?



(A) T is soluble in hot aqueous NaOH

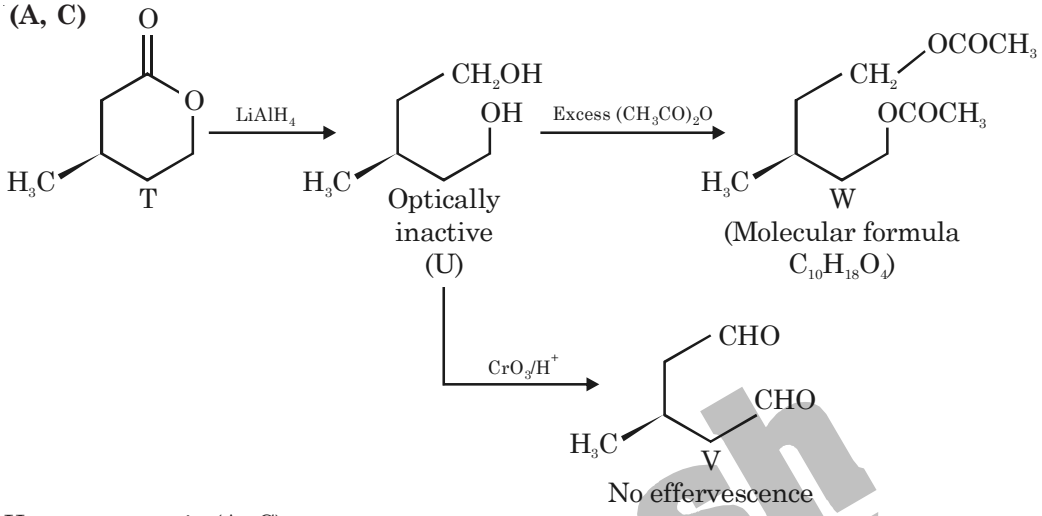
(B) U is optically active

(C) Molecular formula of W is $C_{10}H_{18}O_4$

(D) V gives effervescence on treatment with aqueous $NaHCO_3$

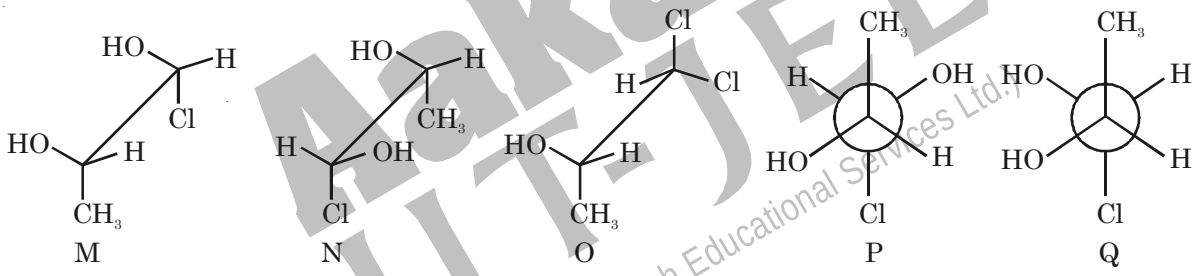
**Answer (A, C)**

**Hints :**




Hence answer is (A, C).

40. Which of the given statement(s) about N, O, P and Q with respect to M is (are) correct?




(A) M and N are non-mirror image stereoisomers (B) M and O are identical

(C) M and P are enantiomers (D) M and Q are identical

**Answer (A, B, C)**

**Hints :** With reference to structure (M), if the chiral carbon bearing the methyl group is C–1 and that bearing the chlorine is C–2 then

| Structure | Configuration of C–1, C–2 respectively |
|---|---|
| M | S, R |
| N | R, R |
| O | S, R |
| P | R, S |
| Q | R, R |

Now from the above configurations we learn that

(A) M and N are diastereomers *i.e.,* non-mirror image stereoisomers.

(B) M and O are surely identical as their configurations are same.

(C) M and P are also enantiomers due to opposite configurations.

But M and Q are again not identical but diastereomeric.

Hence answer is (A, B, C).

# PART–III : MATHEMATICS

## SECTION - I
## (Single Correct Answer Type)

This section contains **8 multiple choice questions.** Each question has 4 choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

41. The equation of a plane passing through the line of intersection of the planes $x + 2y + 3z = 2$ and $x - y + z = 3$ and at a distance $\frac{2}{\sqrt{3}}$ from the point (3, 1, –1) is

(A) $5x - 11y + z = 17$ (B) $\sqrt{2}x + y = 3\sqrt{2} - 1$

(C) $x + y + z = \sqrt{3}$ (D) $x - \sqrt{2}y = 1 - \sqrt{2}$

**Answer (A)**

**Hints :** Required equation of plane is given by

$P_1 + \lambda P_2 = 0$

$\Rightarrow (x + 2y + 3z - 2) + \lambda(x - y + z - 3) = 0$

$\Rightarrow (1 + \lambda)x + (2 - \lambda)y + (3 + \lambda)z - (2 + 3\lambda) = 0$ ...(i)

As, $\left|\frac{3(1+\lambda) + (2-\lambda) - (3+\lambda) - (2+3\lambda)}{\sqrt{(1+\lambda)^2 + (2-\lambda)^2 + (3+\lambda)^2}}\right| = \frac{2}{\sqrt{3}}$

$\Rightarrow \lambda = -\frac{7}{2}$

So, using (i), equation of plane becomes

$\Rightarrow \boxed{5x - 11y + z = 17}$

42. If $\vec{a}$ and $\vec{b}$ are vectors such that $|\vec{a} + \vec{b}| = \sqrt{29}$ and $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = (2\hat{i} + 3\hat{j} + 4\hat{k}) \times \vec{b}$, then a possible value of $(\vec{a} + \vec{b}).(-7\hat{i} + 2\hat{j} + 3\hat{k})$ is

(A) 0 (B) 3 (C) 4 (D) 8

**Answer (C)**

**Hints :** $\vec{a} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) + \vec{b} \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = \vec{0}$

$\Rightarrow (\vec{a} + \vec{b}) \times (2\hat{i} + 3\hat{j} + 4\hat{k}) = \vec{0}$

$\Rightarrow \vec{a} + \vec{b} = \lambda(2\hat{i} + 3\hat{j} + 4\hat{k})$

Given $|\vec{a} + \vec{b}| = \sqrt{29}$

$\Rightarrow \lambda = 1$

So, $\vec{a} + \vec{b} = 2\hat{i} + 3\hat{j} + 4\hat{k}$

Now, $(\vec{a} + \vec{b}) \cdot (-7\hat{i} + 2\hat{j} + 3\hat{k}) = 4$

43. Let $PQR$ be a triangle of area $\Delta$ with $a = 2$, $b = \frac{7}{2}$ and $c = \frac{5}{2}$, where $a$, $b$ and $c$ are the lengths of the sides of the triangle opposite to the angles at $P$, $Q$ and $R$ respectively. Then $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P}$ equals

(A) $\frac{3}{4\Delta}$ (B) $\frac{45}{4\Delta}$ (C) $\left(\frac{3}{4\Delta}\right)^2$ (D) $\left(\frac{45}{4\Delta}\right)^2$

**Answer (C)**

**Hints :** As, $\frac{2\sin P - \sin 2P}{2\sin P + \sin 2P} = \frac{1-\cos P}{1+\cos P} = \tan^2\left(\frac{P}{2}\right)$

Now, $a = 2$, $b = \frac{7}{2}$, $c = \frac{5}{2}$

so, semi perimeter = $s = \frac{a+b+c}{2} = 4$

Now, $\tan\frac{P}{2} = \frac{(s-b)(s-c)}{\Delta} = \frac{3}{4\Delta}$

so, $\tan^2\left(\frac{P}{2}\right) = \left(\frac{3}{4\Delta}\right)^2$

44. Four fair dice $D_1$, $D_2$, $D_3$ and $D_4$, each having six faces numbered 1, 2, 3, 4, 5 and 6, are rolled simultaneously. The probability that $D_4$ shows a number appearing on one of $D_1$, $D_2$ and $D_3$ is

(A) $\frac{91}{216}$ (B) $\frac{108}{216}$ (C) $\frac{125}{216}$ (D) $\frac{127}{216}$

**Answer (A)**

**Hints :** Required probability

= Getting equal number on two dice + equal number on three dice + equal number on four dice

$= \frac{{}^6C_1 \times 3 \times 5 \times 5}{6^4} + \frac{{}^6C_1 \times 3 \times 5}{6^4} + \frac{{}^6C_1}{6^4}$

$= \frac{91}{216}$

45. The value of the integral

$\int_{-\pi/2}^{\pi/2}\left(x^2 + \ln\frac{\pi+x}{\pi-x}\right)\cos x\,dx$ is

(A) 0 (B) $\frac{\pi^2}{2} - 4$ (C) $\frac{\pi^2}{2} + 4$ (D) $\frac{\pi^2}{2}$

**Answer (B)**

**Hints :** $\int_{-\frac{\pi}{2}}^{\frac{\pi}{2}}\left(x^2 + \ln\left(\frac{\pi+x}{\pi-x}\right)\right)\cos x.dx$

$= 2\int_0^{\frac{\pi}{2}} x^2\cos x\,dx$ $\left(\because \ln\left(\frac{\pi-x}{\pi+x}\right)\cos x \text{ is odd function}\right)$

$= 2[x^2 \sin x - 2x.(-\cos x) + 2(-\sin x)]_0^{\frac{\pi}{2}}$

$= \frac{\pi^2}{2} - 4$

46. If $P$ is a 3 × 3 matrix such that $P^T = 2P + I$, where $P^T$ is the transpose of $P$ and $I$ is the 3 × 3 identity matrix, then there exists a column matrix $X = \begin{bmatrix} x \\ y \\ z \end{bmatrix} \neq \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ such that

(A) $PX = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix}$ (B) $PX = X$ (C) $PX = 2X$ (D) $PX = -X$

**Answer (D)**

**Hints :** $P^T = 2P + I$

$\Rightarrow P = 2P^T + I$

$\Rightarrow P = 2[2P + I] + I$

$\Rightarrow P = 4P + 3I$

$\Rightarrow 3P = -3I$

$\Rightarrow PX = -X$

47. Let $a_1, a_2, a_3, \ldots$ be in harmonic progression with $a_1 = 5$ and $a_{20} = 25$. The least positive integer $n$ for which $a_n < 0$ is

(A) 22 (B) 23 (C) 24 (D) 25

**Answer (D)**

**Hints :** As, $\frac{1}{a_{20}} = \frac{1}{25} = \frac{1}{5} + 19d$

$\Rightarrow 19d = \left(\frac{1}{25} - \frac{1}{5}\right)$

$\Rightarrow d = -\frac{4}{25 \times 19}$

Also, $\frac{1}{a_n} = \frac{1}{5} - \frac{4}{25 \times 19}(n-1) < 0$

$\Rightarrow \frac{4(n-1)}{25 \times 19} > \frac{1}{5}$

$\Rightarrow n > \frac{99}{4}$

$\therefore$ The least integral value of n is 25

48. Let $\alpha(a)$ and $\beta(a)$ be the roots of the equation

$(\sqrt[3]{1+a} - 1)x^2 + (\sqrt{1+a} - 1)x + (\sqrt[6]{1+a} - 1) = 0$ where $a > -1$.

Then $\lim_{a \to 0^+} \alpha(a)$ and $\lim_{a \to 0^+} \beta(a)$ are

(A) $-\frac{5}{2}$ and 1 (B) $-\frac{1}{2}$ and –1 (C) $-\frac{7}{2}$ and 2 (D) $-\frac{9}{2}$ and 3

**Answer (B)**

**Hints :** Let $\lim_{a\to 0^+} \alpha(a) = \alpha;\ \lim_{a\to 0^+} \beta(a) = \beta$

as, $\alpha + \beta = \lim_{a\to 0^+} -\left\{\dfrac{(1+a)^{\frac{1}{2}} - 1}{(1+a)^{\frac{1}{3}} - 1}\right\} = \dfrac{-3}{2}$

also, $\alpha\beta = \lim_{a\to 0^+} \dfrac{(1+a)^{\frac{1}{6}} - 1}{(1+a)^{\frac{1}{3}} - 1} = \dfrac{1}{2}$

Therefore, $\alpha = \dfrac{-1}{2}, \beta = -1$

## SECTION - II

## (Paragraph Type)

This section contains **6 multiple choice questions** relating to three paragraphs with two questions on each paragraph. Each question has four choices (A), (B), (C) and (D) out of which **ONLY ONE is correct.**

### Paragraph for Questions 49 and 50

Let $f(x) = (1 - x)^2 \sin^2 x + x^2$ for all $x \in \mathbb{R}$, and let $g(x) = \int_1^x \left(\dfrac{2(t-1)}{t+1} - \ln t\right) f(t)dt$ for all $x \in (1, \infty)$.

49. Which of the following is true?

(A) $g$ is increasing on $(1, \infty)$

(B) $g$ is decreasing on $(1, \infty)$

(C) $g$ is increasing on $(1, 2)$ and decreasing on $(2, \infty)$

(D) $g$ is decreasing on $(1, 2)$ and increasing on $(2, \infty)$

**Answer (B)**

**Hints :** $g'(x) = \left(\dfrac{2(x-1)}{x+1} - \ln x\right) f(x)$

Now clearly $f(x) \geq 0\ \forall\ x \in R$

$\Rightarrow \dfrac{2(x-1) - (x+1)\ln x}{x+1}$

Now as $x > 1$

so, $2(x - 1) - (x + 1)\ln x < 0$

50. Consider the statements

**P:** There exists some $x \in \mathbb{R}$ such that $f(x) + 2x = 2(1 + x^2)$

**Q:** There exists some $x \in \mathbb{R}$ such that $2f(x) + 1 = 2x(1 + x)$

Then

(A) Both **P** and **Q** are true

(B) **P** is true and **Q** is false

(C) **P** is false and **Q** is true

(D) Both **P** and **Q** are false

**Answer (C)**

**Hints :** **P:** $x^2 + 2x + (1 - x)^2 \sin^2 x = 2(1 + x^2)$

$\Rightarrow \sin^2 x(1 - x)^2 = x^2 - 2x + 2$

$\Rightarrow \sin^2 x(1 - x)^2 = (1 - x)^2 + 1$

Which can **never** be true

**Q:** $2x^2 + 2\sin^2 x\,(1 - x)^2 = 2x^2 + 2x$

$\Rightarrow \sin^2 x(1 - x)^2 = 2x$

$\Rightarrow \sin^2 x = \dfrac{2x}{(1-x)^2}$

Clearly, a solution is possible.

**Paragraph for Questions 51 and 52**

Let $a_n$ denote the number of all $n$-digit positive integers formed by the digits 0, 1 or both such that no consecutive digits in them are 0. Let $b_n$ = the number of such $n$-digit integers ending with digit 1 and $c_n$ = the number of such $n$-digit integers ending with digit 0.

51. Which of the following is correct?

(A) $a_{17} = a_{16} + a_{15}$ (B) $c_{17} \neq a_{16} + c_{15}$ (C) $b_{17} \neq b_{16} + c_{16}$ (D) $a_{17} = c_{17} + b_{16}$

**Answer (A)**

**Hints :** Let $k$ be the number of 1's and $r$ be the number of zeros and the first digit obviously has to be "1". Clearly

$a_n = 1 + {}^{n-2}C_1 + {}^{n-3}C_2 + {}^{n-4}C_3 + \ldots\ldots + {}^{k}C_r \quad (k \geq r)$

$\Rightarrow a_{n-1} = 1 + {}^{n-3}C_1 + {}^{n-4}C_2 + {}^{n-5}C_3 + \ldots\ldots$

as $a_{n-2} = 1 + {}^{n-4}C_1 + {}^{n-5}C_2 + {}^{n-6}C_3 + \ldots\ldots$

Clearly, $\boxed{a_{n-2} + a_{n-1} = a_n}$

Hence, $a_{17} = a_{16} + a_{15}$

52. The value of $b_6$ is

(A) 7 (B) 8 (C) 9 (D) 11

**Answer (B)**

**Hints :** So, $b_n = 1 + {}^4C_1 + {}^3C_2 = 8$

**Paragraph for Questions 53 and 54**

A tangent $PT$ is drawn to the circle $x^2 + y^2 = 4$ at the point $P(\sqrt{3},1)$. A straight line $L$, perpendicular to $PT$ is a tangent to the circle $(x - 3)^2 + y^2 = 1$.

53. A common tangent of the two circles is

(A) $x = 4$ (B) $y = 2$ (C) $x + \sqrt{3}y = 4$ (D) $x + 2\sqrt{2}y = 6$

**Answer (D)**

**Hints :** Let m be slope of the common tangent

$y = mx \pm 2\sqrt{1 + m^2}$ ... (i)

$y = m(x - 3) \pm \sqrt{1 + m^2}$ ... (ii)

(i) and (ii) are identical

$$\frac{1}{1}=\frac{m}{m}=\frac{\pm 2\sqrt{1+m^2}}{-3m\pm\sqrt{1+m^2}}$$

$$-3m\pm\sqrt{1+m^2}=\pm 2\sqrt{1+m^2}$$

$\Rightarrow \quad m=\pm\frac{1}{2\sqrt{2}}$ or one tangent is vertical.

$\therefore$ Equation of common tangent is $x = 2$

For $m=-\frac{1}{2\sqrt{2}}$

$$y=\frac{-1}{2\sqrt{2}}x\pm 2\sqrt{1+\frac{1}{8}}$$

$$2\sqrt{2}y+x=6$$

Option (D) is correct

54. A possible equation of $L$ is

(A) $x-\sqrt{3}y=1$ (B) $x+\sqrt{3}y=1$ (C) $x-\sqrt{3}y=-1$ (D) $x+\sqrt{3}y=5$

**Answer (A)**

**Hints :** Equation of tangent at $P(\sqrt{3},1)$ is $\sqrt{3}x+y-4=0$

Equation of L is $y=\frac{1}{\sqrt{3}}(x-3)\pm 1\sqrt{1+\frac{1}{3}}$

$$\sqrt{3}y=+(x-3)\pm 2$$

$\sqrt{3}y-x=-5$ or $\sqrt{3}y-x=-1$

Option (A) is correct

## SECTION - III
## (Multiple Correct Answer(s) Type)

This section contains **6 multiple choice questions.** Each question has four choices (A), (B), (C) and (D) out of which **ONE OR MORE are correct.**

55. Let $X$ and $Y$ be two events such that $P(X\mid Y)=\frac{1}{2}$, $P(Y\mid X)=\frac{1}{3}$ and $P(X\cap Y)=\frac{1}{6}$. Which of the following is (are) correct?

(A) $P(X\cup Y)=\frac{2}{3}$ (B) $X$ and $Y$ are independent

(C) $X$ and $Y$ are not independent (D) $P(X^c\cap Y)=\frac{1}{3}$

**Answer (A, B)**

**Hints :** $P(X)\cdot P(Y/X)=P(Y)\cdot P(X/Y)=P(X\cap Y)$

$$\Rightarrow \quad P(X)=\frac{1/6}{1/3}=\frac{1}{2}$$

$$P(Y)=\frac{1/6}{1/2}=\frac{1}{3}$$

$\therefore \quad P(X \cup Y) = \frac{1}{2} + \frac{1}{3} - \frac{1}{6} = \frac{2}{3}$

$P(X \cap Y) = \frac{1}{6}$

and $P(X) \cdot P(Y) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

$\Rightarrow$ $X$ and $Y$ are independent events

$P(X^C \cap Y) = P(X^C) \cdot P(Y) = \frac{1}{2} \times \frac{1}{3} = \frac{1}{6}$

56. If $f(x) = \int_0^x e^{t^2}(t-2)(t-3)dt$ for all $x \in (0, \infty)$, then

(A) $f$ has a local maximum at $x = 2$

(B) $f$ is decreasing on (2, 3)

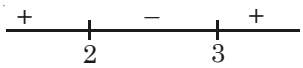
(C) There exists some $c \in (0, \infty)$ such that $f''(c) = 0$

(D) $f$ has a local minimum at $x = 3$

**Answer (A, B, C, D)**

**Hints :** $f(x) = \int_0^x e^{t^2}(t-2)(t-3)\,dt$

$f'(x) = e^{x^2}(x-2)(x-3)$




$f(x)$ has a local maxima at $x = 2$ and minima at $x = 3$

$\Rightarrow$ $f(x)$ is decreasing on (2, 3)

$f''(x) = e^{x^2}(2x-5) + 2xe^{x^2}(x^2 - 5x + 6)$

$= e^{x^2}(2x^3 - 10x^2 + 8x - 5)$

$\because$ $f'(0) = -5$ and $f''(x) \to \infty$ as $x \to \infty$

$\Rightarrow$ $f'(x)$ is zero for some $x \in (0, \infty)$

57. For every integer $n$, let $a_n$ and $b_n$ be real numbers. Let function $f : \mathbb{R} \to \mathbb{R}$ be given by

$$f(x) = \begin{cases} a_n + \sin \pi x, & \text{for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, & \text{for } x \in (2n-1, 2n) \end{cases}, \text{ for all integers } n.$$

If $f$ is continuous, then which of the following hold(s) for all $n$?

(A) $a_{n-1} - b_{n-1} = 0$ (B) $a_n - b_n = 0$

(C) $a_n - b_{n+1} = 1$ (D) $a_{n-1} - b_n = -1$

**Answer (B, D)**

**Hints :** $f(x) = \begin{cases} a_n + \sin \pi x, \text{ for } x \in [2n, 2n+1] \\ b_n + \cos \pi x, \text{ for } x \in (2n-1, 2n) \end{cases}$

$f(x) = \begin{cases} a_n + \sin \pi x, & x \in [2n, 2n+1] \\ b_n + \cos \pi x, & x \in (2n+1, 2n+2) \end{cases}$

For the function to be continuous at $x = 2n$

$a_n + \sin \pi.2n = b_n + \cos \pi.2n$

$a_n = b_n + 1$

$a_n - b_n = 1$

To be continuous at $x = 2n - 1$

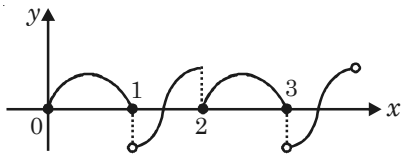
$a_{n-1} + \sin\pi(2n-1) = b_n + \cos\pi(2n-1)$

$a_{n-1} = b_{n-1}$

$a_{n-1} - b_n = -1$

**Alternative method**

$$f(x) = \begin{cases} a_n + \sin \pi x, & x \in [2n, 2n+1] \\ b_n + \cos \pi x, & x = (2n-1, 2n) \end{cases}$$




$\because$ $f(x)$ is continuous

$\therefore$ When $x \to$ odd number

$\Rightarrow$ $a_{n-1} = b_n - 1$

$\Rightarrow$ $a_{n-1} - b_n = -1$

When $x \to$ even number

$a_n = b_n + 1$

$\Rightarrow$ $a_n - b_n = -1$

58. If the straight lines $\frac{x-1}{2} = \frac{y+1}{k} = \frac{z}{2}$ and $\frac{x+1}{5} = \frac{y+1}{2} = \frac{z}{k}$ are coplanar, then the plane(s) containing these two lines is (are)

(A) $y + 2z = -1$ (B) $y + z = -1$ (C) $y - z = -1$ (D) $y - 2z = -1$

**Answer (B, C)**

**Hints :** Given planes are co-planar $\Rightarrow \begin{vmatrix} 2 & 0 & 0 \\ 2 & k & 2 \\ 5 & 2 & k \end{vmatrix} = 0$

$\Rightarrow$ $2(k^2 - 4) = 0$

$\Rightarrow$ $k = \pm 2$

Equation of plane containing, these two lines are

$$\begin{vmatrix} x-1 & y+1 & z \\ 2 & 2 & 2 \\ 5 & 2 & 2 \end{vmatrix} = 0 \text{ and } \begin{vmatrix} x-1 & y+1 & z \\ 2 & -2 & 2 \\ 5 & 2 & -2 \end{vmatrix} = 0$$

$\Rightarrow$ $y - z + 1 = 0$ and $y + 1 + z = 0$

59. If the adjoint of a $3 \times 3$ matrix $P$ is $\begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$, then the possible value(s) of the determinant of $P$ is (are)

(A) –2 (B) –1 (C) 1 (D) 2

**Answer (A, D)**

**Hints :** $\text{adj } P = \begin{bmatrix} 1 & 4 & 4 \\ 2 & 1 & 7 \\ 1 & 1 & 3 \end{bmatrix}$

$|\text{adj } P| = 1(3 - 7) - 4(6 - 7) + 4(2 - 1)$

$= -4 + 4 + 4 = 4$

$\because$ $|\text{adj } P| = |P|^2$

$\Rightarrow$ $|P|^2 = 4$

$\Rightarrow$ $|P| = \pm 2$

60. Let $f:(-1, 1) \to \mathbb{R}$ be such that $f(\cos 4\theta) = \dfrac{2}{2-\sec^2\theta}$ for $\theta \in \left(0, \dfrac{\pi}{4}\right) \cup \left(\dfrac{\pi}{4}, \dfrac{\pi}{2}\right)$. Then the value(s) of $f\left(\dfrac{1}{3}\right)$ is (are)

(A) $1-\sqrt{\dfrac{3}{2}}$ (B) $1+\sqrt{\dfrac{3}{2}}$ (C) $1-\sqrt{\dfrac{2}{3}}$ (D) $1+\sqrt{\dfrac{2}{3}}$

**Answer (A, B)**

**Hints :** Put $\cos 4\theta = \dfrac{1}{3}$

$\Rightarrow \cos 2\theta = \pm\sqrt{\dfrac{2}{3}}$

$2\cos^2\theta = 1 \pm \sqrt{\dfrac{2}{3}}$

$\cos^2\theta = \dfrac{1}{2} \pm \sqrt{\dfrac{1}{6}}$

$$f\left(\frac{1}{3}\right) = \frac{2\left(\frac{1}{2} \pm \sqrt{\frac{1}{6}}\right)}{2\left(\frac{1}{2} \pm \sqrt{\frac{1}{6}}\right) - 1}$$

$$= \frac{\sqrt{6}+2}{2} = 1+\sqrt{\frac{3}{2}}$$

or $\dfrac{-1+\frac{2}{\sqrt{6}}}{+\frac{2}{\sqrt{6}}} = 1-\sqrt{\dfrac{3}{2}}$



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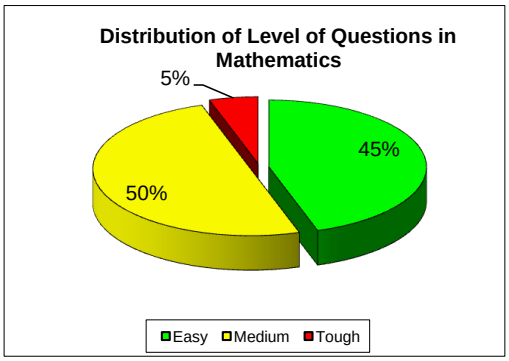
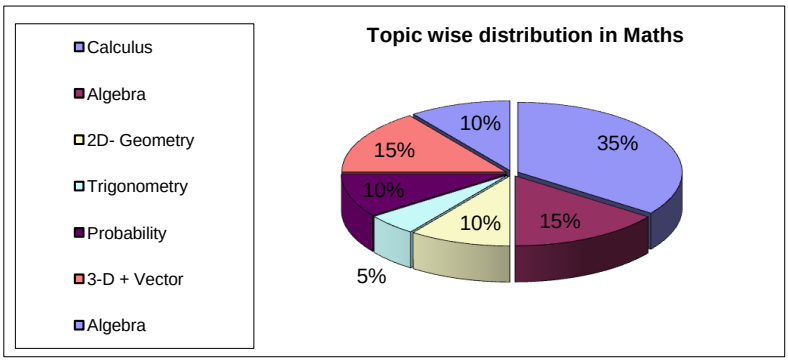
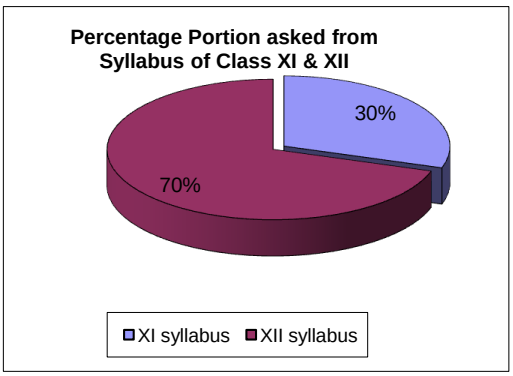
# Topic-wise Analysis of MATHEMATICS - IIT-JEE 2012 (Paper-2)

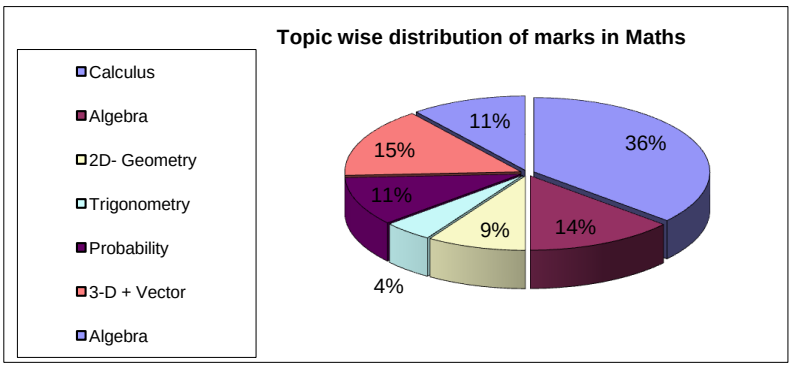
| | XII | XI | XI | XI | XII | XII | XII | |
|---|---|---|---|---|---|---|---|---|
| | **Calculus** | **Algebra** | **2D- Geometry** | **Trigonometry** | **Probability** | **3-D + Vector** | **Algebra** | ***Total*** |
| **Easy** | 2 | 1 | 0 | 1 | 1 | 3 | 1 | *9* |
| **Medium** | 4 | 2 | 2 | 0 | 1 | 0 | 1 | *10* |
| **Tough** | 1 | 0 | 0 | 0 | 0 | 0 | 0 | *1* |
| ***Total*** | *7* | *3* | *2* | *1* | *2* | *3* | *2* | ***20*** |
| **Markwise** | **24** | **9** | **6** | **3** | **7** | **10** | **7** | **66** |

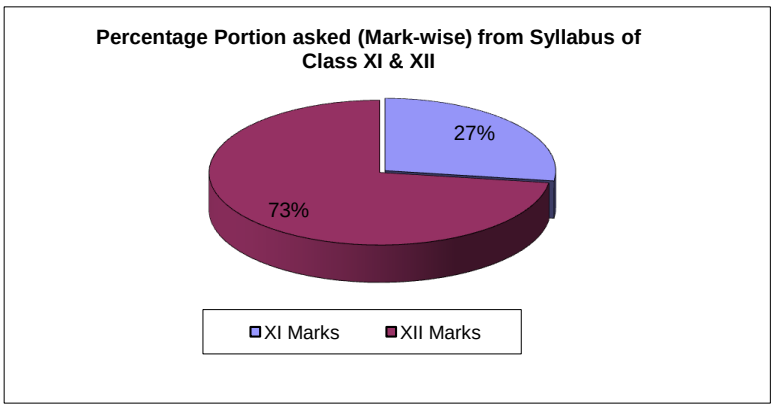
| | | | |
|---|---|---|---|
| **XI Marks** | **18** | **XII Marks** | **48** |
| **XI syllabus** | **6** | **XII syllabus** | **14** |



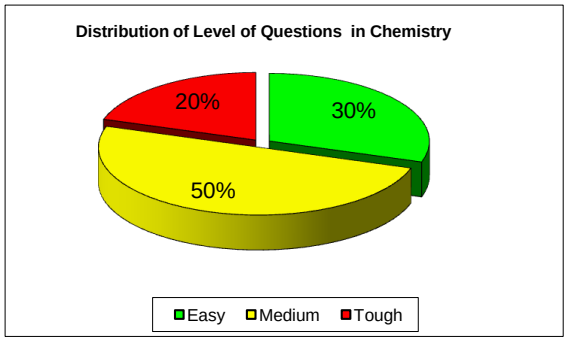



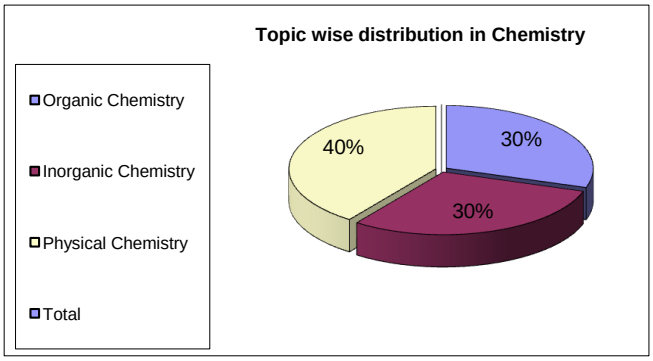



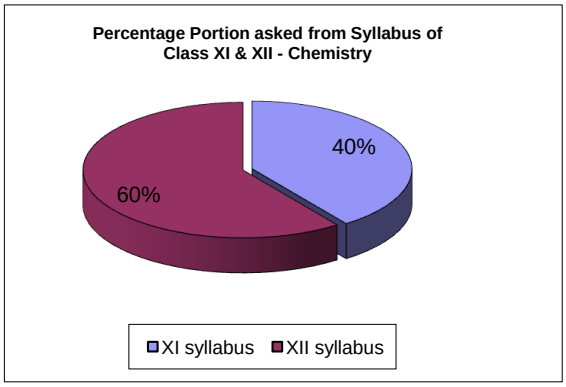



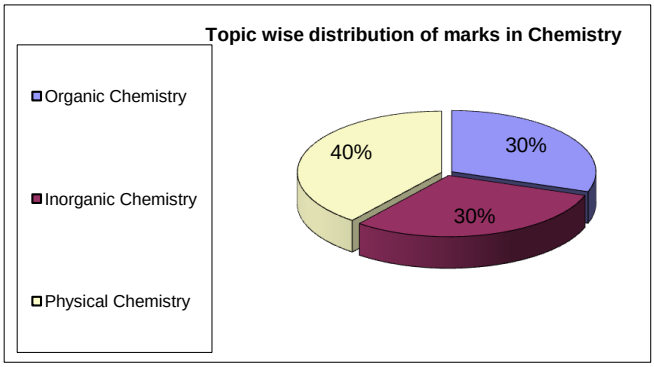
# Topic-wise Analysis of CHEMISTRY - IIT-JEE 2012
# (Paper-2)

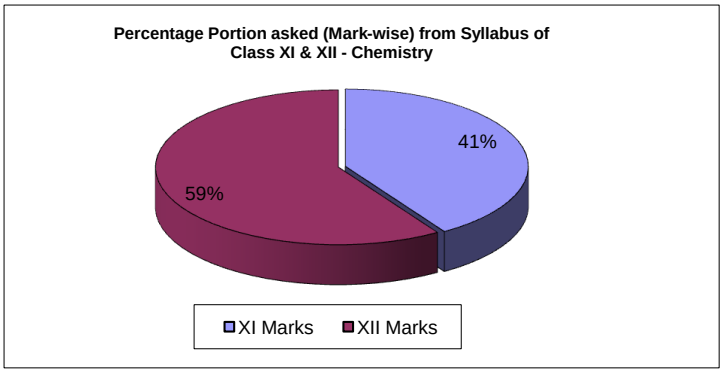
| | Organic Chemistry | Inorganic Chemistry | Physical Chemistry | *Total* |
|---|---|---|---|---|
| **Easy** | 1 | 2 | 3 | *6* |
| **Medium** | 4 | 2 | 4 | *10* |
| **Tough** | 1 | 2 | 1 | *4* |
| ***Total*** | *6* | *6* | *8* | ***20*** |
| **Markwise** | **20** | **20** | **26** | 66 |
| | **XI Marks** | **27** | **XII Marks** | **39** |
| | **XI syllabus** | **8** | **XII syllabus** | **12** |



# Topic-wise Analysis of PHYSICS - IIT-JEE 2012
# (Paper-2)

| | XII | XI | XII | XI | XII | XI | XII | |
|---|---|---|---|---|---|---|---|---|
| | **Electricity** | **Heat & Thermodynamics** | **Modern Physics** | **Mechanics** | **Optics** | **Oscillation & Waves** | **Magnetism** | ***Total*** |
| **Easy** | 1 | 1 | 0 | 0 | 0 | 1 | 2 | *5* |
| **Medium** | 1 | 0 | 2 | 4 | 2 | 0 | 2 | *11* |
| **Tough** | 0 | 0 | 0 | 4 | 0 | 0 | 0 | *4* |
| ***Total*** | *2* | *1* | *2* | *8* | *2* | *1* | *4* | ***20*** |
| **Markwise** | **7** | **3** | **6** | **27** | 6 | 3 | 14 | 66 |

| | | | |
|---|---|---|---|
| **XI Marks** | **33** | **XII Marks** | **33** |
| **XI syllabus** | **10** | **XII syllabus** | **10** |