USN 1 [redacted]



# First Semester B.E. Degree Examination, December 2011
# Engineering Mathematics - I

Time: 3 hrs. Max. Marks:100

**Note: 1.** ***Answer any FIVE full questions, choosing at least two from each part.***
**2.** ***Answer all objective type questions only on OMR sheet page 5 of the answer booklet.***
**3.** ***Answer to objective type questions on sheets other than OMR will not be valued.***



## PART - A

1 a. Choose your answers for the following : **(04 Marks)**

i) If $y = \frac{x}{x-1}$, then $y_n$ is

A) $\frac{(-1)^{n-1}n!}{(x-1)^{n+1}}$ B) $\frac{(-1)^n n!}{(x-1)^{n+1}}$ C) $\frac{(-1)^n (n+1)!}{(x-1)^{n+1}}$ D) $\frac{(-1)^n n!}{(x-1)^n}$

ii) If $y = \log(ax+b)$, then $y_n$ is

A) $\frac{(-1)^n n! a^n}{(ax+b)^n}$ B) $\frac{(-1)^{n-1} n! a^n}{(ax+b)^{n+1}}$ C) $\frac{(-1)^{n-1}(n-1)! a^n}{(ax+b)^n}$ D) $\frac{(-1)^n (n-1)! a^n}{(ax+b)^{n+1}}$

iii) If $f(x) = \sin x$, $x \in (0, \pi)$, then by Rolle's theorem the value of 'x', where the Tangent is parallel to x – axis.

A) 0 B) $\pi/2$ C) $\pi/3$ D) $\pi/4$

iv) Expansion of log (1+x) in powers of x is

A) $x + \frac{x^2}{2} + \frac{x^3}{3} + \ldots$ B) $x - \frac{x^2}{2} + \frac{x^3}{3} - \frac{x^4}{4} + \ldots$

C) $1 - \frac{x}{1!} + \frac{x^2}{2!} - \frac{x^3}{3!} + \ldots$ D) $\frac{x}{1!} - \frac{x^2}{2!} + \frac{x^3}{3!} - \frac{x^4}{4!} + \ldots$

a. If $x = \tan(\log y)$, show that $(1+x^2)y_{n+1} + (2nx-1)y_n + n(n-1)y_{n-1} = 0$. **(04 Marks)**

b. State and prove Cauchy's mean value theorem. **(06 Marks)**

c. Expand $f(x) = \sin(e^x - 1)$ in power's of 'x' upto the terms containing $x^4$. **(06 Marks)**

2 a. Choose your answers for the following : **(04 Marks)**

i) The indeterminate form of $\underset{x \to 1}{\text{Lt}} \left( \frac{x}{x-1} - \frac{(x-1)}{\log x} \right)$ is

A) $\infty - \infty$ B) $\frac{0}{0}$ C) $\frac{\infty}{\infty}$ D) None of these

ii) The angle between the radius vector and the tangent to the curve $r = k\, e^{\theta \cot\alpha}$, where K and α are constants, is :

A) K B) θ C) α D) O

iii) The Pedal equation of the curve $r = a\theta$ is.

A) $p^2 = ar$ B) $\frac{1}{p^2} = \frac{a}{r^2}$ C) $\frac{1}{p^2} = \frac{1}{r^2} + a^2$ D) $\frac{1}{p^2} = \frac{1}{r^2} + \frac{a^2}{r^4}$

iv) The radius of curvature at any point 't' on the curve defined by $x = f(t)$, $y = \phi(t)$ is given by

A) $\frac{[(x')^2 + (y')^2]^{3/2}}{x'y'' - y'x''}$ B) $\frac{x'y'' - y'x''}{[(x')^2 + (y')^2]^{3/2}}$ C) $\frac{(x')^2 + (y')^2}{(x'y'' - y'x'')^{3/2}}$ D) $\frac{(x'y'' - y'x'')^{3/2}}{(x')^2 + (y')^2}$

b. Find the angle of intersection between the curves $r^n \cos(n\theta) = a^n$ and $r^n \sin(n\theta) = b^n$. **(04 Marks)**

c. Show that the radius of curvature at any point '$\theta$' to the curve $x = a(\theta + \sin\theta)$, $y = a(1-\cos\theta)$, is $4a\cos(\theta/2)$. **(06 Marks)**

d. Evaluate $\underset{x\to 0}{\mathrm{Lt}}\left(\frac{a^x+b^x+c^x}{3}\right)^{1/x}$. **(06 Marks)**

3 a. Choose your answers for the following : **(04 Marks)**

i) If $u = x^{y-1}$, then $\frac{\partial u}{\partial y}$ is

A) $x^{y-1}\log x$ B) $(y-1)x^{y-2}$ C) $x^{y-1}\log y$ D) $x^y \log x$

ii) If $Z = f(u, v)$, where $u = x + ct$ and $v = x - ct$, then $\frac{\partial z}{\partial t}$ is given by

A) $\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}$ B) $\frac{\partial z}{\partial u} + \frac{\partial z}{\partial v}$ C) $c\left(\frac{\partial z}{\partial u} - \frac{\partial z}{\partial v}\right)$ D) $c\left(\frac{\partial z}{\partial v} - \frac{\partial z}{\partial u}\right)$

iii) If $x = u(1-v)$, $y = uv$, then $J\left(\frac{x, y}{u, v}\right)$ is equal to

A) $u$ B) $\frac{1}{u}$ C) $uv$ D) $u/v$

iv) The necessary condition for the function $f(x, y)$ to possess extreme values is

A) $f_x = f_y = 0$ B) $f_{xx} - f_{yy} = 0$ C) $(f_{xx})(f_{yy}) - f^2_{xy} = 0$ D) $f_x > 0, f_y > 0$

b. If $u = f\left(\frac{y-x}{xy}, \frac{z-x}{xz}\right)$, find $x^2\frac{\partial u}{\partial x}$. **(04 Marks)**

c. If $x + y + z = u$, $y + z = v$ and $z = uvw$, show that $J\left(\frac{x, y, z}{u, v, w}\right) = uv$. **(06 Marks)**

d. The Horse power required to propel a steamer is proportional to the square of the distance and cube of the velocity. If the distance is increased by 4% and velocity increased by 3%, find the percentage of increase in the Horse power. **(06 Marks)**

4 a. Choose your answers for the following : **(04 Marks)**

i) If $\vec{R} = xi + yj + zk$, $|\vec{R}| = r$, then $\nabla r^2$ is equal to

A) $\frac{\vec{R}}{r^2}$ B) $\frac{-\vec{R}}{2}$ C) $\frac{\vec{R}}{r}$ D) $2\vec{R}$

ii) If $\vec{F} = 3x^2 i - xyj + (a-3)xz\,k$ is solenoidal, then 'a' is equal to

A) 0 B) -2 C) 2 D) 3

iii) If $\vec{A} = x^2 i + y^2 j + z^2 k$, then curl $\vec{A}$ is given by

A) $2xi + 2yj + +2zk$ B) 0 C) $\frac{xi + yj + zk}{2}$ D) $2x + 2y + 2z$

iv) The scale factors for cylindrical coordinate system $(\rho\ \phi\ z)$ are given by

A) $(\rho, 1, 1)$ B) $(1, \rho, 1)$ C) $(1, 1, \rho)$ D) None of these

b. Prove that $\nabla.\phi\vec{F} = \nabla\phi.\vec{F} + \phi(\nabla.\vec{F})$. **(04 Marks)**

c. If $\vec{F} = 2xy^3z^4 i + 3x^2y^2z^4 j + 4x^2y^3z^3 k$, find i) $(\nabla.\vec{F})$ ii) $\nabla\times\vec{F}$. **(06 Marks)**

d. Obtain the expression for $\nabla.\vec{F}$ in orthogonal curvilinear coordinate system $(u_1\ u_2\ u_3)$. **(06 Marks)**

## PART – B

5 a. Choose your answers for the following : (04 Marks)

i) Given $\int_0^1 x^n dx = \frac{1}{x+1}$, then $\frac{d^2}{dx^2}\int_0^1 x^n dx$ gives

A) $\int_0^1 (\log x)^2 x^n dx = \frac{2}{(1+n)^2}$ B) $\int_0^1 (\log x)^2 x^n dx = \frac{2}{(1+n)^3}$

C) $\int_0^1 (\log x)^n x^n dx = \frac{2}{(1+n)^2}$ D) $\int_0^1 (\log x)^2 x^n dx = \frac{-2}{(1+n)^3}$

ii) The value of the integral $\int_0^{\pi} \sin^6 x \cos^5 x \, dx$ is

A) 0 B) $\frac{8}{693}$ C) $\frac{8\pi}{693}$ D) None of these

iii) The volume of the solid generated by revolving the curve r = a (1 + Cosθ) about the line θ = 0 is given by

A) $\frac{2\pi}{3}a^3\int_0^{\pi}(1+\cos\theta)^3 \sin\theta \, d\theta$ B) $\frac{2\pi}{3}a^3\int_0^{\pi}(1+\cos\theta)^3 \cos\theta \, d\theta$

C) $\frac{2\pi}{3}a^3\int_0^{2\pi}(1+\cos\theta)^3 \sin\theta \, d\theta$ D) $\frac{4\pi a^3}{3}$

iv) The entire length of the asteroid $x^{2/3} + y^{2/3} = a^{2/3}$ is

A) 4a B) 8a C) 6a D) 3a

b. Obtain the reduction formula of the integral $\int \cos^n x \, dx$. (04 Marks)

c. Using Leibnitz rule under differentiation under integral sign, evaluate $\int_0^{\pi} \frac{\log(1+2\cos x)}{\cos x} dx$. (06 Marks)

d. Find the surface generated by revolving the cycloid x = a (θ - Sinθ), y = a (1 - Cosθ) about its base, (consider one arc in the 1st quadrant). (06 Marks)

6 a. Choose your answers for the following : (04 Marks)

i) The general solution of the differential equation $\frac{dy}{dx} = \sec\left(\frac{y}{x}\right) + \frac{y}{x}$ is

A) Tan y/x – logx = c B) Sin (y/x) – logx = c
C) Cosec (y/x) – logx = c D) Cos (y/x) – logx = c

ii) Integrating factor for the differential equation $\frac{dx}{dy} + \frac{2x}{y} = y^2$ is

A) $y^2$ B) $e^{x^2}$ C) $e^{2y}$ D) $e^{y^2}$

iii) The general solution of the differential equation (x – y) dx + (y – x) dy = 0 is

A) $\frac{x^2}{2} - y - \frac{y^2}{2} = c$ B) $\frac{x^2}{2} - y + \frac{y^2}{2} = c$ C) $\frac{x^2}{2} - yx + \frac{y^2}{2} = c$ D) None of these

iv) Given the differential equation of f(r, θ, c) = 0, we get differential equation of orthogonal trajectories by changing $r\frac{d\theta}{dr}$ by

A) $\frac{1}{r}\frac{dr}{d\theta}$ B) $-r^2\frac{dr}{d\theta}$ C) $\frac{-1}{r}\frac{dr}{d\theta}$ D) $r\frac{dr}{d\theta}$.

b. Solve $(x^2 - 4xy - 2y^2)\,dx + (y^2 - 4xy - 2x^2)\,dy = 0$. (04 Marks)

c. Solve $(x + 2y^3)\frac{dy}{dx} = y$. (06 Marks)

d. Find the orthogonal trajectories of the family of curves $\frac{x^2}{a^2} + \frac{y^2}{b^2+\lambda} = 1$ ('λ' being the parameter). (06 Marks)

7 a. Choose your answers for the following : **(04 Marks)**

i) The rank of the matrix $\begin{pmatrix} 6 & 1 & 3 & 8 \\ 4 & 2 & 6 & -1 \\ 10 & 3 & 9 & 7 \\ 16 & 4 & 12 & 15 \end{pmatrix}$ is equal to

A) 2 B) 3 C) 4 D) 1

ii) The exact solution of the system of equations $10x + y + z = 12$, $x + 10y + z = 12$, $x + y + 10z = 12$ by inspection is equal to

A) $[0\ 0\ 0]^T$ B) $[1\ 1\ 1]^T$ C) $[1\ 1\ -1]^T$ D) $[-1\ -1\ -1]^T$

iii) If the given system of linear equations in 'n' variables is consistant then the number of linearly independent solution is given by

A) n B) n - 1 C) r - n D) n − r

(Where 'r' stands for rank of co-efficient, matrix).

iv) The trivial solution for the given system of equations $qx - y + 4z = 0$, $4x - 2y + 3z = 0$, $5x + y - 6z = 0$ is

A) (1, 2, 0) B) (0 4 1) C) (0 0 0) D) (1 -5 0)

b. Using elementary row transformations find the rank of the matrix $\begin{pmatrix} 0 & 1 & -3 & -1 \\ 1 & 0 & 1 & 1 \\ 3 & 1 & 0 & 2 \\ 1 & 1 & -2 & 0 \end{pmatrix}$. **(04 Marks)**

c. Test for consistency and solve the system of equations $x + 4 + 3z = 0$, $x - y + z = 0$, $2x - y + 3z = 0$. **(06 Marks)**

d. Applying Gauss Jordan method solve $2x + 3y - z = 5$, $4x + 4y - 3z = 3$, $2x - 3y + 2z = 2$. **(06 Marks)**

8 a. Choose your answers for the following : **(04 Marks)**

i) The linear transformation $y = Ax$ is regular if

A) $|A| = 0$ B) $|A| = 1$ C) $|A| = -1$ D) $|A| \neq 0$

ii) The transformation $\xi = x\,\text{Cos}\alpha - y\,\text{Sin}\alpha$, $\eta = x\,\text{Sin}\alpha + y\,\text{Cos}\alpha$ is orthogonal then the inverse of the transformation matrix is given by

A) $\begin{pmatrix} \text{Cos}\alpha & \text{Sin}\alpha \\ -\text{Sin}\alpha & \text{Cos}\alpha \end{pmatrix}$ B) $\begin{pmatrix} \text{Cos}\alpha & -\text{Sin}\alpha \\ \text{Sin}\alpha & \text{Cos}\alpha \end{pmatrix}$ C) $\begin{pmatrix} \text{Sin}\alpha & \text{Cos}\alpha \\ \text{Cos}\alpha & -\text{Sin}\alpha \end{pmatrix}$ D) $\begin{pmatrix} -\text{Sin}\alpha & \text{Cos}\alpha \\ \text{Cos}\alpha & \text{Sin}\alpha \end{pmatrix}$

iii) The eigen vector 'x' of the matrix 'A' corresponding to eigen value 'λ' satisfy the equation

A) $AX = \lambda X$ B) $\lambda(A - X) = 0$ C) $XA - \lambda A = 0$ D) $|A - \lambda I|X = 0$

iv) Two square matrices A and B are similar if

A) $A = B$ B) $B = P^{-1}AP$ C) $A^1 = B^1$ D) $A^{-1} = B^{-1}$

b. Show that the transformation given below $y_1 = 2x_1 + x_2 + x_3$, $y_2 = x_1 + x_2 + 2x_3$, $y_3 = x_1 - 2x_3$ is regular and find the inverse transformation. **(04 Marks)**

c. Find the matrix P which diagonalizes the matrix $A = \begin{bmatrix} -1 & 1 & 2 \\ 0 & -2 & -1 \\ 0 & 0 & -3 \end{bmatrix}$. **(06 Marks)**

d. Reduce the quadratic form $x_1^2 + 3x_2^2 + 3x_3^2 - 2x_2x_3$ in to canonical form by an appropriate orthogonal transformation which transforms $x_1\ x_2\ x_3$ in terms of new variables $y_1\ y_2\ y_3$. **(06 Marks)**

* * * * *