## EAMCET Maths Practice Questions

## Examples with hints and short cuts from few important chapters

1. If the vectors $\mathrm{pi}-2 \mathrm{j}+5 \mathrm{k}, 2 \mathrm{i}-\mathrm{qj}+5 \mathrm{k}$ are collinear then $(\mathrm{p}, \mathrm{q})=$
1) 0
2) 1
3) -1
4) 2

Hint $: \frac{p}{2}=\frac{-2}{-q}=\frac{5}{5} \Rightarrow p=2, q=2$
2.If the vectors $2 \mathrm{i}-\mathrm{j}+\mathrm{k}, \mathrm{i}+2 \mathrm{j}-3 \mathrm{k}$ and $3 \mathrm{i}+\lambda \mathrm{j}+5 \mathrm{k}$ are coplanar then $\lambda=$

1) 0
2) 1
3) -1
4) 4

Hint :

$$
\left|\begin{array}{ccc}
2 & -1 & 1 \\
1 & 2 & -3 \\
3 & \lambda & 5
\end{array}\right|=0 \Rightarrow \lambda=4
$$

3. If $\bar{a}, \bar{b}, \bar{c}$ are non coplanar unit vector such that

$$
\bar{a} \times(\bar{b} \times \bar{c})=\frac{\bar{b}+\bar{c}}{\sqrt{2}} \text { then angle between } \bar{a}
$$

3) $\frac{\pi}{3}$
4) $\frac{\pi}{6}$

Hint : $\bar{a} \times(\bar{b} \times \bar{c})=(\bar{a} \cdot \bar{c}) \bar{b}-(\bar{a} \cdot \bar{b}) \bar{c}=\frac{\bar{b}}{\sqrt{2}}+\frac{\bar{c}}{\sqrt{2}}$
$\underset{\text { Comparing }}{\bar{a} \cdot \bar{c}=\frac{1}{\sqrt{2}} ; \bar{a} \cdot \bar{b}=\frac{-1}{\sqrt{2}}, ~}$

$$
\Rightarrow(a, b)=\frac{3 \pi}{4}
$$

4.If a,b,c are three non coplanar vectors and , $\quad p=\frac{b \times c}{(a b c)}, q=\frac{c \times a}{(-a b c)}, r=\frac{a \times b}{(a b c)}$ then

1) 3
2) 1
3) -1
4) 2

Hint: $(a \times b) \cdot p+(b+c) \cdot q+(c+a) r=$ $\qquad$

$$
\Sigma(a+b) p=\Sigma(a+b) \cdot \frac{b \times c}{(a b c)}=\sum \frac{(a \cdot b \times c)+b \cdot b \times c}{(a b c)}=\sum \frac{a b c+0}{a b c}=3
$$

5.ABC is equilateral triangle of side a then $\overline{A B} \cdot \overline{B C}+\overline{B C} \cdot \overline{C A}+\overline{C A} \cdot \overline{A B}$ is

1) $\frac{a^{2}}{2}$
2) $-3 \frac{a^{2}}{2}$
3) $-\frac{a^{2}}{2}$
4) 2

Hint: $\overline{A B}+\overline{B C}+\overline{C A}=0$

$$
\begin{aligned}
& \Rightarrow A B^{2}+B C^{2}+C A^{2}+2(A B \cdot B C+B C \cdot C A+C A \cdot A B)=0 \\
& a^{2}+a^{2}+a^{2}+2 \sum A B \cdot B C=0 \\
& \Rightarrow \sum A B \cdot B C=\frac{-3 a^{2}}{2}
\end{aligned}
$$

6.Forces of5,3 units acting along $6 \mathrm{i}+2 \mathrm{j}+3 \mathrm{k}$ and $3 \mathrm{i}-2 \mathrm{j}+6 \mathrm{k}$ respectively on a particle displaced from the point $(2,2,-1)$ to the point $(4,3,1)$. Then total work done.

1) $\frac{148}{7}$
2) $\frac{48}{7}$
3) -1
4) 2

$$
F=5\left(\frac{6 i+2 j+3 k}{\sqrt{36+4+9}}\right)+3\left(\frac{3 i-2 j+6 k}{\sqrt{9+4+36}}\right)
$$

$$
\begin{aligned}
& =\frac{39 i+4 j+33 k}{7} \\
& \bar{d}=(4 i+3 j+k)-(2 i+2 j-k)=2 i+j+2 k \\
& w=F \cdot d=\frac{148}{7}
\end{aligned}
$$

7. $\mathrm{a}, \mathrm{b}, \mathrm{c}$ are unit vectors such that $|a+b+c|=1$ and $a \perp b \bar{c}$ makes $\alpha, \beta$ angle with $\bar{a}, \bar{b}$ then $\cos \alpha+\cos \beta=$
1) 3
2) 1
3) -1
4) 2

Hint: $|a+b+c|=1$

$$
a^{2}+b^{2}+c^{2}+2(a \cdot b+b \cdot c+c \cdot a)=1
$$

$$
\begin{gathered}
1+1+1+2\left[0+1.1 . \cos \alpha+1.1 .^{\cos \beta}\right]=1 \\
\Rightarrow \cos \alpha+\cos \beta=-1
\end{gathered}
$$

8. $(a \times b) \times(c \times d)=5 \bar{c}+6 \bar{d}$ then the value of $\bar{a} \cdot \bar{b} \times(\bar{a}+\bar{c}+2 \bar{d})$
1) 0
2) 1
3) -1
4) 4
$(\bar{a} \times \bar{b}) \times(\bar{c} \times \bar{d})=(a \times b . d) \bar{c}-(a \times b . c) \bar{d}=5 \bar{c}+6 \bar{d}$

Comparing on either sides
$\Rightarrow(\mathrm{abd})=5 \quad \Rightarrow(\mathrm{abc})=-6$
$\bar{a} \cdot \bar{b} \times(\bar{a}+\bar{c}+2 \bar{d})$
$(a \cdot b \times \bar{a})+(\bar{a} \cdot \bar{b} \times c)+(\bar{a} \cdot b \times 2 d)$
$0-6+2(5)=-6+10=4$
9.. If the sum of the squares of the perpendicular distances of ' $p$ ' from coordinate axes is ' 12 ' then locus of ' $p$ ' is

1) 6
2) 13
3) -1
4) 2

Hint: $\left(\sqrt{y_{1}^{2}+z_{1}^{2}}\right)+\left(\sqrt{z_{1}^{2}+x_{1}^{2}}\right)+\left(\sqrt{x_{1}^{2}+y_{1}^{2}}\right)^{2}=12$

$$
\Rightarrow x_{1}^{2}+y_{1}^{2}+z_{1}^{2}=6
$$

10The ratio in which of plane divides the line segment joining $(-3,4,2)(2,1,3)$ is

1) $1: 2$
2) $1: 3$
3) $3: 2$
4) $2: 2$

Hint: ${ }^{-x_{1}}: x_{2}=3: 2$
11. If the extremities of a diagonal of a square are $[1,-2,3][2-3,5]$ then length of side

1) $\sqrt{6}$
2) 13
3) $\sqrt{3}$
4) 2

Hint: diagonal $\sqrt{(1-2)^{2}+(-2+3)^{2}+(3-5)^{2}}=\sqrt{1+1+4}=\sqrt{6}$

$$
\text { Side }=\frac{d}{\sqrt{2}}=\frac{\sqrt{6}}{\sqrt{2}}=\sqrt{3}
$$

12 If a line makes an angle of $\frac{\pi}{4}$ with positive direction of each x - axis and y -axis then the angle made by the line with z - axis is

1) $\frac{\pi}{2}$
2) $\frac{3 \pi}{4}$
3) $\frac{\pi}{3}$
4) $\frac{\pi}{6}$

Hint: $\cos ^{2} \alpha+\cos ^{2} \beta+\cos ^{2} \gamma=1$

$$
\begin{gathered}
\frac{1}{2}+\frac{1}{2}+\cos ^{2} \gamma=1 \\
\gamma=90^{\circ}
\end{gathered}
$$

13. If the plane $7 x+11 y+13 z=3003$ meets the coordinate axes at $A, B, C$ then the centroid of the $\triangle A B C$ is---
1) $(143,91,77)$
2) $0,273,0$
3) $(1,1,1)$
4) $(3,3,3)$

Hint : intercepts form $\frac{7 x}{3003}+\frac{11 y}{3003}+\frac{13 z}{3003}=1$

$$
A[429,0,0] B[0,273,0] C[0,0,231]
$$

centroid of triangle $\left(\frac{429+0+0}{3}, \frac{273}{3}, \frac{237}{3}\right)$

$$
\Rightarrow(143,91,77)
$$

14..If $P(1,1,0), Q(1,0,1)$ then the projection of $P Q$ on the plane $x+y+z=3$ is $\qquad$

1) $\sqrt{6}$
2) 13
3) $\sqrt{2}$
4) 2

Hint : $\mathrm{PQ}=\mathrm{OQ}-\mathrm{OP}=(0,-1,1)$
Plane drs $=(1,1,1)$
Angle $\cos \theta=\frac{0-1+1}{\sqrt{2} \sqrt{3}}=0 \Rightarrow \theta=90^{\circ}$
Projection of 'PQ' on the plane $P Q \sin \theta=$

$$
=\sqrt{0+1+1} \cdot \sin 90^{\circ}=\sqrt{2}
$$

15. Let $\vec{a}=\hat{i}-2 \hat{j}+3 \hat{k}, \vec{b}=2 \hat{i}+3 \hat{j}-\hat{k}$ and $\vec{c}=\lambda \hat{i}+\hat{j}+(2 \lambda-1) \hat{k}$. If $\vec{c}$ is parallel to the plane containing $\vec{a}, \vec{b}$, then $\lambda$ is equal to
1) 0
2) 1
3) -1
4) 2

Sol. Given $\vec{a}=\hat{i}-2 \hat{j}+3 \hat{k}, \vec{b}=2 \hat{i}+3 \hat{j}-\hat{k}$ and $\vec{c}=\lambda \hat{i}+\hat{j}+(2 \lambda-1) \hat{k}$
so vector $(\vec{a} \times \vec{b})$ also perpendicular to the vector $\vec{c}$, i.e, $\left(\theta=90^{\circ}\right)$
So, $(\vec{a} \times \vec{b}) \cdot \vec{c}$ should be equal to zero or $(\vec{a} \times \vec{b}) \cdot \vec{c}=0$

$$
\begin{aligned}
& \vec{a} \times \vec{b}=\left|\begin{array}{ccc}
\hat{i} & \hat{j} & \hat{k} \\
1 & -2 & 3 \\
2 & 3 & -1
\end{array}\right|=(2-9) \hat{i}+(6+1) \hat{j}+(3+4) \hat{k} \\
& =-7 \hat{i}+7 \hat{j}+7 \hat{k}
\end{aligned}
$$

Then from Equation (i) $(-7 \hat{i}+7 \hat{j}+7 \hat{k}) \cdot(\lambda \hat{i}+\hat{j}+(2 \lambda-1) \hat{k})=0$
$\Rightarrow \lambda=0$
Hence, the value of $\lambda$ is 0
16 If three unit vectors $\vec{a}, \vec{b}, \vec{c}$ satisfy $\vec{a}+\vec{b}+\vec{c}=\overrightarrow{0}$, then the angle between $\vec{a}$ and $\vec{b}$

1) $\frac{2 \pi}{3}$
2) $\frac{5 \pi}{6}$
3) $\frac{\pi}{3}$
4) $\frac{\pi}{6}$

Sol. Given condition is $\vec{a}+\vec{b}+\vec{c}=\overrightarrow{0}$ and $\vec{a}, \vec{b}, \vec{c}$ are unit vectors then $|\vec{a}|=|\vec{b}|=|\vec{c}|=1$

Let the angle between $\vec{a}$ and $\vec{b}$ is $\theta$
Now, from Equation $(\vec{a}+\vec{b})=-\vec{c}$

Squaring on both sides $(\vec{a}+\vec{b})=(\vec{c})^{2} \quad\left[\because(\vec{c})^{2}=|\vec{c}|^{2}\right]$
$\Rightarrow(\vec{a})^{2}+(\vec{b})^{2}+2(\vec{a}) \cdot(\vec{b})=|\vec{c}|^{2}$
$\Rightarrow|\vec{a}|^{2} \cdot|\vec{b}|^{2}+2 \vec{a} \cdot \vec{b}=|\vec{c}|^{2}$

$$
\Rightarrow \quad 2\{1.1 \cdot \cos \theta\}=-1 \Rightarrow \theta=\frac{2 \pi}{3}
$$

17. $(\vec{a}+2 \vec{b}-\vec{c}) \cdot(\vec{a}-\vec{b}) \times(\vec{a}-\vec{b}-\vec{c})$ is equal to
1) $-[\vec{a} \vec{b} \vec{c}]$
2) $2[\vec{a} \vec{b} \vec{c}]$
3) $3[\vec{a} \vec{b} \vec{c}]$
4) $\overrightarrow{0}$

Sol. $\quad \mathrm{V}=[\vec{a} \vec{b} \vec{c}]\left|\begin{array}{ccc}1 & 2 & -1 \\ 1 & -1 & 0 \\ 1 & -1 & -1\end{array}\right|=3[\vec{a} \vec{b} \vec{c}]$
18. If $\vec{u}=\vec{a}-\vec{b}, \vec{v}=\vec{a}+\vec{b},|\vec{a}|=|\vec{b}|=2$, then $|\vec{u} \times \vec{v}|$ is equal to

1) $2 \sqrt{16-(\vec{a} \cdot \vec{b})^{2}}$
2) $\sqrt{16-(\vec{a} \cdot \vec{b})^{2}}$
3) $2 \sqrt{4-(\vec{a} \cdot \vec{b})^{2}}$
4) $\sqrt{4-(\vec{a} \cdot \vec{b})^{2}}$

Sol. We have, $\vec{u}=\vec{a}-\vec{b}, \vec{v}=\vec{a}+\vec{b}$

$$
\begin{aligned}
& \Rightarrow \vec{u} \times \vec{v}=(\vec{a}-\vec{b}) \times(\vec{a}+\vec{b}) \\
& 0-\vec{b} \times \vec{a}+\vec{a} \times \vec{b}-0=-2 \vec{a} \times \vec{b} \Rightarrow|\vec{u} \times \vec{v}|=2|\vec{a} \times \vec{b}| \\
& =2 \sqrt{|\vec{a} \times \vec{b}|^{2}}=2 \sqrt{|\vec{a}|^{2}|\vec{b}|^{2} \sin ^{2} \theta|\hat{n}|^{2}}\{\because \hat{n}=\text { unit vector }|\hat{n}|=1\} \\
& =2 \sqrt{4.4 \sin ^{2} \theta .1}
\end{aligned}
$$

$=2 \sqrt{16-16\left(\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}\right)^{2}} \Rightarrow 2 \sqrt{16-(\vec{a} \cdot \vec{b})^{2}}$
19. If the angle $\theta$ between the vectors $\vec{a}=2 x^{2} \hat{i}+4 x \hat{j}+\hat{k}$ and $\vec{b}=7 \hat{i}-2 \hat{j}+x \hat{k}$ is such that $90^{\circ}<\theta<180^{\circ}$, then x lies in the interval

1) $\left(0, \frac{1}{2}\right)$
2) $\left(\frac{1}{2}, 1\right)$
3) $\left(1, \frac{3}{2}\right)$
4) $\left(\frac{1}{2}, \frac{3}{2}\right)$

Sol. Given $\vec{a}=2 x^{2} \hat{i}+4 x \hat{j}+\hat{k}, \vec{b}=7 i-2 \hat{j}+x \hat{k}$, also $90^{\circ}<\theta<180^{\circ}$

We know that, $\cos \theta=\frac{\vec{a} \cdot \vec{b}}{|\vec{a}||\vec{b}|}$

$$
\cos \theta=\frac{\left(2 x^{2} \hat{i}+4 x \hat{j}+\hat{k}\right) \cdot(7 \hat{i}-2 \hat{j}+x \hat{k})}{\sqrt{4 x^{4}+16 x^{2}+1 \cdot \sqrt{49+4+x^{2}}}}
$$

$\because \quad \theta$ lies between $\left(90^{\circ}, 180^{\circ}\right)$
i.e, $\cos \theta$ is negative in IInd quadrant

So, RHS is also negative i.e, $\frac{7 x(2 x-1)}{\sqrt{4 x^{4}+16 x^{2}+1} \cdot \sqrt{53+x^{2}}}<0$
$7 x(2 x-1)<0$

So, $x \in\left(0, \frac{1}{2}\right)$
20. If $x+y=1$, then $\sum_{r=0}^{n} r^{2} .{ }^{n} C_{r} x^{r} \cdot y^{n-r}$ equal to
a) $n x y$
b) $n x(x+y n)$
c) $n x(n x+y)=$
d) None of these
$=$
Hint: $\sum_{r=0}^{n} r^{2} \cdot{ }^{n} C_{r} x^{r} \cdot y^{n-r}$

$$
\sum_{r=0}^{n}[r(r-1)+r]{ }^{n} C_{r} x^{r} \cdot y^{n-r}
$$

$$
=n x(n x-x+1)=n x(n x+y)
$$

21. Coefficient of $\mathrm{t}^{24}$ in $\left(1+\mathrm{t}^{2}\right)^{12}\left(1+\mathrm{t}^{12}\right)\left(1+\mathrm{t}^{24}\right)$ is
a) $2+{ }^{12} \mathrm{C}_{6}$
=b) ${ }^{12} \mathrm{C}_{6}$
c) $1+{ }^{12} \mathrm{C}_{6}$
d) $3+{ }^{12} \mathrm{C}_{6}$

Hint: Write general term is $\left(1+t^{2}\right)^{12}$ and observe $t^{24}$ coefficient in multiplication
22. Sum of the coefficients of the terms of degree $m$ in the expansion of
$(1+\mathrm{x})^{\mathrm{n}}(1+\mathrm{y})^{\mathrm{n}}(1+\mathrm{z})^{\mathrm{n}}$ is
a) $\left({ }^{n} \mathrm{C}_{\mathrm{m}}\right)^{3}$
b) $3\left({ }^{n} C_{m}\right)$
c) ${ }^{\mathrm{n}} \mathrm{C}_{3 \mathrm{~m}}$
d) ${ }^{3 n} C_{m}$

Hint: $\mathrm{r}+\mathrm{s}+\mathrm{t}=\mathrm{m}$ and required is $(\mathrm{n}+\mathrm{n}+\mathrm{n}) \mathrm{C}_{\mathrm{m}}={ }^{3 n} \mathrm{C}_{\mathrm{m}}$
23. Find the term independent in $\left(x^{2}-\frac{1}{x}\right)^{6}$

1) 15
2) 12
3) 1
4) 14

Hint: $\frac{n p}{p+q}+1 \Rightarrow \frac{6.2}{2+1}+1=5^{\text {th }} \Rightarrow T_{5}={ }^{6} C_{4}=15$
24. In $(\sqrt[5]{3}+\sqrt[7]{2})^{24}$ Number of Rational terms are $=$

1) 3
2) 1
3) -1
4) 2

Hint: $\left[\frac{24}{L C M ~ 5,7}\right]+1 \Rightarrow 1$

25 Find the number of terms in $(x+y+z)^{10}$ is

1) 33
2) 31
3) 66
4) 26


Hint: ${ }^{n+r-1} C_{r-1} \Rightarrow{ }^{10+3-1} C_{3-1}={ }^{12} C_{2}=66$

$262^{\text {nd }}, 3^{\text {rd }}, 4^{\text {th }}$ terms of $(1+x)^{n}$ are in AP then n is $=$

1) 3
2) 7
3) 1
4) 2

Hint: $(n-2 r)^{2}=n+2$, substitute $r=2$ gives $n=7$
27.If the coefficient of ' $x$ ' in the expansion of $\left(x^{2}+\frac{k}{x}\right)^{5}$ is 270 then $\mathrm{k}=$

1) 3
2) 1
3) 21
4) 2

Hint: $\frac{n p-s}{p+q}+1$

$$
\frac{5.2-1}{2+1}+1=4^{\text {th }} \quad \text { term } \Rightarrow k=3
$$

28 For natural numbers m, n if $(1-y)^{m}(1+y)^{n}=1+a_{1} y+a_{2} y^{2}+\ldots$. and $a_{1}=a_{2}=10$ then m $\qquad$ , n $\qquad$

1) $40-2) 45 \quad 3) 20$
2) 35

Hint: $\left({ }^{m} C_{o}-{ }^{m} C_{1} y+{ }^{m} C_{2} y^{2}+\ldots.\right) \times\left({ }^{n} C_{o}+{ }^{n} C_{1} y+{ }^{n} C_{2} y^{2}\right)$
Collect $\mathrm{y}_{1}, \mathrm{y}_{2}$ terms and comparing equation $\Rightarrow m=35 ; n=45$
29. Coefficient of $x^{9}$ in $(x+1)(x+2) \ldots(x+10)$ is

1) 30
2) 10
3) 51
4) 55

Hint : $\frac{n(n+1)}{2}=\frac{10.11}{2}=55$
30.The integral part of $(\sqrt{2}+1)^{6}$ is $=$

1) 198
2) 115
3) 120
4) 178

Hint: $(\sqrt{2}+1)^{6}+(\sqrt{2}-1)^{6}=2\left[{ }^{6} C_{o}(\sqrt{2})^{6}+{ }^{6} C_{2}(\sqrt{2})^{4} \cdots\right]$

$$
\begin{aligned}
& (\sqrt{2}+1)^{6}+(\sqrt{2}-1)^{6}=198 \\
& (\sqrt{2}+1)^{6}=198-(\sqrt{2}-1)^{6}=198-f=197+F
\end{aligned}
$$

Where F and f are fractions

$$
I=197
$$

31. Number of terms in $(x+a)^{100}+(x-a)^{100}$ are $=$
1) 102$) 11$
2) 12
3) 51

Hint : $\Rightarrow \frac{100}{2}+1 \Rightarrow 51_{\text {terms }}$
32. Non zero terms in $(1+x)^{82}+(1-x)^{82}+(1+i x)^{82}+(1-i x)^{82}$

Hint : $\frac{n+2}{4}=\frac{82+2}{4}=21$ terms

1) 212$) 11$
2) 20
3) 13
33. 

$$
(1+x)^{15}=a_{o}+a_{1} x+\ldots+a_{15} x^{15}=\sum_{r=1}^{15} r \frac{a^{r}}{a_{r-1}}=
$$

1) 110
2) 115
3) 120
4) 135
Hint : It is in the form of (15)th synopsis $\frac{n(n+1)}{2}=\frac{15(15+1)}{2}=15.8=120$
$34|x|<\frac{1}{2}$ then the coefficient of $x^{r}$ in $\frac{1+2 x}{(1-2 x)^{2}}$ is
5) $r .2^{r}$
6) $(2 r-1) \cdot 2^{r}$
7) $r .2^{2 r+1}$
8) $(2 x+1) 2^{r}$

Hint:
$(1+2 x)(1-2 x)^{-2}=(1+2 x)\left[1+2(2 x)+3(2 x)^{2} \ldots ..\right]=(1+2 x)\left[1+4 x+12 x^{2}+\ldots.\right]$
Put $\quad r=2$ Collect coefficient $x^{2} \Rightarrow 8 x^{2}+12 x^{2}=20$
(4) option $r=2 \Rightarrow(2.2+1) 2^{2}=20$

35
$C_{o}+2 C_{1}+3 C_{2}+\ldots .+(n+1) C_{n}=$

1) $2^{n}+n \cdot 2^{n-1}$
2) $2^{n-1}+n \cdot 2^{n}$
3) $2^{n}+(n+1) \cdot 2^{n-1}$
4) $2^{n-1}$

Hint: Put $\mathrm{n}=2$
${ }^{n} C_{o}+2 \cdot{ }^{2} C_{1}+3 \cdot{ }^{2} C_{2}=1+2.2+3=8$
Option (1) $2^{2}+2 \cdot 2^{2-1}=8$
36.. If $x$ is positive, the first negative term in the expansion of $(1+x)^{27 / 5}$ is

1) $7^{\text {th }}$ term
2) $5^{\text {th }}$ term
3) $8^{\text {th }}$ term
4) $6^{\text {th }}$ term

Hint: $[27 / 5]+3=8$ th term
37. If ${ }^{\alpha, \beta}$ are roots of the equation $2 \mathrm{x}^{2}+6 \mathrm{x}+\mathrm{b}=0(\mathrm{~b}<0)$ then $\frac{\alpha}{\beta}+\frac{\beta}{\alpha}$ is less than
a) 2
b) -2
c) 18
d) None of these

Hint: $\alpha+\beta=-3, \alpha \beta=b / 2$

$$
D=36-4 b>0(\because b<0)
$$

$$
\frac{\alpha}{\beta}+\frac{\beta}{\alpha}=\frac{\alpha^{2}+\beta^{2}}{\alpha \beta}=\frac{(\alpha+\beta)^{2}-2 \alpha \beta}{\alpha \beta}
$$

39.If $\alpha, \beta$ are the roots of $a x^{2}+b x+c=0 ; \alpha+h, \beta+h$ are the roots of $p x^{2}+q x+r=0$; and $D_{1}, D_{2}$ the respective discriminants of these equations, then $D_{1}: D_{2}=$
a)
b) $\frac{b^{2}}{q^{2}}$
c) $\frac{c^{2}}{\mathrm{r}^{2}}$
d) None
of
these

Hint: Let $^{\mathrm{A}}=\alpha+\mathrm{h}, \mathrm{B}=\beta+\mathrm{h}$

$$
A-B=\alpha-\beta \Rightarrow
$$

$$
(A-B)^{2}=(\alpha-\beta)^{2}
$$

$$
\frac{\mathrm{D} 1}{\mathrm{D} 2}=\frac{\mathrm{a}^{2}}{\mathrm{P}^{2}}
$$

40.The values of $\mathbf{m}$; for which one of the roots of $x^{2}-3 x+2 m=0$ is double of one of the roots of $x^{2}-x+m=0$ is (are)
a) $0,-1$
b) $0,-2$
c) 1,2
d) None of these

Hint: be the root of $x^{2}-x+m=0$ and $2 \alpha$ be the root of $x^{2}-3 x+2 m=0$

$$
\begin{aligned}
& x^{2}-\alpha+m=0 \text { and } 4 \alpha^{2}-6 \alpha+2 m=0 \\
& \frac{\alpha^{2}}{-m}=\frac{\alpha}{-m}=\frac{1}{2} \Rightarrow m^{2}=-2 m
\end{aligned}
$$

$$
\mathrm{m}=0 \mathrm{~m}=-2
$$

41. $\lim _{x \rightarrow 0} \frac{\tan x-\sin x}{x^{2}}$ is equal to
1) 0
2) 1
3) $\frac{1}{2}$
4) $-\frac{1}{2}$

Sol. $\lim _{x \rightarrow 0} \frac{\tan x-\sin x}{x^{2}} \quad\left(\right.$ form $\left.\frac{0}{0}\right)$

By 'L' Hospital Rule
$\lim _{x \rightarrow 0} \frac{\sec ^{2} x-\cos x}{2 x}$
Again, by 'L' Hospital Rule $\lim _{x \rightarrow 0} \frac{2 \sec x \cdot \sec x \cdot \tan x+\sin x}{2}=\frac{2 \cdot 1 \cdot 1 \cdot 0+0}{2}=\frac{0}{2}=0$
42. If $f: R \rightarrow R$ defined by $f(x)=\left\{\begin{array}{c}\frac{1+3 x^{2}-\cos 2 x}{x^{2}}, \text { for } x \neq 0 \\ k, \quad \text { for } x=0\end{array}\right.$ is continuous at $x=0$,
then k is equal to

1) 1
2) 5
3) 6
4) 0

Sol. $f(x)=\left\{\begin{array}{c}1+3 x^{2}-\cos 2 x, \text { for } x \neq 0 \\ k, \quad \text { for } x=0\end{array}\right.$

RHL $\quad f(0+h)=\lim _{h \rightarrow 0} \frac{1+3\left(0+h^{2}\right)-\cos (0+h)}{\left(0+h^{2}\right)}$

$$
\begin{aligned}
& =\lim _{h \rightarrow 0} \frac{1+3 h^{2}-\cos 2 h}{h^{2}}=\lim _{h \rightarrow 0} \frac{1+3 h^{2}-\left(1-2 \sin ^{2} h\right)}{h^{2}} \\
& =\lim _{h \rightarrow 0} \frac{1+3 h^{2}-1+2 \sin ^{2} h}{h^{2}} \\
& =\lim _{h \rightarrow 0}\left\{3+2\left(\frac{\sin ^{2} h}{h^{2}}\right)\right\}=3+2 \cdot \lim _{h \rightarrow 0}\left(\frac{\sin ^{2} h}{h^{2}}\right)^{2}=3+2=5 \\
& \text { LHL } f(0-h)=\lim _{h \rightarrow 0} \frac{1+3(0-h)^{2}-\cos 2(0-h)}{(0-h)^{2}}=5
\end{aligned}
$$

43. If $f(x)=(\cos x)(\cos 2 x) \ldots(\cos n x)$, then $f^{1}(x)+\sum_{r=1}^{n}(r \tan r x) f(x)$ is equal to
1) $f(x)$
2) 0
3) $-f(x)$
4) $2 f(x)$

Sol. $\quad f(x)=(\cos x)(\cos 2 x) \ldots(\cos n x)$
$f^{\prime}(x)=-\sin x \cdot \cos 2 x \ldots . \cos n x+\cos x \frac{d}{d x}\{\cos 2 x \cdot \cos 3 x \ldots \cos n x\}$
$f^{\prime}(x) \Rightarrow-(\sin x \cdot \cos 2 x \ldots . \cos n x)-(2 \cos x \cdot \sin 2 x \ldots \cos 3 x)$
$-(3 \cos x \cdot \cos 2 x \cdot \sin 3 x \ldots \cdot \cos n x)-(n \cos x \cdot \cos 2 x \ldots \sin n x)$

So, $\Rightarrow f^{\prime}(x)+\sum_{r=1}^{n}(r \tan r x) f(x)$
$=f^{\prime}(x)+\{\tan x+2 \tan 2 x+3 \tan 3 x+\ldots .+n \tan n x\} f(x)$

$$
\begin{aligned}
& =f^{\prime}(x)+f(x) \tan x+2 f(x) \tan 2 x+\ldots+n f(x) \tan n x \\
& =f^{\prime}(x)+[(\sin x \cdot \cos 2 x \ldots \cos n x)+(2 \cos x \cdot \sin 2 x \ldots \cos n x)+\ldots+(n \cos x \cdot \cos 2 x \ldots \sin n x)] \\
& =f^{\prime}(x)-f^{\prime}(x) \Rightarrow 0
\end{aligned}
$$

Hence, $f^{\prime}(x)+\sum_{r=1}^{n}(r \tan r x) f(x)=0$
44. If $y=\cos ^{-1}\left(\frac{a^{2}-x^{2}}{a^{2}+x^{2}}\right)+\sin ^{-1}\left(\frac{2 a x}{a^{2}+x^{2}}\right)$, then $\frac{d y}{d x}$ is equal to

1) $\frac{a}{x^{2}+a^{2}}$
2) $\frac{2 a}{x^{2}+a^{2}}$
3) $\frac{4 a}{x^{2}+a^{2}}$
4) $\frac{4 a^{2}}{x^{2}+a^{2}}$

Sol. $\quad y=\cos ^{-1}\left(\frac{a^{2}-x^{2}}{a^{2}+x^{2}}\right)+\sin ^{-1}\left(\frac{2 a x}{a^{2}+x^{2}}\right)$

Put $x=a \tan \theta$
$\Rightarrow \theta=\tan ^{-1}\left(\frac{x}{a}\right) \Rightarrow y=\cos ^{-1}(\cos 2 \theta)+\sin ^{-1}(\sin 2 \theta) \Rightarrow y=4 \theta$
$\Rightarrow y=4 \tan ^{-1}\left(\frac{x}{a}\right) \Rightarrow \frac{d y}{d x}=\frac{4 a^{2}}{a^{2}+x^{2}}$
45. If $f(x)=\sin x+\cos x$ then $f\left(\frac{\pi}{4}\right) f^{(i v)}\left(\frac{\pi}{4}\right)$ is equal to

1) 1
2) 2
3) 3
4) 4

Sol. $f(x)=\sin x+\cos x, f^{\prime}(x)=\cos x-\sin x$

$$
\left.f^{\prime \prime}(x)=-\sin x-\cos x \text { So, } f\left(\frac{\pi}{4}\right)=f^{" "( } \frac{\pi}{4}\right)=\sin \left(\frac{\pi}{4}\right)+\cos \left(\frac{\pi}{4}\right)
$$

$$
=\frac{1}{\sqrt{2}}+\frac{1}{\sqrt{2}}=\frac{2}{\sqrt{2}}=\sqrt{2} \text { Then, } f\left(\frac{\pi}{4}\right) f^{\prime \prime \prime \prime} f\left(\frac{\pi}{4}\right)=\sqrt{2} \times \sqrt{2}=2
$$

46. If $y=\sin \left(m \sin ^{-1} x\right)$, then $\left(1-x^{2}\right) y_{2}-x y_{1}$ is equal to (Here, $y_{n}$ denotes $\frac{d^{n} y}{d x^{n}}$
1) $m^{2} y$
2) $-m^{2} y$
3) $2 m^{2} y$
4) $-2 m^{2} y$

Sol. $y=\sin \left(m \sin ^{-1} x\right)$
$y_{1}=\cos \left(m \sin ^{-1} x\right) \cdot m \cdot \frac{1}{\sqrt{1-x^{2}}}$ where $\left(y_{1}=\frac{d y}{d x}\right)$
$y_{1} \sqrt{1-x^{2}}=m \cos \left(m \sin ^{-1} x\right) \Rightarrow y_{2} \sqrt{1-x^{2}}+y_{1} \frac{1}{2 \sqrt{1-x^{2}}} \cdot(-2 x)$

$$
\begin{aligned}
& =-m \sin \left(m \sin ^{-1} x\right) \cdot \frac{m}{\sqrt{1+x^{2}}} \quad\left(\because y_{2}=\frac{d^{2} y}{d x^{2}}\right) \\
& =\frac{-m^{2}}{\sqrt{1+x^{2}}} \sin \left(m \sin ^{-1} x\right) \Rightarrow y_{2}\left(1-x^{2}\right)-x y_{1}=m^{2} y
\end{aligned}
$$

47. If $u=\sin ^{-1}\left(\frac{x^{4}+y^{4}}{x+y}\right)$, then $x \frac{\partial u}{\delta x}+y \frac{\partial u}{\partial y}$ is equal to
1) $3 u$
2) $4 u$
3) $3 \sin u$
4) $3 \tan u$

Sol. $u=\sin ^{-1}\left(\frac{x^{4}+y^{4}}{x+y}\right)$

Let $v=\sin u=\frac{x^{4}+y^{4}}{x+y}$, here degree is homogeneous, so $\mathrm{n}=4-1=3$

By Euler's theorem,

$$
\begin{aligned}
& x \frac{\partial v}{\partial x}(\sin u)+y \frac{\partial v}{\partial y}(\sin u)=3 \sin u, \quad x \cos u \frac{\partial v}{\partial x}+y \cos u \frac{\partial v}{\partial y}=3 \sin u \\
& x \frac{\partial v}{\partial x}+y \frac{\partial v}{\partial y}=3 \tan u
\end{aligned}
$$

48. If $f_{n}(x)=\log \log \log \ldots \log x$ (log is repeated $\mathrm{n}-$ times), then $\int\left(x f_{1}(x) f_{2}(x) \ldots f_{n}(x)\right)^{-1}$
1) $f_{n+1}(x)+c$
2) $\frac{f_{n+1}(x)}{n+1}+c$
3) $n f_{n}(x)+c$
4) 

$\frac{f_{n}(x)}{n}+c$

Sol. $\quad f_{n}(x)=\log \cdot \log \cdot \log \ldots \log x$ (upto n terms)

$$
f_{1}(x)=\log x
$$

$$
f_{n-1}(x)=\log \log \log \ldots \log x \text { (upto }(n-1) \text { times) }
$$

Now, $\int\left(x f_{1}(x) f_{2}(x) \ldots f_{n}(x)\right)^{-1} d x$

$$
=\int \frac{d x}{\left[x f_{1}(x) f_{2}(x) \ldots f_{n}(x)\right]}=\int \frac{\left[x f_{1}(x) f_{2}(x) \ldots f_{n-1}(x)\right] d t}{\left[x f_{1}(x) f_{2}(x) \ldots f_{n-1}(x)\right] . t}=\int \frac{d t}{t} 4
$$

49. If $\int(1-\cos x) \operatorname{cosec}^{2} x d x=f(x)+c$, then $f(x)$ is equal to
1) $\tan \frac{x}{2}$
2) $\cot \frac{x}{2}$
3) $2 \tan \frac{x}{2}$
4) $\frac{1}{2} \tan \frac{x}{2}$

Sol. $\quad \int(1-\cos x) \operatorname{cosec}{ }^{2} x d x$

$$
\begin{aligned}
& =\int\left(2 \sin ^{2} \frac{x}{2}\right) \cdot \frac{1}{\sin ^{2} x} d x\left[\because \cos x=1-2 \sin ^{2} \frac{x}{2}\right] \\
& =\frac{1}{2} \int \sec ^{2} \frac{x}{2} d x=\frac{1}{2} \tan \frac{x}{2} \cdot 2+c=\tan \frac{x}{2}+c=f(x)+c \Rightarrow f(x)=\tan \frac{x}{2}
\end{aligned}
$$

50. If $I_{n}=\int_{o}^{\pi / 4} \tan ^{n} x d x$, then $I_{2}+I_{4}, I_{3}+I_{5}, I_{4}+I_{6}+\ldots$, are in
1) Arithmetic progression
2) geometric progression
3) Harmonic progression
4) arithmetic - geometric
progression

Sol. $\quad I_{n}=\int_{o}^{\pi / 4} \tan ^{r+2} d x$
We have, $I_{r}+I_{r+2}=\frac{1}{r+1}$
i.e, $I_{2}+I_{4}=\frac{1}{3}$
$I_{3}+I_{5}=\frac{1}{4}$

$$
I_{4}+I_{6}=\frac{1}{5}
$$

Which are clearly in HP
51. If a straight line L is perpendicular to the line $4 x-2 y=1$ and forms a triangle of area 4 square units with the coordinate axes, then the equation of the line L is

1) $2 x+4 y+7=0$
2) $2 x-4 y+8=0$
3) $2 x+4 y+8=0$
4) $4 x-2 y-8=0$

Ans. 3

Sol: Perpendicular to the line $4 x-2 y=1$ is $x+2 y+\lambda=0$ and area is $\frac{c^{2}}{2 a b}=\frac{\lambda^{2}}{4}=4$ $\lambda=4 \Rightarrow x+2 y+4=2 x+4 y+8=0$
52. The image of the point $(4,-13)$ with respect to the line $5 x+y+6=0$ is

1) $(-1,-14)$
2) $(3,4)$
3) $(1,2)$
4) $(-4,13)$

Ans. 1

Sol: Using formula $\frac{h-x_{1}}{a}=\frac{k-y_{1}}{b}=\frac{a\left(h-x_{1}\right)+b\left(k-y_{1}\right)}{a^{2}+b^{2}}$
53. The image of the line $x+y-2=0$ in the $y-$ axis is

1) $x-y+2=0$
2) $y-x+2=0$
3) $x+y+2=0$
4) $x+y-2=0$

Ans. 1

Sol :Intercepts of given line are $\mathrm{A}(2,0), \mathrm{B}(0,-2)$ thus w.r.t y -axes $\mathrm{A}=(-2,0)$ and $\mathrm{B}(0,-2)$ Thus by using two point formula image is $x-y+2=0$
54. The distance between the two lines represented by $8 x^{2}-24 x y+18 y^{2}-6 x+9 y-5=0$ is

1) 0
2) $\frac{3}{4 \sqrt{13}}$
3) $\frac{6}{\sqrt{13}}$
4) $\frac{7}{2 \sqrt{13}}$

Ans. 4

Sol: distance between the two lines $=2 \sqrt{\frac{g^{2}-a c}{a(a+b)}}=2 \sqrt{\frac{9+40}{8(8+18)}}=\frac{7}{2 \sqrt{13}}$
55. A pair of perpendicular lines passes through the origin and also through the points of intersection of the curve $x^{2}+y^{2}=4$ with $x+y=a$, where $a>0$. Then a is equal to

1) 2
2) 3
3) 4
4) 5

Ans. 1

Sol: Solving two equations $x^{2}+(a-x)^{2}=4$

Gives $\mathrm{x}=\frac{a \pm \sqrt{8-a^{2}}}{2}$ as point of intersection is real number $\begin{aligned} & \sqrt{8-a^{2}} \geq 0 \\ & \\ & a \leq 2 \sqrt{2}\end{aligned}$ thus a=2
56. If the angle $2 \theta$ is acute, then the acute angle between the pair of straight lines $x^{2}(\cos \theta-\sin \theta)+2 \mathrm{xy} \cos \theta+y^{2}(\cos \theta+\sin \theta)=0$ is

1) $2 \theta$
2) $3 \theta$
3) 0
4) $\theta$

Ans. 4

Sol: If $\alpha$ is the angle between the lines then

$$
\begin{aligned}
& \operatorname{Cos} \alpha=\frac{\cos \theta-\sin \theta+\cos \theta+\sin \theta}{\sqrt{(\cos \theta-\sin \theta-\cos \theta-\sin \theta)^{2}+4 \cos ^{2} \theta}}=\frac{2 \cos \theta}{2}=\cos \theta \\
& \Rightarrow \alpha=\theta
\end{aligned}
$$

57. If the slope of one of the lines is twice the slope of the other in the pair of straight lines $a x^{2}$ $+2 \mathrm{hxy}+b y^{2}=0$ then $8 h^{2}=$
1) 2 ab
2) 3 ab
3) $4 a b$
4) $9 a b$

Ans. 4

Sol: Let the slopes be m, $2 \mathrm{~m} \quad \therefore$ Their ratio is $1: 2$
Required condition is $a b(1+m)^{2}=4 h^{2} l m \Rightarrow \mathrm{ab}(1+2)^{2}=4 h^{2}$ (1) (2)

$$
\Rightarrow 8 h^{2}=9 \mathrm{ab}
$$

58 The condition that one of the lines $a x^{2}+2 \mathrm{hxy}+b y^{2}=0$ will bisect the angle between the coordinate axes i

1) $24 h^{2}$
2) $34 h^{2}$
3) $4 h^{2}$
4) $14 h^{2}$

Ans. 3

Sol: Equation of the angle bisector of the coordinate axes are $\mathrm{y}= \pm \mathrm{x}$.

$$
\begin{aligned}
& a x^{2}+2 \mathrm{hxy}+b y^{2}=0 \Rightarrow a x^{2}+2 \mathrm{hx}( \pm \mathrm{x})+\mathrm{b}( \pm \mathrm{x})^{2}=0 \Rightarrow(\mathrm{a}+\mathrm{b})\left(x^{2}\right)=( \pm 2 h) x^{2} \\
& \Rightarrow \mathrm{a}+\mathrm{b}= \pm 2 \mathrm{~h} \Rightarrow(a+b)^{2}=4 h^{2}
\end{aligned}
$$

59 The area (in square units) of the triangle formed by the lines $x=0, y=0$ and

$$
3 x+2 y=7 \text { is }
$$

Ans. 4


Sol: Area of the t1) 2
2) 3
3) 4
4) $\frac{49}{12}$
riangle $=\frac{(7)^{2}}{2|3 \times 2|}=\frac{49}{12}$ square unit.
60 The area of the triangle formed by the axes and the line $(\cosh \alpha-\sinh \alpha) \mathrm{x}+$ $(\cosh \alpha-\sinh \alpha) y=2$ in sq. unit, is

1) 2
2) 3
3) 4
4) 5

Ans. 1

Sol : Area of the triangle $=\frac{(-2)^{2}}{2|(\cosh \alpha-\sinh \alpha)(\cosh \alpha+\sinh \alpha)|}=\frac{(-2)^{2}}{\cosh ^{2} \alpha-\sinh ^{2} \alpha}=2$
61. If $A$ and $B$ are square matrices of the same order and $A$ is non-singular then for a positive integer $\mathrm{n},\left(\mathrm{A}^{-1} \mathrm{BA}\right)^{n}$ is equal to
a) $A^{-n} B^{n} A^{n}$
b) $A^{n} B^{n} A^{-n}$
c) $A^{-1} B^{n} A$
d) $n\left(\mathrm{~A}^{-1} \mathrm{BA}\right)$

HINT: If $\mathrm{n}=2=\left(A^{-1} B A\right)^{2}=\left(A^{-1} B A\right)\left(A^{-1} B A\right)$


Generally

$$
\left(A^{-1} B A\right)^{\prime}=A^{-1} B^{n} A
$$

Hence 3rd options is correct
62. If $\left|\begin{array}{ccc}x^{n} & x^{n+2} & x^{n+3} \\ y^{n} & y^{n+2} & y^{n+3} \\ z^{n} & z^{n+2} & z^{n+3}\end{array}\right|=(x-y)(y-z)(z-x)\left(\frac{1}{x}+\frac{1}{y}+\frac{1}{z}\right)$ then value of n is
a) -1
b) -2
c) 1
d) 2

HINT: Order of determinant is $n+n+2+n+3=3 n+5$
Order of R.H.S $=1+1-1=2$
$\Rightarrow 3 n+5=2 \Rightarrow 3 n=-3 \Rightarrow n=-1$
Hence $1^{\text {st }}$ options is correct
63. If $\mathrm{f}(\mathrm{x})=\tan \mathrm{x}$ and A, B, C are the angles of $\triangle A B C$, then $\left|\begin{array}{ccc}f(A) & f(\pi / 4) & f(\pi / 4) \\ f(\pi / 4) & f(B) & f(\pi / 4) \\ f(\pi / 4) & f(\pi / 4) & f(C)\end{array}\right|=$
a) 0
b) -2
c) 2
d) 1
HINT: $\quad\left|\begin{array}{ccc}\operatorname{Tan} A & 1 & 1 \\ 1 & \operatorname{Tan} B & 1 \\ 1 & 1 & \operatorname{TanC}\end{array}\right|$
$\operatorname{Tan} A(\operatorname{Tan} B \operatorname{Tan} C-1)-1(\operatorname{Tan} C-1)+1(1-\operatorname{Tan} B)$
$\operatorname{Tan} A \operatorname{Tan} B \operatorname{Tan} C-\operatorname{Tan} A-\operatorname{Tan} B-\operatorname{Tan} C+2=2$
$\because \sum \operatorname{Tan} A=\pi \operatorname{Tan} A$
Hence 3rd options is correct
64. If $\alpha, \beta, \gamma$ are the roots of $x^{3}+p x^{2}+q=0$, where $\mathrm{q} \neq 0$, then $\Delta=\left|\begin{array}{lll}1 / \alpha & 1 / \beta & 1 / \gamma \\ 1 / \beta & 1 / \gamma & 1 / \alpha \\ 1 / \gamma & 1 / \alpha & 1 / \beta\end{array}\right|$ equals
a) $-p / q$
b) $1 / q$
c) $p^{2 / q}$
d) 0

HINT:.

$$
R_{1}+R_{2}+R_{3} \Delta=\left|\begin{array}{ccc}
\sum \frac{1}{\alpha} & \Sigma \frac{1}{\beta} & \sum \frac{1}{\gamma} \\
\frac{1}{\beta} & \frac{1}{\gamma} & \frac{1}{\alpha} \\
\frac{1}{\gamma} & \frac{1}{\alpha} & \frac{1}{\beta}
\end{array}\right| \Delta=\left|\begin{array}{ccc}
0 & 0 & 0 \\
\frac{1}{\beta} & \frac{1}{\gamma} & \frac{1}{\alpha} \\
\frac{1}{\gamma} & \frac{1}{\alpha} & \frac{1}{\beta}
\end{array}\right|=0
$$

Hence 4th options is correct
65. The characteristic roots of the matrix $A=\left[\begin{array}{ccc}1 & 2 & 3 \\ 0 & -4 & 2 \\ 0 & 0 & 7\end{array}\right]$ is/are given by
a) $1,-4$
b) $1,-4,7$
c) $-1,4,-7$
d) None of these

HINT: Characteristic equation of $\mathbf{A}$ is $|A-\lambda I|=0$.

Hence 2nd options is correct
66. If $\mathrm{a}=1+2+4+\ldots \ldots .$. . upto n terms

$$
\mathrm{b}=1+3+9+\ldots \ldots . . . . \text { upto } \mathrm{n} \text { terms }
$$

$$
\mathrm{c}=1+5+25+\ldots . . . . . \text { upto } \mathrm{n} \text { terms }
$$



Then $\left|\begin{array}{ccc}a & 2 b & 4 c \\ 2 & 2 & 2 \\ 2^{n} & 3^{n} & 5^{n}\end{array}\right|=$
a) $(30)^{n}$
b) $(10)^{n}$
c) 0
d) $2^{n}+3^{n}+5^{n}$

HINT: $\Delta=\frac{1}{2}\left|\begin{array}{ccc}2^{n}-1 & 3^{n}-1 & 5^{n}-1 \\ 1 & 1 & 1 \\ 2^{n} & 3^{n} & 5^{n}\end{array}\right| R_{3}-R_{1} \Rightarrow$ two rows are equal $\Delta=0$

Hence 3rd options is correct

67 Let $\mathrm{a}, \mathrm{b}$, c be positive real numbers. The following system of equations in $x, y, z$ $\frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}-\frac{z^{2}}{c^{2}}=1 \quad \frac{x^{2}}{a^{2}}-\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1 \quad \frac{x^{2}}{a^{2}}+\frac{y^{2}}{b^{2}}+\frac{z^{2}}{c^{2}}=1$ has

1) No solution
2) unique solution
3) Infinitely many solution
4) finitely many solution

Sol. Let $\frac{x^{2}}{a^{2}}=X, \frac{y^{2}}{b^{2}}=Y, \frac{z^{2}}{c^{2}}=Z$

$$
\mathrm{X}+\mathrm{Y}-\mathrm{Z}=1, \mathrm{X}-\mathrm{Y}=\mathrm{Z}=1,-\mathrm{X}+\mathrm{Y}+\mathrm{Z}=1
$$

## Determinant of coefficient $\neq 0$

Hence 4th options is correct
68 If A is non-singular and $(\mathrm{A}-2 \mathrm{I})(\mathrm{A}-4 \mathrm{I})=0$, then $\frac{1}{6} A+\frac{4}{3} A^{-1}=$
a) I
b) 0
c) 2 I
d) 6 I
I

HINT: $A^{2}-6 A I+8 I=0$ Taking $A^{-1}$ on both the sides

$$
\begin{aligned}
A+8 A^{-1} & =6 I \\
\frac{A}{6}+\frac{4}{3} A^{-1} & =I
\end{aligned}
$$

Hence 1st options is correct
69. The mean and the variance of a binomial distribution are 4 and 2 respectively. Then the probability of 2 successes is

1) $\frac{28}{256}$
2) $\frac{2}{256}$
3) 4
4) 8

Sol: In a B.D. : mean $=n p=4$ $\qquad$ (1), variance $=n p q=2$
(2) $\div(1)::^{q=\frac{2}{4}=\frac{1}{2}} \therefore p=\frac{1}{2} \quad \therefore n=8$
$\therefore$ Probability of 2 successes (B.D) $={ }^{8} C_{2}\left(\frac{1}{2}\right)^{8}=\frac{8.7}{2 \times 4 \times 4 \times 4 \times 4}=\frac{28}{256}$
70.A pair of fair dice is thrown independently three times. The probability of getting a score of exactly 9 twice is

1) $8 / 729$
2) $8 / 243$
3) $1 / 729$
4) $8 / 9$

Sol. Probability of getting a score 9 when a pair of dice is thrown $=$
$\frac{4}{36}=\frac{1}{9}=P$
$\therefore q=1-\frac{1}{9}=\frac{8}{9}$
$\therefore$ Required probability $={ }^{3} C_{2}\left(\frac{8}{9}\right)^{1} \cdot\left(\frac{1}{9}\right)^{2}=\frac{8}{243}$
71.It is given that the events $A$ and $B$ are such that

$$
P(A)=\frac{1}{4}, P\left(\frac{A}{B}\right)=\frac{1}{2}
$$

$P\left(\frac{B}{A}\right)=\frac{2}{3}$ Then $\mathrm{P}(\mathrm{B})=$

1) $\begin{array}{llll}\frac{1}{6} & \text { 2) } \frac{1}{3} & \text { 3) } \frac{2}{3} & \text { 4) } \frac{1}{2}\end{array}$

Sol.
$P\left(\frac{A}{B}\right)=\frac{P(A \cap B)}{P(B)}=\frac{1}{2} \Rightarrow P(A \cap B)=\frac{1}{2} P(B)$
$P\left(\frac{B}{A}\right)=\frac{P(A \cap B)}{P(A)}=\frac{2}{3} \Rightarrow P(A \cap B)=\frac{2}{3} \times P(A)=\frac{2}{3} \times \frac{1}{4}=\frac{1}{6}$
From (1) and (2) : $\frac{1}{2} \mathrm{P}(\mathrm{B})=\frac{1}{6} \Rightarrow P(B)=\frac{1}{3}$.
72. Four number are chosen at random from $\{1,2,3, \ldots . .40\}$. The probability that they are not consecutive is]

1) $\frac{1}{2470}$
2) $\frac{4}{7969}$
3) $\frac{2469}{2470}$
4) $\frac{7965}{7969}$

Sol. 4 consecutive numbers are $1,2,3,4 ; 2,3,4,5 ; \ldots \ldots . ; 37,38,39,40$
$\therefore$ Probability for the 4 numbers to be consecutive $={ }^{\frac{37}{40} C_{4}}=\frac{37 \times 4 \times 3 \times 2 \times 1}{40 \times 39 \times 38 \times 37}=\frac{1}{2470}$
73. If three six faced fair dice are thrown together then the probability that the sum of the number appearing on the dice is $K(3 \leq K \leq 8)$ is

1) $\frac{(K-1)(K-2)}{432}$
2) $\frac{K(K-1)}{432}$
3) $\frac{K^{2}}{432}$
4) $\frac{K^{2}}{216}$

Hint: Coefficient of $\mathrm{X}^{\mathrm{k}}$ in $\left(\mathrm{X}+\mathrm{X}^{2} \ldots+\mathrm{X}^{6}\right)^{3} \Rightarrow{ }^{K-1} C_{2}$
74. If four people are chosen at Random. Then the probability that no two of them were fore on the same day of the week is

1) 30
2) $203 / 225$
3) $120 / 343$
4) $6 / 49$

$$
\text { Hint }: \frac{7 \cdot 6 \cdot 5 \cdot 4}{7 \cdot 7 \cdot 7 \cdot 7}=\frac{120}{343}
$$

75. A man throw a die until he gets a number bigger than 3 . The probability that he gets a 5 in the last throw
1) $1 / 2$
2) 

$11 / 3$
3) $2 / 3$
4) $3 / 5$

Hint: $=P(5)=1 / 6 \quad P(1$ or 2 or 3$) 3 / 6=1 / 2$

$$
\Rightarrow \frac{1}{6}+\frac{1}{2}\left(\frac{1}{6}\right)+\left(\frac{1}{2}\right)^{2} 1 / 6 \ldots=1 / 3
$$

76. On a toss a two dice. A throw a total of ' 5 ' then the probability that he will throw another 5 before he throws ' 7 ' is
1) $\frac{1}{9}$
2) $\frac{1}{6}$
3) $2 / 5$
4) $5 / 36$
Hint:
$P(5)=4 / 36=\frac{1}{9} \quad P(\neq 5$ or $\neq 7)=\frac{36-4-6}{36}=\frac{13}{18}$

$$
\begin{aligned}
& P(7)=6 / 36=\frac{1}{6} \\
& \Rightarrow \frac{1}{9}+\frac{13}{18} \cdot \frac{1}{9}=\frac{a}{1-r}=2 / 5
\end{aligned}
$$

77. In a Poisson on distribution variance is ' $m$ '. The sum of the term in odd places in this distribution
1) $e^{-m}$
2) $e^{m}$
3) $e^{-m} \cosh m$
4) $e^{-m} \sin m$

Hint : $\mathrm{P}(\mathrm{x}=0)+\mathrm{P}(\mathrm{x}=2) \ldots . .=e^{-m} \cosh m$
78. If the mean of Binomial distribution with 9 Trials is ' 6 ' then its variance.

1) 2
2) 3
3) 4
4) $\sqrt{2}$

Hint: $\mathrm{np}=6 ; \sigma^{2}=n p q$
79. In a book of 500 pages, it is found that there are 250 typing errors. Assume that poisson law holds for the number of errors per page. Then the probability that a random sample of 2 pages will contain no error is :

1) $e^{-0.3}$
2) $e^{-0.5}$
Sol: Here $\lambda=2 \times \frac{250}{500}=1$
3) $e^{-1}$
4) $e^{-2}$
$\therefore$ The probability that a random sample of 2 pages will contain no error is $\frac{\lambda^{0}}{0!} e^{-\lambda}=e^{-1}$
80.In a polygon of ' n ' sides has 275 diagonals then $\mathrm{n}=$
5) 20
6) 30
7) 4
8) 25

Hint: $\frac{n(n-3)}{2}=275 \Rightarrow n=25$
81. Number of ways of distinctly 8 identical balls ' $m$ ' 3 distinct boxes. So that none of boxes is empty

1) 20
2) 21
3) 4
4) 25

Hint: ${ }^{n-1} c_{r-1}=8-1 c_{3-1}=7 c_{2}=21$
82. The number of ways of selecting 10 balls out of unlimited number of white, red, green and blue balls.

1) 200
2) 309
3) 40
4) 286

Hint: ${ }^{n+r-1 c_{r-1}=10+4-1 c_{4-1}=13 c_{3}=286}$
83. The number of positive divisor of $2^{3} \times 3^{6} \times 7^{2}$

Hint: Positive divisor $(3+1)(6+1)(2+1)=84$
Proper divisor $(3+1)(6+1)(2+1)-2=82$
Odd divisor $\quad(6+1)(2+1)=21$

1) 20
2) 30
3) 21
4) 25
84. The number of three digit numbers having only two consecutive digits identical is
a) 153
b) 162
c) 168
d) 163

Hint: $9 \times 9 \times 1+9 \times 1 \times 9=162$
85.Given five line segments of length $2,3,4,5,6$ units. Then the number of triangles that can be formed by joining these lines is
a) ${ }^{5} \mathrm{C}_{3}-3$
b) ${ }^{5} \mathrm{C}_{3}-1$
c) ${ }^{5} \mathrm{C}_{3}$
d) ${ }^{5} \mathrm{C}_{3}-2$

Hint: $5_{c_{3}}-3$. since $2,3,5 ; 2,4,6 ; 2,3,6$ does not form a triangle
86. Let $\mathbf{T}_{\mathbf{n}}$ denote the number of triangles which can be formed using the vertices of a regular polygon of $\mathbf{n}$ sides. If $\mathbf{T}_{\mathbf{n + 1}}-\mathbf{T}_{\mathbf{n}}=\mathbf{2 1}$, the $\mathbf{n}$ equals
a) 5
b) 7
c) 6
d) 4

Hint: $(\mathrm{n}+1)_{\mathrm{c}_{3}}-\mathrm{n}_{\mathrm{c}_{3}}=21$ thus $\mathrm{n}=7$
87.In a 12 - storey house ten people enter a lift cabin. It is known that they will leave in groups of 2,3 and 5 people at different stories. The number of ways they can do so if the lift does not stop upto the second storey is
a) 78
b) 112
c) 720
d) 132 .

Hint: $10_{p_{3}}=720$
88. The sum of integers from 1 to 100 that are divisible by 2 or $\mathbf{5}$ is
a) 3000
b) 3050
c) 3600
d) 3250

Hint: Required sum $=(2+4+6+\ldots . .+100)$

$$
\begin{aligned}
& +(5+10+15+\ldots . .100) \\
& -(10+20+\ldots . .+20)=3600
\end{aligned}
$$

89. The number of distinct rational numbers $n$ such that $0<n<1$ and $n=\frac{p}{q}$, where

$$
\mathbf{p}, \mathbf{q} \in\{1,2,3,4,5,6\} \text { is }
$$

a) 15
b) 13
c) 12
d) 11

Hint: From 6 digits 2 digits can be selection ${ }^{6} \mathrm{C}_{2}$ ways \& they can be arranged in only one way. Out of these ways $\frac{1}{2}, \frac{2}{4}, \frac{3}{6}$ represent same number $\frac{2}{3}, \frac{4}{6}$ represent same number $\frac{1}{3}, \frac{2}{6}$ represent same number $\therefore$ No. of numbers $=15-4=11$
90. In a chess tournament, where the participants were to play one game with another. Two chess players fell ill, having played 3 games each. If the total number of games played is 84 , the number of participants at the beginning was
a) 15
b) 16
c) 20
d) 21

Hint: $(\mathrm{n}-2)_{\mathrm{C}_{2}}+6=84$ then $\mathrm{n}=15$

