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[Total No. of Questions : 09] [Total No. of Pages : 03

Paper ID [AM101]

(Please fill this Paper ID in OMR Sheet)

B.Tech. (Sem. - 1st/2nd)

MATHEMATICS - I (AMA - 101)

Time : 03 Hours **Maximum Marks : 60**

Instruction to Candidates:

1) Section - A is **Compulsory.**
2) Attempt any **Five** questions from Section - B & C.
3) Select atleast **Two** questions from Section - B & C.

Section - A

Q1) *(2 Marks Each)*

a) If $u = x\psi\left(\frac{y}{x}\right)$ then find $x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$.

b) State the method to find maxima and minima of $z = f(x, y)$ using partial derivatives.

c) Find the equation of the right circular cone with vertex at origin and passing through circle $x^2 + y^2 + z^2 = 3$, $x + y + z = 1$.

d) State De-Moivre's theorem.

e) Discuss convergence of $\sum\left(1+\frac{1}{\sqrt{n}}\right)^{-n^{3/2}}$.

f) Explain Bisection method to solve equations.

g) Draw the rough sketch of $y^2 = x + 5$ in plane.

h) If $z = \sin^{-1}\left(\frac{x+y}{\sqrt{x}+\sqrt{y}}\right)$ then prove that $x\frac{\partial z}{\partial x} + y\frac{\partial z}{\partial y} = \frac{1}{2}\tan z$.

i) State perpendicular axis theorem to find moment of inertia.

j) Prove that e^z is periodic function; z is complex number.

Section - B

(8 Marks Each)

Q2) (a) If $u = \sin^{-1}\left(\dfrac{x^3+y^3+z^3}{ax+by+cz}\right)$ then prove that $xu_x + yu_y + zu_z = 2\tan u$.

(b) Expand $\tan^{-1}\left(y/x\right)$ in powers of $x-1$ and $y-1$ using Taylor's theorem.

Q3) (a) A rectangular box open at the top is to have the volume of 32 cubic feet. Find the dimensions of the box requiring least material for its construction.

(b) Draw the rough sketch of $x^3 - 12x - 16 = y$.

Q4) (a) Find the volume generated by revolving the area enclosed by loop of the curve $y^4 = x(4-x)$ about x-axis.

(b) Find $\iiint \dfrac{dxdydz}{\sqrt{1-x^2-y^2-z^2}}$ where integration is taken over sphere $x^2+y^2+z^2=1$ in positive octant.

Q5) (a) Find the moment of inertia of area bounded by curve $r^2 = a^2\cos 2\theta$ about its axis.

(b) Using double integration find volume generated by revolution of cardioid $r = a(1-\cos\theta)$ about its axis.

Section - C

(8 Marks Each)

Q6) (a) Discuss the convergence of the series $x + \dfrac{2^2x^2}{2!} + \dfrac{3^3x^3}{3!} + \dfrac{4^4x^4}{4!} + ---$.

(b) Show that sequence converges to unique limit point if convergent.

Q7) (a) Show that

$$(1+\sin\theta + i\cos\theta)^n + (1+\sin\theta - i\cos\theta)^n = 2^{n+1}\cos^n\left(\frac{\pi}{4}-\frac{\theta}{2}\right)\cos\left(\frac{n\pi}{4}-\frac{n\theta}{2}\right).$$

(b) Sum the series

$$\sin\alpha + \sin(\alpha+\beta) + \sin(\alpha+2\beta) + \sin(\alpha+3\beta) + --- + \sin(\alpha+(n-1)\beta).$$

Q8) (a) Solve $x + 2y + z - w = -2$, $2x + 3y - z + 2w = 7$, $x + y + 3z - 2w = -6$, $x + y + z + w = 2$ by Gauss Jordan Method.

(b) Find a real root of $x^3 - 4x - 9 = 0$ by Regula-Falsi Method.

Q9) (a) Find the equation of sphere having its centre on plane $4x - 5y - z = 3$ and passing through the circle $x^2 + y^2 + z^2 - 2x - 3y + 4z + 8 = 0$, $x - 2y + z = 8$.

(b) Find the equation of right circular cylinder whose generating lines have d. cosines <1, m, n> and passes through the circumference of the circle $x^2 + z^2 = a^2$ in zox-plane.

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