USN

06CS34

Third Semester B.E. Degree Examination, June-July 2009
Discrete Mathematical Structures

Time: 3 hrs. Max. Marks:100

Note: ***Answer any FIVE full questions, choosing at least Two from each part.***

PART - A

1 a. Define inverse, converse and contra positive of a conditional statement. **(06 Marks)**

b. Find the possible truth value p, q, and r if

i) $P \to (q \vee r)$ is FALSE

ii) $P \wedge (q \to r)$ is TRUE. **(06 Marks)**

c. By constructing truth tables

i) S.T. $[(p \vee q) \to r] \Leftrightarrow [(p \to r) \wedge (q \to r)]$

ii) Examine whether

$[(p \vee q) \to r] \leftrightarrow [\neg r \to \neg (p \vee q)]$ is a tautology. **(08 Marks)**

2 a. When is a conclusion q is said to follow from the premises $H_1, H_2, \ldots\ldots, H_n$?

Let p, q, r be the primitive statements

p : Ragu studies.

q : Ragu plays tennis.

r : Ragu passes in Discrete Mathematics.

Let H_1, H_2 and H_3 be the premises

H_1 : If Ragu studies, then he will pass in Discrete Mathematics.

H_2 : If Ragu does not play tennis, then he will study.

H_3 : Ragu failed in Discrete mathematics. Show that q follows from H_1, H_2 and H_3. **(08 Marks)**

b. Show that rvs follows from cvd, $cvd \to \neg h$, $\neg h \to a \wedge \neg b$ and $a \wedge \neg b \to r \vee s$. **(06 Marks)**

c. Let $p(x) : x \geq 0$

$q(x) : x^2 \geq 0$ and $r(x) = x^2 - 3x - 4 = 0$.

Then for the universe comprising of all real numbers, find the truth values of

i) $(\exists x)\ [p(x) \wedge q(x)]$

ii) $(\forall x)\ [p(x) \to q(x)]$

iii) $(\exists x)\ [p(x) \wedge r(x)]$. **(06 Marks)**

3 a. Define the power set of a set. Obtain all the power sets of A_2 { 1, 2, 3, 4 }. **(04 Marks)**

b. For any sets A and B prove that $A \times (B \cap C) = (A \times B) \cap (A \times C)$. **(06 Marks)**

c. Prove that $\dfrac{1^2 + 3^2 + 5^2 + \ldots\ldots + (2n-1)^2}{3} = \dfrac{n(2n+1)(2n-1)}{3}$ by mathematical induction. **(04 Marks)**

d. A Computer services company has 300 Programmers. It is known that 180 of these can program in Pascal, 120 in FORTRAN, 30 in C++, 12 in Pascal and C++, 18 in FORTRAN and C++, 12 in Pascal and FORTRAN and 6 in all three languages.

i) If a programmer is selected at random, what is the probability that she can program in exactly two languages?

ii) If two programmers are selected at random, what is the probability that they can both program in Pascal? **(06 Marks)**

4 a. State the pigeon hole principle. If five coloures are used to paint 26 doors, show that at least six doors will have the same colour. **(06 Marks)**

b. Solve $a_n - 5a_{n-1} + 6a_{n-2} = 0$ where $a_0 = 2$ and $a_1 = 5$ by characteristic root method. **(06 Marks)**

c. For the Fibonacci sequence show that: $F_n = \left[\left(\frac{\sqrt{5}+1}{2}\right)^n - \left(\frac{\sqrt{5}-1}{2}\right)^n\right]$. **(08Marks)**

PART - B

5 a. Define a matrix and digraph of a relation with example. **(04 Marks)**

b. Show that congruence modulo m is an equivalence relation. **(06 Marks)**

c. If A = { 1, 2, 3, 4 }, B = { 2, 5 } and C = { 3, 4, 7 }, determine (AUB) x C and A x (BUC). **(04**

d. Let R = {(1, 1), (1, 2), (2,3), (3, 3), (3, 4)} be a relation on A = { 1, 2, 3, 4 }.

i) Draw the graph of R.

ii) Obtain R^2 and draw graph of R^2. **(06 Marks)**

6 a. Define a Stirling's Number of second kind. **(06 Marks)**

b. Let A = { 1, 2, 3, 4, 5, 6, 7 } and B = { w, x, y, z }. Find the number of on to functions from A to B. **(06 Marks)**

c. Define the partition of a set. If R = { (1, 1), (1, 2), (2, 1), (2, 2), (3, 4), (4, 3), (3, 3), (4, 4) } defined on the set A = { 1, 2, 3, 4 }, determine the partition induced. **(08 Marks)**

7 a. Define an Abelian group with examples. **(08 Marks)**

b. Define homomorphism and isomorphism. **(04 Marks)**

c. If G is a cyclic group, then show that:

i) If G is of infinite order, then G is isomorphic to (2, +).

ii) If G is finite order with $|G| = n$, then G is isomorphic to (2n, +). **(08 Marks)**

8 a. Define : **(04 M**

i) Ring with unity

ii) Ring with two divisor.

b. Prove that set Z with binary operation ⊕ and ⊙ defined by

x ⊕ y = + y – 1

x ⊙ y = x + y – xy, is a commutative ring with unity. **(10 Marks)**

c. State and prove Lagrange's theorem. **(06 Marks)**

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