Study Adda

CAT

Chapter wise solved paper

CHAPTER 10 FUNCTIONS

- **Directions (Qs. 1–2): Read the information given below and answer the Questions that follows'.**

If *md* (i) = | x |, *mn* (x, y) = minimum of *x* and *y* and *Ma* (*a*, *b*, *c*,....) = maximum of *a*, *b*, *c*, ...

1. **Value of Ma [md (*a*), mn (md (*b*), *a*), mn (*ab*, md (*ac*))] where** $a = -2, b = -3, c = 4$ **is**
(a) 2 (b) 6 (c) 8 (d) –2 (1994)

2. **Give that** $a > b$ **then the relation Ma [md (** a **). mn (** a, b **)] = mn [** a **, md (Ma (** a, b **))] does not hold if**
(a) $a < 0, b < 0$ (b) $a > 0, b > 0$
(c) $a > 0, b > 0, |a| < |b|$ (d) $a > 0, b < 0, |a| > |b|$ (1994)

- **Directions (Qs. 3–6) : Read the information given below and answer the questions that follows :**

If $f(x) = 2x + 3$ and $g(x) = \frac{x-3}{2}$, that

3. $fog(x) =$
(a) 1 (b) $gof(x)$ (c) $\frac{15x+9}{16x-5}$ (d) $\frac{1}{x}$ (1994)

4. **For what value of** $x; f(x) = g(x-3)$
(a) –3 (b) 1/4 (c) –4 (d) None of these (1994)

5. **What is value of (gofofogogof) (x) (fogofog) (x)**
(a) x (b) x^2 (c) $\frac{5x+3}{4x-1}$ (d) $\frac{(x+3)(5x+3)}{(4x-5)(4x-1)}$ (1994)

6. **What is the value of fo <fog) o (gof) (x)**
(a) x (b) x^2 (c) $2x + 3$ (d) $\frac{x+3}{4x-5}$

- **Directions (Qs. 7—10): Read the information given below and answer the questions that follows :**

le (x, y) = least of (x, y), mo (x) = |x|, me (x, y)= maximum of (x, y)

7. **Find the value of me (*a* + mo (le (*a*, *b*)); mo (*a* + me (mo (*a*) mo (*b*)), at** $a = -2$ **and** $b = -3$
(a) 1 (b) 0 (c) 5 (d) 3 (1995)

8. **Which of the following must always be correct for *a*, *b* > 0**
(a) mo (le (*a*, *b*)) ≥ (me (mo (*a*), mo (*b*)))
(b) mo (le (*a*, *b*)) > (me (mo (*a*), mo (*b*))
(c) mo (le (*a*, *b*)) < (le (mo (*a*)), mo (*b*))
(d) mo (le (*a*, *b*)) = le (mo (*a*), mo (*b*)) (1995)

9. **For what values of *a* is me** $(a^2 - 3a, a - 3) < 0$ **?**
(a) $1 < a < 3$ (b) $0 < a < 3$ (c) $a < 0$ and $a < 3$ (d) $a < 0$ or $a < 3$ (1995)

10. **For what values of *a* le** $(a^2 - 3a, a - 3) < 0$
(a) $1 < a < 3$ (b) $0 < a < 3$ (c) $a < 0$ and $a < 3$ (d) $a < 0$ or $a < 3$ (1995)

- **Directions (Qs. 11): Answer the questions independently**

11. **Largest value of min** $(2 + x^2, 6 - 3x)$**, when** $x > 0$ **is**
(a) 1 (b) 2 (c) 3 (d) 4 (1995)

- **Directions (Qs. 12-13): Read the information given below and answer the questions that/allows :**

A, S, M and D are functions of x and y, and they are defined as follows :
A(x, y) = x + y
S (x, y) = x – y
M (x, y) = x y
D (x, y) = x/y where $y \neq 0$

12. What is the value of M(M(A (M (*x, y*),S (*y, x*), A (*y, x*)) for x = 2,y = 3
(a) 50 (b) 140 (c) 25 (d) 70 (1996)

13. What is the value of S[M (D (A (*a, b*), 2), D(A (*a, b*), 2)), M(D (S (*a, b*), 2), D (S (*a, b*), 2))]
(a) a^2+b^2 (b) ab (c) a^2-b^2 (d) a/b (1996)

- **Directions (Qs. 14-16): Read the information given below and answer the questions that follows :**

The following functions have been defined :
la (x, y, z) = mm (x + y, y + z)
Ie (x, y, z) == max (x - y, y - z)
ma (x, y, z) = (1/2) [Ie (x, y, z) + la (x, y, z)]

14. Given that x > y > z > 0, which of the following is necessarily true ?
(a) la (x, y, z) < Ie (x, y, z) (b) ma (x, y, z) < la (x, y, z)
(c) ma (x, y, z) < Ie (x, y, z) (d) None of these (1997)

15. What is the value of ma (10,4, le (la (10,5,3), 5,3))
(a) 7·0 (b) 6·5 (c) 8·0 (d) 7·5 (1997)

16. For $x=15, y=10$ and $z=9$, find the value of: Ie (x , min $(y, x-z)$. Ie (9, 8, ma (x,y,z)))
(a) 5 (b) 12 (c) 9 (d) 4 (1997)

- **Directions (Qs. 17-19) : Read the information given below and answer the questions that follows :**

The following operations are defined for real numbers $a\#b=a+b$ if a and b both are positive else $a\#b=1$. $a\nabla b=(ab)^{a+b}$ if ab is positive else $a\nabla b=1$.

17. $(2\#1)/(1\nabla 2)=$
(a) 1/8 (b) 1 (c) 3/8 (d) 3 (1998)

18. $\{(((1\#1)\#2)-(10^{1.3}\nabla\log_{10}0\cdot 1)\}/(1\nabla 2)=$
(a) 3/8 (b) $4\log_{10}0\cdot 1/8$ (c) $(4+10^{1.3})/8$ (d) None of these (1998)

19. $((X\#-Y)/-X\nabla Y))=3/8$, then which of the following must be true?
(a) X = 2, Y = 1 (b) X > 0, Y < 0 (c) X, Y both positive (d) X, Y both negative (1998)

- **Directions (Qs. 20-22) : Read the information given below and answer the questions that follows :**

If x and y are real numbers, the functions are defined as $f(x,y)=|x+y|, F(x,y)=-f(x,y)$ and $G(x,y)=-F(x,y)$. Nom with the help of this information answer the following questions :

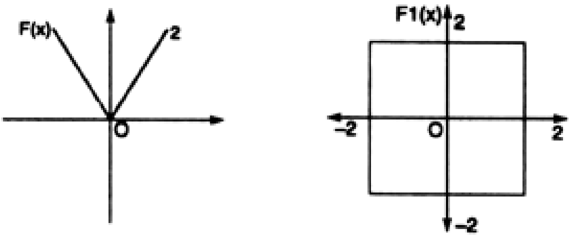
20. Which of the following will be necessarily true
(a) $G(f(x,y),F(x,y))>F(f(x,y),G(x,y))$ (b) $F(F(x,y),F(x,y))=F(G(x,y),G(x,y))$
(c) $F(G(x,y),(x+y)\neq G(F(x,y),(x-y))$ (d) $f(f(x,y),F(x-y))=G(F(x,y),f(x-y))$

21. If $y=$ which of the following will give x^2 as the final value
(a) $f(x,y)G(x,y)4$ (b) $G(f(x,y)f(x,y))F(x,y)/8$
(c) $-f(x,y)G(x,y)/\log_2 16$ $-f(x,y)G(x,y)F(x,y)/F(3x,3y)$

22. What will be the final value given by the function $G(f(G(F(2,-3),0)-2),0))$
(a) 2 (b) -2 (c) 1 (d) -1 (1999)

- **Directions (Qs. 2J-26) : Read the information given below and answer the questions that follows;**
  Any function has been defined fm a variable x, whew range of $x \in (-2, 2)$.
  Mark (a) if $F1(x) = -F(x)$
  Mark (b) if $F1(x) = F(-x)$
  Mark (c) if $F1(x) = -F(-x)$
  Otherwise mark (d).

23.



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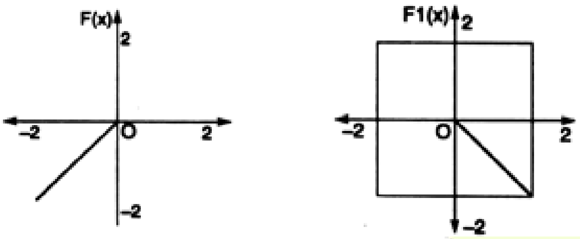

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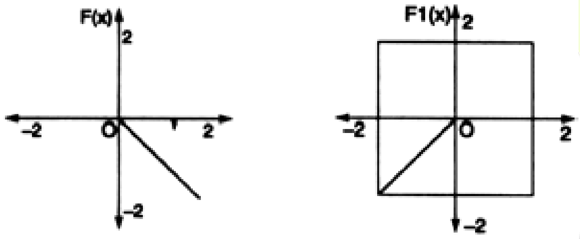

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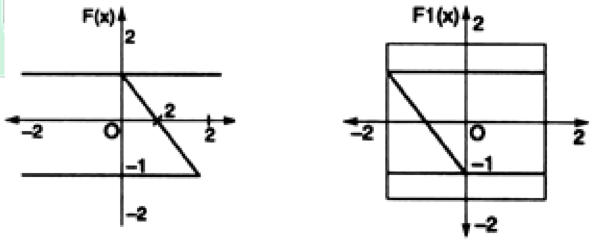


- **Directions (Qs. 27-28) : Read the information given below and answer the questions that follows :**
  Certain relation is defined among variable A and B.
  Using the relation answer the questions given below
  @ (A, B) = average of A and B
  ∴ (A, Byproduct of A and B,
  x (A, B) = the result when A is divided by B

27. The sum of A and B is given by
(a) \(@(A, B,), 2) (b) @(\(A, B,), 2) (c)' @(X(A, B,), 2) (d) none of these (2000)

28. The average of A, B and C is given by
(a) @(×(\(@(A, B,), 2)C), 3) (b) @(x(\(@(A, B,)), C2))

(c) $X(@(\backslash(@(A,B,)2),C,3))$ (d) $X(\backslash(@(\backslash @(A,B,)2),C)2),3)$ (2000)

- **Directions (Qs. 29–31) : Read the information given below and answer the questions that follows :**

x and y non-zero real numbers

$f(x,y) = +(x+y)^{0.5}$, if $(x+y)^{0.5}$ is real otherwise $= (x+y)^2$

$g(x,y) = (x+y)^2$ if $(x+y)^{0.5}$ is real, otherwise $= -(x+y)$

29. For which of the following is f (x, y) necessarily greater than g (x, y)

(a) x and y are positive (b) x and y are negative
(c) x and y are greater than –1 (d) none of these (2000)

30. Which of the following is necessarily false ?

(a) $f(x,y) \geq g(x,y)$ for $0 \leq x, y < 0 \cdot 5$ (b) $f(x,y) > g(x,y)$ when $x, y < -1$
(c) $f(x,y) > g(x,y)$ for x, y > 1 (d) None of these

31. If $f(x,y) = g(x,y)$ then

(a) x = y (b) x + y = 1 (c) x + y = - 2 (d) Both b and c (2000)

- **Directions (Qs. 32-33) : Answer the questions independent of each other.**

32. Which of the following equation will be best fit for above data ?

X	1	2	3	4	5	6
Y	4	8	14	22	32	44

(a) $y = ax + b$ (b) $y = a + bx + cx^2$ (c) $y = e^{ax+b}$ (d) none of these (2000)

33. If $f(0,y) = y+1$, and $f(x+1,y) = f(x, f(x,y))$ .Then, what is the value of ?

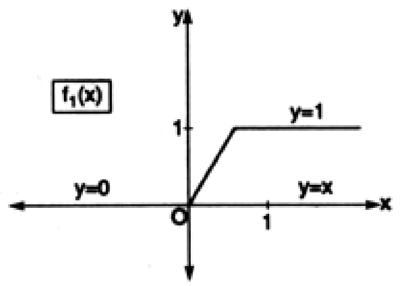
(a) 1 (b) 2 (c) 3 (d) 4 (2000)

- **Directions (Qs. 34-36) ; Read the information given below and answer the questions that follows :**

Graphs of some functions are given mark the options.

(a) If $f(x) = 3f(-x)$ (b) If $f(x) = f(-x)$ (c) If $f(x) = -f(-x)$ (d) If $3f(x) = 6f(-x)$ for $x > 0$

34.



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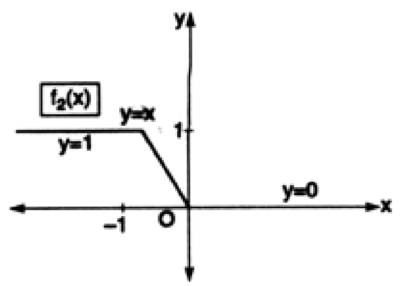

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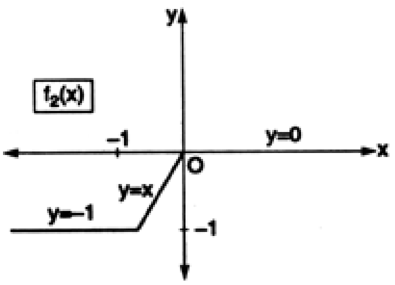


- **Directions (Qs. 37-39): Read the information given below and answer the awstions that follows :**

  Functions m and M are defined as follows :

  m (a, b, c) = min (a + b, c, a)

  M (a, b, c) = max (a + b, c, a)

37. If $a=-2, b=-3$ and $c=2$ what is the maximum between $[m(a,b,c,)+M(a,b,c]/2$ and $[m(a,b,c,)-M(a,b,c]/2$

(a) 3/2 (b) 7/2 (c) $-3/2$ (d) $-7/2$ (2000)

38. If a and b, c are negative, then what gives the minimum of a and b

(a) $m(a,b,c)$ (b) $-M(-a,a,-b)$ (c) $m(a+b,b+c)$ (d) none of these (2000)

39. What is $m(M(a,b,c)), m(a+b,c,b), -M(a,b,c)$ for $a=2, b=4, c=3$?

(a) –4 (b) 0 (c) –6 (d) 3 (2000)

- **Directions (Qs. 40-41): Read the information given below and answer the questions that follows :**

  $f(x)=1/(1+x)$ if x is positive $=1+x$ if x is negative or zero

  $f^n(x)=f(f^{n-1}(x))$

40. If $x=1$ find $f^1(x)f^2(x)f^3(x)f^4(x).\ f^9.(x)$

(a) 1/5 (b) 1/6 (c) 1/7 (d) 1/8 (2000)

41. If $x=-1$ what will $f(x)$ be

(a) 2/3 (b) 1/2 (c) 8/5 (d) 1/8 (2000)

- **Directions (Qs. S2-43) : Read the information given below and answer the questions that follows :**

  The batting average (BA) of a test batsman is computed from runs scored and innings played-completed innings and incomplete innings (not out) in the following manner :

  r_1 = number of runs scored in completed innings

  n_1 = number of completed innings

  r_2 = number of runs scored in incomplete innings

  n_2 = number of incomplete innings

  $$BA=\frac{r_1+r_2}{n_1}$$

  TO better assess a batsman's accomplishments, the ICC is considering two other measures MB A_1 and MB A_2 defined as follows:

  $$MBA_1=\frac{r_1}{n_2}+\frac{r_2}{n_2}\max\times\left[0,\left(\frac{r_2}{n_2}-\frac{r_1}{n_2}\right)\right]: MBA_2=\frac{r_1+r_2}{n_1+n_2}$$

42. Based on the information provided which of the following is true ?

(a) $MB\ A_1 \le BA \le MBA_2$ (b) $BA \le MB\ A_2 \le MB\ A_1$

(c) $MB\,A_2 \le BA \le MB\,A_1$ (d) None of these (2000)

43. An experienced cricketer with no incomplete innings has a BA of 50. The next time he bats, the innings is incomplete and he scores 45 runs. It can be inferred that

(a) BA and MB Ai will both increase

(b) BA will increase and MB A_2 will decrease

(c) BA will increase and not enough data is available to assess change in MB A_1 and MB A_2

(d) None of these

- **Directions (Qs. 44-19): Answer the questions independent of each other.**

44. If $f(x) = \log\left\{\frac{1+x}{1-x}\right\}$, then $f(x)+f(y)$ is :

(a) $f(x+y)$ (b) $f\left\{\frac{x+y}{1+xy}\right\}$ (c) $(x+y)f\left\{\frac{1}{1+xy}\right\}$ (d) $\frac{f(x)+f(y)}{1+xy}$ (2002)

45. Suppose, for any real number $x,[x]$ denotes the greatest integer less than or equal to x. Let $L(x,y)=[x]+[y]+[x+y]$ and $R(x,y)=[2x]+[2y]$. Then it's impossible to find any two positive real numbers x and y for which

(a) $L(x,y)=R(x,y)$ (b) $L(x,y)=R(x,y)$ (c) $L(x,y)<R(x,y)$ (d) $L(x,y)>R(x,y)$ (2002)

46. Let $g(x)=\max(5-x, x+2)$. The smallest possible value of $g(x)$ is

(a) $4\cdot0$ (b) $4\cdot5$ (c) $1\cdot5$ (d) None of these (2003)

47. Let $f(x)=|x-2|+|2\cdot5-x|+|3\cdot6-x|$, where x is a real number, attains a minimum at

(a) $x=2\cdot3$ (b) $x=2\cdot5$ (c) $x=2\cdot7$ (d) None of these (2003)

48. When the curves $y=\log_{10}x$ and $y=x^{-1}$ are drawn in the $x-y$ plane, how many times do they intersect for values $x \ge 1$?

(a) Never (b) Once (c) Twice (d) More than twice. (2003)

49. Consider the following two curves in the $x-y$ plane; $y=x^3+x^2+5; y=x^2+x+5$ (2003)

Which of the following statements is true for $-2 \le x \le 2$

(a) The two curves intersect once (b) The two curves intersect twice

(c) The two curves do not intersect (d) The two curves intersect thrice

- **Directions (Qs.50-52) : Answer the question on the basis of the table given below.**

Two binary operations ⊕ and * are defined over the set (a,e,f,g,h) as per the following tables :

	a	e	f	g	h
a	a	e	f	g	h
e	e	f	g	h	a
f	f	g	h	a	e
g	g	h	a	e	f
h	h	a	e	f	g

*	a	e	f	g	h
a	a	a	a	a	a
e	a	e	f	g	h
f	a	d	h	e	g
g	a	g	e	h	f
h	a	h	g	f	e

Thus, according to the first table $f \oplus g - a$ a, while according to the second table $g*h=f$, and so on. Also, let $f^2=f*f, g^3=g*g*g$, and so on.

50. What is the smallest positive integer n such that $g^n=e$?

(a) 4 (b) 5 (c) 2 (d) 3 (2003)

51. Upon simplification, $f\oplus[f*\{f\oplus(f*f(\}]$ equals :

(a) e (b) f (c) g (d) h (2003)

52. Upon simplification, $\{a^{10}*(f^{10}\oplus g^9)\}\oplus e^8$ equals :

(a) e (b) f (c) g (d) h (2003)

53. Let $f(x) = ax^2 - b|x|$, where a and b are constants. Then at $x = 0, f(x)$ is :

(a) maximized whenever a > 0, b > 0
(b) maximized whenever a > 0, b < 0
(c) minimized whenever a > 0, b > 0
(d) minimized whenever a > 0, b < 0.

54. If $f(x) = x^3 - 4x + p$, and $f(0)$ and $f(1)$ are of opposite signs, then which of the following is necessarily true ?

(a) $-1 < p < 2$ (b) $0 < p < 3$ (c) $-2 < p < 1$ (d) $-3 < p < 0$ (2004)

- **Directions (Qs. 55–56) ; Answer the questions on the basis of the information given below :**

$f_1(x) = x \quad 0 \le x \le 1$
$= 1 \quad x \ge 1$
$= 0$ otherwise
$f_2(x) = f_1(-x)$ for all x
$f_3(x) = -f_2(x)$ for all x
$f_4(x) = f_3(-x)$ for all x

55. How many of the following products are necessarily zero for every x :

$f_1(x)f_2(x), f_2(x)f_3(x), f_2(x)f_4(x)$

(a) 0 (b) 1 (c) 2 (d) 3 (2004)

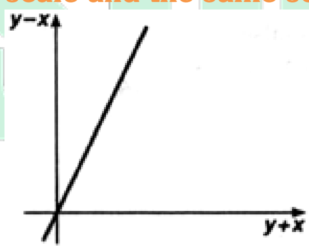
56. Which of the following is necessarily true ?

(a) $f_4(x) = f_1(x)$ for all x (b) $f_1(x) = -f_3(-x)$ for all x
(c) $f_2(-x) = f_4(x)$ for all x (d) $f_1(x) + f_3(x) = 0$ for all x (2004)

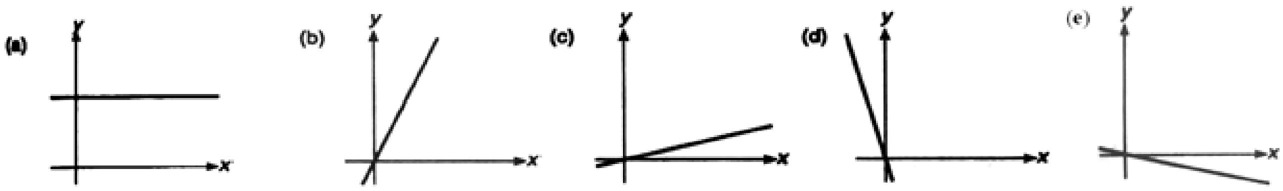
57. Let g (x) be a function such that $g(x+1) + g(x-1) = g(x)$ for every real x. Then for what value of p is the relation $g(x+p) = g(x)$ necessarily true for every real x ?

(a) 5 (b) 3 (c) 2 (d) 6 (2005)

58. The graph of $y - x$ against $y + x$ is as shown below. (All graphs in this question are drawn to scale and the same seal has been used on each axis.)



Which of the following shows the graph of y against x?



59. Let $f(x) = \max(2x+1), 3-4x$, where x is any real number. Then, the minimum possible value of $f(x)$ is ;

(a) $\frac{1}{3}$ (b) $\frac{1}{2}$ (c) $\frac{2}{3}$ (d) $\frac{4}{3}$ (e) $\frac{5}{3}$ (2006)

ANSWERS

1. B	2. A	3. B	4. C	5. B	6. C	7. A	8. D
9. B	10. C	11. C	12. D	13. B	14. D	15. B	16. C
17. C	18. A	19. B	20. B	21. C	22. B	23. D	24. D
25. D	26. C	27. A	28. D	29. D	30. C	31. B	32. B
33. D	34. B	35. D	36. C	37. C	38. C	39. C	40. D
41. C	42. D	43. B	44. B	45. D	46. D	47. B	48. B
49. D	50. A	51. D	52. A	53. D	54. B	55. B	56. C
57. D	58. D	59. E	60.	61.	62.	63.	64.

Solutions

1. **(b)** $Ma[md(a), mn(md(b), (a), mn(ab, md(ab, md(ac))]$

$Ma[|-2|, mn(|-3|, -2), mn(6, |-8|)]$

$Ma[2, mn(3, -2), mn(6, 8)]$

$Ma[2, -2, 6] = 6$

2. **(a)** $Ma[md(a), mn(a,b)] = mn[a, md(Ma(a,b)]$

$Ma[2, -3] = mn[-2, md(-2)]$

$2 = -2$

relation does not hold for $a = -2$ and $b = -3$

or $a < 0, b < 0$

3. $fo\ g(x) = f\{g(x)\} = f\left(\frac{x-3}{2}\right) = 2\left(\frac{x-3}{2}\right) + 3 = x$

$gof(x) = g\{f(x)\} = g(2x+3) = \frac{2x+3-3}{2} = x$

$\therefore\ fog(x) = gof(x)$

4. **(c)** $f(x) = g(x-3)$

$2x + 3 = \frac{x-3-3}{2} = \frac{x-6}{2}$

$4x + 6 = x - 6$

$3x = -12$

$x = -4$

5. **(b)** $\{go\ fo\ go\ go\ f(x)\}\{fo\ go\ g(x)\}$ from Q.3, we have

$fog(x) go\ f(x) = x$

therefore above expression becomes $(x)\cdot(x) = x^2$

6. (c) $fo(fog)0(gof)(x)$

We have, $fog(x) = gof(x) = x$

So give expression reduces to $f(x)$ that is $2x+3$

7. (a) $me(a + mo(le(a,b)), mo(a + me(mo(a), mo(b)))$

Given $a = -2, b = -3$

$= -2 + mo(-3)$

$= -2+3=1$

$mo(a+me(mo(a),mo(b)))$

$= mo(a+me(mo(a)(-2),mo(-3)))$

$= mo(-2+me(2,3)) = mo(-2+3) = mo(1) = 1$

$\Rightarrow\ me(1,1)=1$

8. **(d)** (a) $mo(le(a,b)) \geq me(mo(a),mo(b))$

$\equiv le(a,b) > me(a,b)$ as $a,b>0$ which is false.

(b) $mo(le(a,b)) > me(mo(a),mo(b))$ which is again false.

(c) $mo(le(a,b)) < le(mo(a),mo(b))$

or $le(a,b) < le(a,b)$ which is false

(d) $mo(le(a,b)) = le(mo(a),mo(b))$

or $le(a,b) = le(a,b)$ TRUE

9. **(b)** $me(a^2-3a,a-3)<0$ or $me[a(a-3),a-3]<0$

Case I. $a<0, a^3-3a > a-3 \Rightarrow a(a-3)<0$ or v

Which is not true.

Case II. $0<a<3, a(a-3)<0$ or $0<a<3$ which is true.

Case III. $a=3, me(0,0)<0$ not true.

Case IV. $a>3, a(a-3)<0$ or $0<a<3$ not true.

Alternatively, it can also be found by putting some values of a, say $a=-1$ in case I.

$a=1$ in case II and $a=4$ in case IV.

10. **(b)** $le(a(a-3),(a-3))<0$

Again in **case I,** $a<0; a-3<0$ or $a<3$

(from last Question) can be true)

In **case II,** $0<a<3; a-3<0$ or $a<3$ can be true

In **case III,** $a=3, le(0,0)=0<0$, not true

In **case IV,** $a>3, a-3<0$ or $a<3$ not true

Hence (b) and (c) are correct.

11. **(c)** Equating $2+x^2=6-3x$

$\Rightarrow x^2+3x-4=0 \Rightarrow x^2+4x-x-4=0$

or $(x+4)(x-1)=0$

$\Rightarrow\ x=-4$ or 1

But $x>0$ so $x=1$, so $LHS=RHS=2+1=3$

It means the largest value of function

$\min(2+x^2, 6-3x)$ is 3.

12. **(d)** $M(M(A(M(x,y),S(y,x)),x)A(y,x)$

$M(M(A(6,1),2),A(3,2)$

$M(M(7,2),A(3,2))$

$M(14,5)=70$

13. **(b)** $S[M(D(A(a,b),2)D(A(a,b),2)),M(D(S(a,b),2),D(S(a,b),2))]$

$\Rightarrow S[M(D(a+b,2),D(a+b,2)),M(D(a-b,2),D(a-b,2))]$

$$\Rightarrow S\left[M\left(\left(\frac{a+b}{2}\right)\left(\frac{a+b}{2}\right)\right),M\left(\frac{a-b}{2},\frac{a-b}{2}\right)\right]$$

$$\Rightarrow S\left[\left(\frac{a+b}{2}\right)^2,\left(\frac{a-b}{2}\right)^2\right]=\frac{(a+b)^2-(a-b)^2}{2^2}=\frac{(2a)(2b)}{4}=ab$$

14. **(d)** Since $x > y > z > 0$

$\therefore\ la(x,y,z) = y+z$

and $le = \max(x-y, y-z)$

we cannot find the value of le. Therefore we can't say whether $la > le$ or $le > la$.
Hence we can't comment, as data is insufficient.

15. (b) $la(10,5,3)=8$

$le(8,5,3)=3$

$ma(10,4,3)=\frac{1}{2}[7+6]=\frac{13}{2}=6\cdot 5$

16. **(c)** $ma(15,10,9)=\frac{1}{2}[19+15]=12$

$\min(10,6)=6$

$le(9,8,12)=1$

$le(15,6,1)=9$

17. **(c)** $(2\#1)/(1/\Delta 2)=\frac{2+1}{2^2+1}=\frac{3}{8}$

18. **(a)** Numerator $=4-[(10^{1\cdot 3}\Delta\log_{10})0\cdot 1)]$

$=4-(10^{1\cdot 3}\Delta(-1))=4-1=3$

Denominator $=1\nabla 2=2^{1+2}=8$

Hence answer $=\frac{3}{8}$

19. **(b)** Try for $(a),(c)$ and (d) all give numerator and denominators as 1 i.e., $\frac{Num}{Den}=\frac{1}{1}=1$

Hence (b) is the answer.

20. **(b)** Going by option elimination.

(a) will be invalid when $x+y=0$

(b) is the correct option as both sides gives $-2|x+y|$ as the result.

(c) will be equal when $(x+y)=0$

(d) is not necessarily equal (plug values and check)

21. **(c)** Consider option (c) as

$-F(x,y).G(x,y)=-[-|x+y|.|x+y|]=4x^2$ for $x=y$.

And $\log_2 16=\log_2 2^4=4,$ which gives value of option (c) as x^2.

22. **(b)** Solve sequentially from innermost bracket to get the answer. Answer is (b).

23. **(d)** From the graph $F1(x)=F(x)$ for $x\in(-2,0)$ but, $F1(x)=-F(x)$ for $x\in(0,2)$.

24. **(d)** From the graphs, $F1(x)=-F(x)$ and also $F1(x)=F(-x)$. So both (a) and (b) are satisfied which is not given in any of the option.

25. **(d)** By observation $F1(x)=-F(x)$ and also $F1(x)=F(-x)$. So both (a) and (b) are satisfied. Since no option is given mark (d) as the answer.

26. **(c)** By observation $F1(x)=-F(-x)$. This can be checked by taking any value of x say 1, 2. So answer is (c).

27. **(a)** $@(A,B)=\dfrac{A+B}{2}$

$\backslash(@(A,B),2)=\left(\dfrac{A+B}{2}\right)\times 2=A+B$

28. **(d)** $X(\backslash(@(\backslash(@(A,B),2),C),2),3)$

$=\left(\left(\left(\left(\dfrac{(A+B)}{2}*2\right)+C\right)/2\right)*2\right)/3=\dfrac{A+B+C}{3}$

= average of A,B and C.

29. **(d)** $\begin{Bmatrix} x^2<x, & 0<x<1 & f(x,y)=(x+y)^{0\cdot 5} \\ x^2>x & 1<x & g(x,y)=(x+y)^2 \end{Bmatrix}$ when x and y are positive thus for $x+y>1,(x+y)^{0\cdot 5}<(x+y)^2$ therefore, $f(x,y)<g(x,y)$

we can therefore eliminate answer option a if x and y are both negative then $f(x,y)=(x+y)^2$ and $g(x,y)=-(x+y)$ now for $-1<x+y<0$, then $(x+y)^2<-1x+y$

therefore $f(x,y)<g(x,y)$

thus answer option b is eliminated. As evident from the above discussion, for x and $y>-1$, we cannot again guarantee that $f(x,y)>g(x,y)$.

30. **(c)** When $0\le x,y<0\cdot 5,x+y$ may be <1 or 1, so given statement (a) can be true or false.

When $x,y<-1$, again statement (b) can be true of false.

When $x,y>1,x+y>1$ hence $f(x,y)<g(x,y)$.

$f(x,y)>g(x,y)$

Thus statements (c) given is necessarily false.

31. **(b)** When $x+y=1$ we have $(x+y)^2=(x+y)^{0\cdot 5}$

i.e., $f(x,y)=g(x,y)$

Thus answer is (b)

32. **(b)** It is not liner in x and y that's way option (a) is neglected. It also can't be exponential. By substituting X and Y in $y=a+bx+cx^2$ we see that it gets satisfied,.

33. **(d)** $f(x+1,y)=f[f,f(x,y)]$

put $x=0,f(1,y)=f[0,f(0,y)]=f[0,y+1]$

$=y+1+1=y+2$

put $y=2,f(1,2)=4$

34. **(b)** As graph is symmetrical about y-axis, we can say function is even, so $f(x)=f(-x)$.

35. **(d)** We see from the graph. Value of $f(x)$ in the left region is twice the value of $f(x)$ in the right region.

so $2f(x)=f(-x)$ or $6f(x)=3f(-x)$

36. **(c)** $f(-x)$ is replication of $f(x)$ abut y axis $-f(x)$ is replication of $f(x)$ about $x-$axis and $-f(-x)$ is replication of $f(x)$ about $y-$axis followed by replication about $x-$axis. Thus given graph is of $f(x)=-f(-x)$.

37. **(c)** Putting the actual values in the functions, we get the required answers.

$m(a,b,c)=-5,M(a,b,c)=2$

so $[m(a,b,c)+MM(a,b,c)]/2$ is maximum.

38. **(c)** $m(a,b,c)=\min(a+b,c,a)$;

$-M(-a,a,-b)$
$=-\max(0,-b-a);$
$m(a+b,b,c)=\min(a+2b,c,a+b)$

39. **(c)** $m(M(a-b,b,c),m(a+b,c,b),-M(a,b,c))=m(3,4-6)=-6$

40. **(d)** $f(1)=\frac{1}{1+1}=\frac{1}{2}$ as x is positive.

$f^2(1)=f(f(1))=\frac{1}{1+1/2}=\frac{2}{3}$

$f^3(1)=f(f^2(1))=f[2/3]=\frac{3}{5}$

$f^4(1)=\frac{5}{8}$ thus $f^1(1)f^2(x)f^3(1)...f^9(1)=\frac{1}{8}$

41. **(c)** When x is negative, $f(x)=1+x$

$f(-1)=1-1=0;$

$f^2(-1)=f(f(-1))=f(0)=1;$

$f^3(-1)=f(f^2(-1))f(1)=\frac{1}{1+1}=\frac{1}{2};$

$f^4(-1)=f(f^4(-1))f(1/2)=2/3$ and $f^5(-1)=3/5$

42. **(d)** Clearly $BA\geq MB\,A_1$ and $MB\,A_2\leq BA$ as $n_1>n_1+n_2$.

So option (a), (b) and (c) are neglected.

see $BA=\frac{r_1}{n_1}+\frac{r_2}{n_1}\geq\frac{r_1}{n_1}+\frac{n_2}{n_1}\max\times\left[0,\frac{r_2}{n_2}-\frac{r_1}{n_1}\right]$

because $\frac{r_2}{n_1}\geq 0$ and

$\frac{r_2}{n_1}\geq\left(\frac{n_2}{n_1}\times\frac{r_2}{n_2}-\frac{n_2}{n_1}\times\frac{r_1}{n_1}\right)$ or $\frac{r_2}{n_1}\geq\frac{r_2}{n_1}-\frac{n_2r_1}{n_1^2}$

So none of the answers match.

43. **(b)** Initial $Ba=50, BA$ increases as numerator increases with denominator ruminator remaining the same

$MB\,A_2=\frac{r_1+r_2}{n_1+n_2}$ decreases as average of total runs decreases form 50, as runs scored in this inning are less than 50.

44. **(b)** $f(x)=\log\left(\frac{1+x}{1-x}\right)$ and $f(y)=\log\left(\frac{1+y}{1-y}\right)$

$\therefore\ f(x)+f(y)=\log\left(\frac{1+x}{1-x}\right)+\log\left(\frac{1+y}{1-y}\right)$

$=\log\left\{\left(\frac{1+x}{1-x}\right)\left(\frac{1+y}{1-y}\right)\right\}=\log\left(\frac{1+x+y+xy}{1-x-y+xy}\right)$

$$= \log \frac{(1+xy)\left(1+\frac{x+y}{1+xy}\right)}{(1+xy)\left(1-\frac{x+y}{1+xy}\right)}$$

[Divide the Nr and Dr by $(1+xy)$]

$$= \log \frac{1+\frac{x+y}{1+xy}}{1-\frac{x+y}{1+xy}} = f\left(\frac{x+y}{1+xy}\right)$$

45. (d) $[x]$ means if $x = 5\cdot5$, then $[x] = 5$

$L[x, y] = [x] + [y] + [x + y]$

$R(x, y) = [2x] + [2y]$

Relationship between $L(x, y)$ and $R(x, y)$ can be found by putting various values of x and y.

Put $x = 1\cdot6$ and $y = 1\cdot8$

$L(x, y) = 1 + 1 + 3 = 5$ and $R(x, y) = 3 + 3 = 6$

So (b) and (c) are wrong.

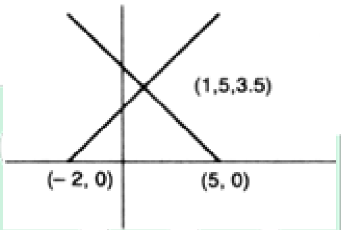
If $x = 1\cdot2$ and $y = 2\cdot3$

$L(x, y) = 1 + 2 + 3 = 6$ and $R(x, y) = 2 + 4 = 6$

or $R(x, y) = L(x, y)$, so (a) is not true.

We see that (d) will never be possible.

46. (d) $g(x) = \max(5 - x, x + 2)$. Drawing the graph



The dark lines represent the function $g(x)$. It clearly shoes the smallest value of $g(x) = 3\cdot5$.

47. (b) $f(x) = |x-2| + |2\cdot5 - x| + |3\cdot6 - x|$ can attain minimum value when either of the terms = 0.

Case I :

When $|x-2| = 0 \Rightarrow x = 2$, value of $f(x) = 0\cdot5 + 1\cdot6 = 2\cdot1$.

Case II :

When $|2\cdot5 - x| = 0 \Rightarrow x = 2\cdot5$ value of $f(x) = 0\cdot5 + 0 + 1\cdot1 = 1\cdot6$

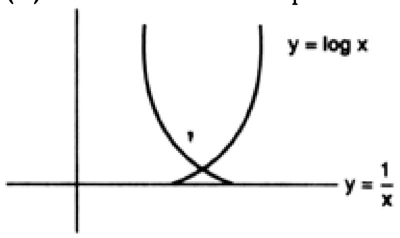
Case III :

When $|3\cdot6 - x| = 0 \Rightarrow x = 3\cdot6$

$\Rightarrow f(x) = 1\cdot6 + 1\cdot1 + 0 = 2\cdot7$

Hence the minimum value of $f(x)$ is $1\cdot6$ at $x = 2\cdot5$.

48. (b) The curved can be plotted as follows :



We see that they meet once.

49. **(d)** Substitute values $-2 \le x \le 2$ in the given curves. We find the curves will intersect at $x = 0, 1$ and -1.

50. **(a)** From the table, we have $g * g = h$ (this is g squared)

$h * g = f$ (this is g cubed)

$h * g = e$ (this is g to power 4)

51. **(d)** $f \oplus [f * \{f \oplus (f * f)\}]$ is to be simplified. So we start from the innermost bracket.

$f * f = h$

$f \oplus h = e$

$f * e = f$

$f \oplus f = h$

52. **(a)** $\{a^{10} * (f^{10} \oplus g^{9})\} \oplus e^{8}$

$f * f = hg * g = ha * a = ae * e = e$

$h * f = gh * g = fa^{10} = ae^{8} = e$

$g * f = ef * g = e$

$e * f = ef * g = e$

$e * f = fe * g = g$

$f^{5} = fg^{5} = g$

So, $f^{10} = f^{5}$ & $f^{5} = f * f = h$ So, $g^{9} = g^{5} * g^{4} = g * e = g$

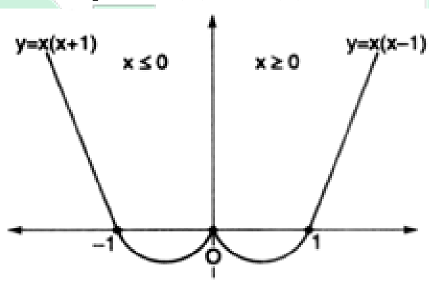
$\because$ $\{a^{10} * (f^{10} \oplus g^{9})\} \oplus e^{8}$

$\{a * (h \oplus g)\}e$

$\{a * f\} \oplus e \Rightarrow e$.

53. **(d)** $y = ax^2 - b|x|$

As the options (a) and (c) include $a > 0, b > 0$



We take $a = b = 1$

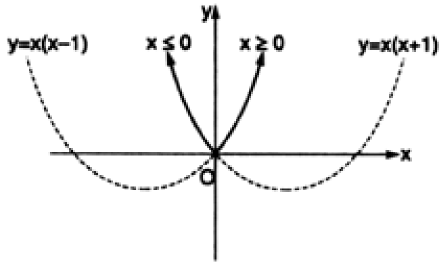
Accordingly the equation becomes $y = x^2 - |x|$.

A quick plot gives us.

So at $x = 0$ we neither have a maximum nor a minima.

As the option (b) and (d) include $a > 0, b > 0$

We take $a = 1, b = -1$



According the equation becomes $y = x^2 + |x|$

So at $r = 0$, we have a minima.

54. **(b)** $f(x) = x^3 - 4x + p$

$f(0) = p, f(1) = p - 3$

Given $f(0)$ and $f(1)$ are of opposite signs.

$p(p-3) < 0$

If $p < 0$ then $p - 3$ is also less than 0.

$\therefore$ $p(p-3) > 0$ i.e., p cannot be negativce.

$\therefore$ choice (a), (c) and (d) are eliminated

$0 < p < 3$

55. **(b)** Consider the product $f_1(x)f_2(x)$;

for $x \geq 0, f_2(x) = 0$ hence $f_1(x)f_2(x) = 0$

and for $x < 0 f_1(x) = 0,$ hence $f_1(x)f_2(x) = 0$

Consider the product $f_2(x)f_3(x)$;

for $x \geq 0, f_2(x) = 0, f_3(x) = 0$, hence $f_2(x)f_3(x) < 0$

for $x < 0, f_2(x) > 0, f_3(x) < 0$, hence $f_2(x)f_3(x) < 0$

Consider the product $f_2(x)f_4(x)$

for $x \geq 0, f_2(x) = 0, f_3(x) = 0$, hence $f_2(x)f_3(x) = 0$

for $x < 0, f_4(x) = 0,$ hence $f_2(x).f_4(x) = 0$

$\therefore$ $f_1(x).f_2(x)$ and $f_2(x).f_4(x)$ always take a zero value.

56. **(b)** choice (a) : from the graph it can be observed that $f_1(x) = f_4(x),$ for $x \leq 0$ but $f_1(x) \neq f_4(x)$ for $x > 0$.

Choice (b) : The graph of $f_3(x)$ is to be reflected x-axis flowed by a reflection in y-axis (in either order), to obtain the graph of $-f_3(-x)$ this would give the graph of $f_1(x)$.

Choice (c) : The graph of $f_2(-x)$ is obtained by the reflection of the graph of $f_2(x)$ in y-axis, which gives us the graph of $f_1(x)$ and not $f_4(x)$ hence option 3 is ruled out.

Choice (d) : for $x > 0 f_1(x) > 0$ and $f_3 = 0$ hence $f_1 = (x) + f_3(x) > 0$

57. **(d)** $g(x+1) + g(x-1) = g(x)$

$g(x+2) + g(x) = g(x+1)$

Adding these two equations, we get

$g(x+2) + g(x-1) = 0$

$\Rightarrow$ $g(x+3) + g(x) = 0$

$\Rightarrow g(x+4) + g(x+1) = 0$

$\Rightarrow g(x+5) + g(x+2) = 0$

$\Rightarrow g(x+6)+g(x+3)=0$

$\Rightarrow g(x+6)-g(x)=0$

58. **(d)** From the graph of $(y-x)$ versus $(y+x)$, it is obvious that inclination is more than 45^o.

Slope of line $=\dfrac{y-x}{y+x}=\tan(45^o+\theta)$

$\Rightarrow \dfrac{y-x}{y+x}=\dfrac{1+\tan\theta}{1-\tan\theta}$

By componendo-dovidendo $\dfrac{y}{x}=\dfrac{-1}{\tan\theta}$ which is nothing but the slope of the line that shows the graph of y versus x. And as $0^o<\theta<45^o$, absolule value of $\tan\theta$ is less than 1.

$\dfrac{-1}{\tan\theta}$ is negative and also greater than 1.

$\Rightarrow$ The slope of the graph y versus x must be negative and greater than.1. accordingly, only option (d) satisfies. This can also be tried by putting the value of $(y+x)=2$ (say) and $(y-x)=4$

Hence, we can solve for value of y and x and cross-check with the given options.

59. **(e)** $f(x)=\max(2x+1,3-4x)$

Therefore, the two equations are

$y=2x+1$ and $y=3-4x$

Now, $y-2x=1$

$\Rightarrow \dfrac{y}{1}+\dfrac{x}{-1/2}=1$

Similarly, $y+4x=3$

$\Rightarrow \dfrac{y}{3}+\dfrac{x}{3/4}=1$

$\therefore$ Their point of intersection would be

$2x+1=3-4x$

$\Rightarrow 6x=2\Rightarrow x=\dfrac{1}{3}$

So, when $x\le\dfrac{1}{3}$, then $f(x)_{\max}=3-4x$

and when $x\ge\dfrac{1}{3}$, then $f(x)_{\max}=2x+1$

Hence, the minimum of this would be at $x=\dfrac{1}{3}$

i.e, $y=\dfrac{5}{3}$

Alternative method :

As $f(x)=\max(2x+1,3-4x)$

We know that $f(x)$ would be minimum at the point of intersection of these curves.

i.e, $2x+1=3-4x$

i.e., $6x=2\Rightarrow x=\dfrac{1}{3}$

Hence, minimum value of $f(x)$ is $\frac{5}{3}$.