Study Adda

CAT

Chapter wise solved paper

CAHPTER 8 ALGEBRA

- **Directions (Qs. 1–18) : Answer the questions independent of each other.**

1. **If** $\log_7 \log_5(\sqrt{x+5}+\sqrt{x})=0,$ **find the value of x**
(a) 1 (b) 0 (c) 2 (d) None of these (1994)

2. If a + b + c = 0, where $a \neq b \neq c$ c, then $\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ac}+\frac{c^2}{2c^2+ab}$ is equal to
(a) zero (b) 1 (c) –1 (d) ate (1994)

3. **If the harmonic mean between two positive numbers is to their geometric mean as 12 : 13; then the numbers could be in the ratio**
(a) 12 : 13 (b) 1/12 : 1/13 (c) 4 : 9 (d) 2 : 3 (1994)

4. **If one root of** $x^2+px+12=0$ **is 4, while the equation** $x^2+px+q=0$ **has equal roots, then the value of if is**
(a) 49/4 (b) 4/49 (c) 4 (d) 1/4 (1994)

5. **Fourth term of an arithmatic progression is 8. What is the sum of the first 7 terms of the arithmatic progression;**
(a) 7 (b) 64 (c) 56 (d) Can't be determined (1994)

6. **What is the value of m which satisfies** $3m^2-21m+30<$
(a) $m<2$, or m > 5 (b) m > 2 (c) 2 < m < 5 (d) m < 5 (1995)

7. **The value of** $\frac{(55)^3+(45)^3}{(55)^2-55\times 45+(45)^2}$ **is**
(a) 100 (b) 105 (c) 125 (d) 75 (1995)

8. 5^6-1 is divisible by
(a) 13 (b) 31 (c) 5 (d) None of these (1995)

9. **One root of** $x^2+kx-8=0$ **is square of the other, then, the value of k is**
(a) 2 (b) 8 (c) –8 (d) –2 (1995)

10. **Once I had been to the post-office to buy stamps of five rupees, two rupees and one rupee. I paid the clerk Rs. 20, and since he did not have change, he gave me three more stamps of one rupee. If the number of stamps of each type that I had ordered initially was more than one, what was the total number of stamps that 1 bought.**
(a) 10 (b) 9 (c) 12 (d) 8 (1994)

11. **Given the quadratic equation x² - (A - 3) x - (A - 2), for what value of A will the sum of the squares of the roots be zero**
(a) –2 (b) 3 (c) 6 (d) None of these (1996)

12. **Which of the following values of x do not satisfy the inequality** $(x^2-3x+2>0)$ **at all ?**
(a) $1 \le x \le 2$ (b) $-1 \ge x \ge -2$ (c) $0 \le x \le 2$ (d) $0 \ge x \ge -2$ (1996)

13. $\log_2[\log_7(x^2-x+37)]=1$, then what could be the value of x ?
(a) 3 (b) 5 (c) 4 (d) None of these (1996)

14. **P and Q are two integers such that (PQ) = 64. Which of the following cannot be the value of P + Q ?**
(a) 20 (b) 65 (c) 16 (d) 35 (1997)

15. **If the roots,** x_1 **and** x_2**, of the quadratic equation** $x^2-2x+c=0$ **also satisfy the equation** $7x_2-4x_1=47$ **, then which of the following is true ?**

(a) $c = -15$ (b) $x_1 = -5, x_2 = 3$ (c) $x_1 = 4\cdot5, x_2 = -2\cdot5$ (d) None of these (1997)

16. One year payment to the servant is Rs. 90 plus one turban. The servant leaves after 9 months and receives Rs. 65 and a turban. Then find the price of the turban

(a) Rs. 10 (b) Rs. 15 (c) Rs. $7\cdot5$ (d) Cannot be determined (1998)

17. You can collect rubies and emeralds as many as you can. Each ruby is worth Rs. 4 crores and each emerald is wroth of Rs. 5 crore. Each ruby weights $0\cdot3$ kg and each emerald weighs $0\cdot4$ kg. Your bag can carry at the most 12 kg. What you should collect to get the maximum wealth ?

(a) 20 rubies and 15 emeralds (b) 40 rubies
(c) 28 rubies and 9 emeralds (d) None of these (1998)

18. For the given pair (x, y) of positive integers, such that $4x - 17y = 1$ and $x \le 1{,}000$, how many integer values of y satisfy the given conditions

(a) 55 (b) 56 (c) 57 (d) 58 (1999)

- **Directions (Qs. 19–21) : Read the information given below and answer the questions that/allows:**

These are m vessels with known volumes $V_1, V_2, \ldots, V_m$ arranged in ascending order of volumes, where V_1 is greater than $0\cdot5$ litre and V_m is less than 1 litre. Each of these is full of water. The water is emptied into a minimum number of white empty vessels each having volume 1 litre. If the volumes of the vessels increases with the value of lower bound 10^{-1}.

19. What is the maximum possible value of m

(a) 7 (b) 6 (c) 5 (d) 8 (1999)

20. If m is maximum, then what is minimum number of white vessels required to empty it ?

(a) 7 (b) 6 (c) 5 (d) 8 (1999)

21. If m is maximum, then what is range of the volume remaining empty in the vessel with the maximum empty space

(a) $0\cdot45 - 0\cdot55$ (b) $0\cdot55 - 0\cdot65$ (c) $0\cdot1 - 0\cdot75$ (d) $0\cdot75 - 0\cdot85$ (1999)

22. Find the following sum

$1/(2^2-1) + 1/(4^2-1) = 1/(6^2-1) + \ldots + 1/(20^2-1)$

(a) 9/10 (b) 10/11 (c) 19/21 (c) 10/21 (2000)

23. $x > 2, y > -1$ then which of the following holds good ?

(a) $xy > -2$ (b) $xy < -1$ (c) $x > -2/y$ (d) None of these

24. A, B and C are 3 cities that form a triangle and where every city is connected to every other one by at least one direct root. There are 33 routes direct and indirect from A to C and there are 23 direct routes from B to A. How many direct routes are there from A to C ?

(a) 15 (b) 10 (c) 20 (d) 25 (2000)

25. If the equation $x^3 - ax^2 + bx - a = 0$ has three real roots then the following is true

(a) $a = 11$ (b) $a \ne 11$ (c) $b = 1$ (d) $b \ne 1$ (2000)

26. $|x^2 + y^2| = 0\cdot1$ **and** $|x - y| = 0\cdot2$**, then the value of** $|x| + |y|$ **is**

(a) $0\cdot6$ (b) $0\cdot2$ (c) $0\cdot36$ (d) $0\cdot4$ (2000)

27. Let x, y and 2 be distinct integers, x and y are odd positive, and 2 is even and positive. Which one of the following statements cannot be true ?

(a) $(r-z)^2 y$ is even (b) $(x-z)y^2$ is odd (c) $(x-z)y$ is odd (d) $(x-y)^2 z$ is even (2001)

28. If $x > 5$ and $y < -1$, then which of the following statements is true ?

(a) $(x+4y) > 1$ (b) $x > -4y$ (c) $-4x < 5y$ (d) None of these (2001)

29. **Two men X and Y started working for a certain company at similar jobs on January 1, 1950. X asked for an initial salary of Rs. 300 with an annual increment of Rs. 30. Y asked for an initial salary of Rs. 200 with a rise of Rs. 15 every six months. Assume that the arrangements remained unaltered till December 31, 1959. Salary is paid on the last day of the month. What is the total amount paid to them as salary during the period ?**
(a) Rs. 93,300 (b) Rs. 93,200 (c) Rs. 93,100 (d) None of these (2001)

30. **x and y are real numbers satisfying the conditions $2 < x < 3$ and $-8 < y < -7$. Which of the following expression will have the least value ?**
(a) x^2y (b) xy^2 (c) $5xy$ (d) None of these (2001)

31. **m is the smallest positive integer such that for any integer $n \le m$, the quantity $n^3 - 7n^2 + 11n - 5$ is positive. What is the value of m ?**
(a) 4 (b) 5 (c) 8 (d) None of these (2001)

32. **All the pages numbers from a book are added, beginning at page 1. However, one page number was mistake added twice. The sum obtained was 1000. Which page number was added twice ?**
(a) 44 (b) 45 (c) 10 (d) 12 (2001)

33. **If a, b, c and d are four positive real numbers such that $abcd = 1$, what is the minimum value of $(1+a)(1+b)(1+c)(1+d)$?**
(a) 4 (b) 1 (c) 16 (d) 18 (2001)

34. **For a Fibonacci sequence, from the third term onwards, each term in the sequence is the sum of the previous two terms in that sequence. If the difference in squares of seventh and sixth terms of this sequence is 517, what is the tenth term of this sequence ?**
(a) 147 (b) 76 (c) 123 (d) Cannot be determined (2001)

35. **Let x, y be two positive numbers such that $x + y = 1$. Then, the minimum value of $\left(x+\frac{1}{x}\right)^2 + \left(y+\frac{1}{y}\right)^2$ is**
(a) 12 (b) 20 (c) $12\cdot5$ (d) $13\cdot3$ (2001)

36. **Let b be a positive integer and $a = b^2 - b$. If $b \le 4$ then $a^2 - 2a$ is divisible by**
(a) 15 (b) 20 (c) 24 (c) None of these (2001)

37. **Ujakar and Keshab attempted to solve a quadratic equation. Ujakar made a mistake in writing down the consist term. He ended up with the roots (4, 3). Keshab made a mistake in writing down the coefficient of x. He got Ih roots a5 (3, 2). What will be the exact roots of the original quadratic equation ?**
(a) (6, 1) (b) (−3, −4) (c) (4, 3) (d) (−4, −3) (2001)

38. **The nth element or a series is represented as $X_a = (-1)^n X_n - 1$. If $X_0 = x$ and $x > 0$ then the following is always true**
(a) X_n is positive if n is even (b) X_n is positive if n is odd
(c) X_n is negative if n is even (d) None of these (2002)

39. **If x, y and z are real numbers such that, x + y + z = 5 and xy + yz + zx = 3. What is the largest value that x can have ?**
(a) $\frac{5}{3}$ (b) $\sqrt{19}$ (c) $\frac{13}{3}$ (d) None of these (2002)

40. **Let S denote the infinite sum $2 + 5x + 9x^2 + 14x^3 + 20x^4 + \ldots$, where $|x| < 1$ and the coefficient of**

x^{n-1} **is** $\frac{1}{2}n(n+3),(n=1,2,....)$ **Then S equals**

(a) $\frac{2-x}{(1-x)^3}$ (b) $\frac{2-x}{(1+x)^3}$ (c) $\frac{2+x}{(1-x)^3}$ (d) $\frac{2+x}{(1+x)^3}$ (2002)

41. If $x^2+5y^2+z^2=2y(2x+z)$ then which of the following statements are necessarily true ?

A. $x=2y$ B. $x=2z$ C. $2x=z$

(a) Only A (b) Only B and C (c) Only A and B (d) None of these (2002)

42. Amol was asked to calculate the arithmetic mean of ten positive integers each of which had two digits. By mistake, he interchanged the two digits, say a and b, in one of these ten integers. As a result, his answer for the arithmetic mean was $1\cdot 8$ more than what it should have been. Then $b-a$ equals :

(a) 1 (b) 2 (c) 3 (d) None of these (2002)

43. **A child was asked to add first few natural numbers (that is, 1+2+3+...) so long his patience permitted. As he stopped he gave the sum as 575. When the teacher declared the result wrong the child discovered, he had missed one number in the sequence during addition, The number he missed was**

(a) less than 10 (b) 10 (c) 15 (d) more than 15 (2002)

44. **The number of real roots of the equation** $\frac{A^2}{x}+\frac{B^2}{x-1}=1$ **where A and B are real numbers not equal to zero simultaneously is**

(a) None (b) 1 (c) 2 (d) 1 or 2 (2002)

45. **If** $pqr=1$, **the value of the expression** $\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}$ **is equal to**

(a) $p+q+r$ (b) $\frac{1}{p+q+r}$ (c) 1 (d) $p^{-1}+q^{-1}+r^{-1}$ (2002)

46. **Which one of the following conditions must p, if and r satisfy so that the following system of linear simultaneous equations has at least one solution, such that** $p+q+r\neq 0$

$x+2y-3z=p$

$2x+6y-11z=q$

$x-2y+7z=r$

(a) $5p-2q-r=0$ (b) $5p+2q+r=0$ (c) $5p+2q-r=0$ (d) $5p-2q+r=0$ (2003)

47. **The sum of 3rd and 15th elements of an arithmetic progression is equal to the sum of 6th, 11th and 13th elements of the progression. Then which element of the series should necessarily be equal to zero**

(a) 1 st (b) 9 th (c) 12 th (d) None of these (2003)

48. **The number of non-negative real roots of** $2^x-x-1=0$ **equals**

(a) 0 (b) 1 (c) 2 (d) 3 (2003)

49. **Let** a,b,c,d **be four integers such that** $a+b+c+d=4m+1$ **where m is a positive integer. Given m, which one of the following is necessarily true ?**

(a) The minimum possible value of $a^2+b^2+c^2+d^2$ is $4m^2-2m+1$

(b) The minimum possible value of $a^2+b^2+c^2+d^2$ is $4m^2+2m+1$

(c) The maximum possible value of $a^2+b^2+c^2+d^2$ is $4m^2-2m+1$

(d) The maximum possible value of $a^2+b^2+c^2+d^2$ is $4m^2+2m+1$ (2003)

50. **The 288th term of the series a, b, b, c, c, c, d, d, d, d, e, e, e, e, e, f, f, f, f, f, f, ... is**

(a) u (b) υ (c) ω (d) x (2003)

51. Let p and q be the roots of the quadratic equation $x^2-(\alpha-2)x-\alpha-1=0$. What is the minimum possible value of p^2+q^2?
(a) 0 (b) 3 (c) 4 (d) 5 (2003)

52. $\log_3 2, \log_3(2^x-5), \log_3(2^x-7/2)$ are in arithmetic progression, then the value of x is equal to
(a) 5 (b) 4 (c) 2 (d) 3 (2003)

53. **There are 8436 steel balls, each with a radius of 1 centimetre, stacked in a pile, with 1 ball on top, 3 balls in the second layer, 6 in the third layer, 10 in the fourth, and so on. The number of horizontal layers in the pile is**
(a) 34 (b) 38 (c) 36 (d) 32 (2003)

54. **If the product of n positive real numbers is unity, then their sum is necessarily**
(a) a mutliple of n (b) equal to $n+\frac{1}{n}$ (c) never less than n (d) a positive integer (2003)

55. **Given that** $-1\le \upsilon \le 1, -2\le u\le -0\cdot5$ **and** $-2\le z\le -0\cdot5$ **and** $\omega=\upsilon z/u$, **then which of the following is necessarily true ?**
(a) $-0\cdot5\le\omega\le2$ (b) $-4\le\omega\le4$ (c) $-4\le\omega\le2$ (d) $-2\le\omega\le-0\cdot5$ (2003)

56. **If x, y, z are distinct positive real numbers then** $\frac{x^2(y+z)+y^2(x+z)+z^2(x+y)}{xyz}$ **would be**
(a) greater than 4 (b) greater than 5 (c) greater than 6 (d) None of these (2003)

57. **In a certain examination paper, there are n questions. For j = 1, 2,...., n, there are** 2^{n-j} **students who answered j or more questions wrongly. If the total number of wrong answers is 4095, then the value of n is**
(a) 12 (b) 11 (c) 10 (d) 9 (2003)

58. **The infinite sum** $1+\frac{4}{7}+\frac{9}{7^2}+\frac{16}{7^3}+\frac{25}{7^4}+....$ **equals**
(a) $\frac{27}{14}$ (b) $\frac{21}{13}$ (c) $\frac{49}{27}$ (d) $\frac{256}{147}$ (2003)

59. **The number of roots common between the two equations** $x^3+3x^2+4x+5+0$ **and** $x^3+2x^2+7x+3=0$ **is**
(a) 0 (b) 1 (c) 2 (d) 3 (2003)

60. **A real number x satisfying** $1-\frac{1}{n}<x\le3+\frac{1}{n}$, **for every positive integer n, is best described by**
(a) $1<x<4$ (b) $1<x\le3$ (c) $0<x\le4$ (d) $1\le x\le3$

61. **If x and y are integers then the equation 5x+ 19y = 64 has**
(a) no solution for x < 300 and y < 0 (b) no solution for x > 250 and y > –100
(c) a solution for 250 < x < 300 (d) a solution for – 59 < y < – 56 (2003)

62. **If both n and b belong to the set (1, 2, 3, 4), then the number of equations of the form** $ax^2+bx+1=0$ **having real roots is**
(a) 10 (b) 7 (c) 6 (d) 12 (2003)

63. **What is the sum of 'n' terms in the series :** $\log m+\log(m^2/n)+\log(m^3/n^2)+\log(m^4/n^3)+...$ **log**
(a) $\log\left[\frac{n^{(n-1)}}{m^{(n+1)}}\right]^{n/2}$ (b) $\log\left[\frac{m^m}{n^n}\right]^{n/2}$ (c) $\log\left[\frac{m^{(1-n)}}{n^{(1-m)}}\right]^{n/2}$ (d) $\log\left[\frac{m^{(1+n)}}{n^{(n-1)}}\right]^{n/2}$ (2003)

64. **If three positive real numbers x, y, z satisfy y - x = z - y and xyz = 4, then what is the minimum possible value y ?**

(a) $2^{1/3}$ (b) $2^{2/3}$ (c) $2^{1/4}$ (d) $2^{3/4}$ (2003)

65. If n is such that $36 \le n \le 72$ then $x = \dfrac{n^2 + 2\sqrt{n}(n+4)+16}{n+4\sqrt{n}+4}$ satisfies

(a) $20 < x < 54$ (b) $23 < x < 58$ (c) $25 < x < 64$ (d) $28 < x < 60$ (2003)

66. If $13x + 1 < 2z$ and $z + 3 = 5y^2$,then

(a) x is necessarily less than y (b) x is necessarily greater than y

(c) x is necessarily equal to y (d) None of these

67. If $|b| \ge 1$ and $x = -|a|b$, then which one of the following is necessarily true ?

(a) $a - xb < 0$ (b) $a - xb \ge 0$ (c) $a - xb > 0$ (d) $a - xb \le 0$ (2003)

68. Let a, b, c, d and e be integers such that a = 6b = 12c, and 2b = 9d = 12e. Then which of the following pairs contains a number that is not an integer ?

(a) $\left|\dfrac{a}{27}, \dfrac{b}{e}\right|$ (b) $\left|\dfrac{a}{36}, \dfrac{c}{e}\right|$ (c) $\left|\dfrac{a}{12}, \dfrac{bd}{18}\right|$ (d) $\left|\dfrac{a}{6}, \dfrac{c}{d}\right|$ (2003)

69. Consider the set $T_n = \{n, n+1, n+2, n+3, n+4$ where n = 1, 2, 3,...., 96. How many of these sets contain 6 or any integral mutliple thereof (i.e., any one of the numbers 6, 12, 18, ...) ?

(a) 80 (b) 81 (c) 82 (d) 83 (2003)

70. If $\dfrac{1}{3}\log_3 M + 3\log_3 N = 1 + \log_{0.008} 5$, then :

(a) $M^9 = \dfrac{9}{N}$ (b) $N^9 = \dfrac{9}{M}$ (c) $M^3 = \dfrac{3}{N}$ (d) $N^9 = \dfrac{3}{M}$ (2003)

71. If x and y are integers then the equation 5x + 19y = 64 has :

(a) no solution for x < 300 and y < 0 (b) no solution for x > 250 and y > —100

(c) a solution for 250 < x < 300 (d) a solution for – 59 < y < – 56 (2003)

72. If $\log_{10} x - \log_{10}\sqrt{x} = 2\log_x 10$, then a possible value of x is given by :

(a) 10 (b) $\dfrac{1}{100}$ (c) $\dfrac{1}{1000}$ (d) None of these (2003)

73. Let S_1 be a square of side fl. Another square S_2 is formed by joining the mid-points of the sides of S_1. The same process is applied to S_2 to form yet another square S_3, and so on. If $A_1, A_2, A_3, \ldots$ be the areas and $P_1, P_2, P_3, \ldots$ be the perimeters of $S_1, S_2, S_3, \ldots$ respectively, then the ratio $\dfrac{P_1 + P_2 + P_3 + \ldots}{A_1 + A_2 + A_3 + \ldots}$ equals ;

(a) $\dfrac{2(1+\sqrt{2})}{a}$ (b) $\dfrac{2(2-\sqrt{2})}{a}$ (c) $\dfrac{2(2+\sqrt{2})}{a}$ (d) $\dfrac{2(1+2\sqrt{2})}{a}$ (2003)

74. The total number of integer pairs (x, y) satisfying the equation x + y = xy is :

(a) 0 (b) 1 (c) 2 (d) None of these (2004)

75. Suppose n is an integer such that the sum of the digits of n is 2, and $10^{10} < n < 10^{11}$. The number of different values for n is :

(a) 11 (b) 10 (c) 9 (d) 8 (2004)

76. If $\dfrac{a}{b+c} = \dfrac{b}{c+a} = \dfrac{c}{a+b} = r$ then r cannot take any value except :

(a) $\dfrac{1}{2}$ (b) -1 (c) $\dfrac{1}{2}$ or -1 (d) $-\dfrac{1}{2}$ or -1 (2004)

77. Let $u = (\log_2 x)^2 - 6\log_2 + 12$ where x is a real number. Then the equation $x^n = 256$, has :
(a) no solution for x (b) exactly one solution for x
(c) exactly two distinct solutions for x (d) exactly three distinct solutions for x (2004)

78. For a positive integer n, let P_n denote the product of the digits of n and S_n denote the sum of the digits of n. The number of integers between 10 and 1000 for which $P_n + S_n = n$ is
(a) 81 (b) 16 (c) 18 (d) 9 (2005)

79. In the $X - Y$ plane, the area of the region bounded by the graph of $|x+y|+|x-y|=4$ is
(a) 8 (b) 12 (c) 16 (d) 20 (2005)

80. If $x \geq y$ and y > 1, then the value of the expression $\log_x\left(\frac{x}{y}\right)+\log_y\left(\frac{y}{x}\right)$ can never be
(a) 1 (b) $-0\cdot5$ (c) 0 (d) 1 (2005)

81. A telecom service provider engages male and female operators for answering 1000 calls per day. A male operator can handle 40 calls per day whereas a female operator can handle 50 calls per day. The male and the female operators get a fixed wage of Rs. 250 and Rs. 300 per day respectively. In addition, a male operator gets Rs. 15 per call he answers and a female operator gets Rs. 10 per call she answers. To minimize the total cost, how many male operators should the service provider employ assuming he has to employ more than 7 of the 12 female operators available for the job ?
(a) 15 (b) 14 (c) 12 (d) 10 (2005)

82. Three Englishmen and three Frenchmen work for the same company. Each of them knows a secret not known to others. They need to exchange these secrets over person-to-person phone calls so that eventually each person knows all six secrets. None of the Frenchmen knows English and only one Englishman knows French. What is the minimum number of phone calls needed for the above purpose ?
(a) 5 (b) 10 (c) 9 (d) 15 (2005)

83. Consider a triangle drawn on the X - Y plane with its three vertices at (41, 0), (0, 41) and (0,0) each vertex being represented by its (X, Y) coordinates. The number of points with integer coordinates inside the triangle (excluding all the points on the boundary) is
(a) 780 (b) 800 (c) 820 (d) 741 (2005)

84. The digits of a three-digit number A are written in the reverse order to form another three digit number B. If B > A and B - A is perfectly divisible by 7, then which of the following is necessarily true ?
(a) $100 < A < 299$ (b) $106 < A < 305$ (c) $112 < A < 311$ (d) $118 < A < 317$ (2005)

85. If $a_1 = 1$ and $a_{n+1} - 3a_n + 24n$ for every positive integer n, then 0100 equals
(a) $3^{99} - 200$ (b) $3^{99} + 200$ (c) $3^{100} - 200$ (d) $3^{100} + 200$ (2005)

86. What are the values of x and y that satisfy both the equations ?

$$2^{0.7x}.3^{-1.25y} = \frac{8\sqrt{6}}{27}$$

$$4^{0.3x}.9^{0.2y} = 0.(81)^{1/5}$$

(a) x = 2, y = 5 (b) x = $2\cdot5$, y = 6 (c) x = 3, y = 5 (d) x = 3, y = 4
(e) x = 5, y = 2 (2006)

87. The number of solutions of the equation $2x + y = 40$, where both x and y are positive integers and x≤y is
(a) 7 (b) 13 (c) 14 (d) 18 (e) 20 (2006)

88. Consider the set $S=\{1,2,3,....,1000\}$. How many arithmetic progressions can be formed from the elements of S that start with 1 and end with 1000 and have at least 3 elements ?

(a) 3 (b) 4 (c) 6 (d) 7 (e) 8 (2006)

89. What values of x satisfy $x^{2/3}+x^{1/3}-2\le 0$?

(a) $-8\le x\le 1$ (b) $-1\le x\le 8$ (c) $1<x<8$ (d) $1\le x\le 8$ (e) $-8\le x\le 8$ (2006)

90. If $\log_x x=(a\cdot\log_z y)=(b\cdot\log_x z)=ab$, then which of the following pairs of values for (a, b) is not possible ?

(a) $\left(-2,\frac{1}{2}\right)$ (b) $(1,1)$ (c) $(0.4,2.5)$ (d) $\left(\pi-\frac{1}{\pi}\right)$ (e) $(2,2)$ (2006)

ANSWERS

1.	B	2.	B	3.	C	4.	A	5.	C	6.	C	7.	A	8.	B
9.	D	10.	A	11.	D	12.	A	13.	C	14.	D	15.	A	16.	A
17.	B	18.	D	19.	C	20.	D	21.	C	22.	D	23.	D	24.	B
25.	D	26.	D	27.	A	28.	D	29.	A	30.	C	31.	D	32.	C
33.	C	34.	C	35.	C	36.	C	37.	A	38.	D	39.	C	40.	A
41.	C	42.	B	43.	D	44.	D	45.	C	46.	A	47.	C	48.	C
49.	B	50.	D	51.	D	52.	D	53.	C	54.	C	55.	B	56.	C
57.	A	58.	C	59.	A	60.	C	61.	C	62.	B	63.	D	64.	B
65.	D	66.	D	67.	B	68.	D	69.	A	70.	B	71.	C	72.	B
73.	C	74.	C	75.	A	76.	C	77.	B	78.	A	79.	C	80.	A
81.	D	82.	C	83.	A	84.	B	85.	C	86.	E	87.	B	88.	D
89.	A	90.	E												

SOLUTIONS

1. **(b)** $\log_7\log_5(\sqrt{x+5}+\sqrt{x})=0$

$\log_5(\sqrt{x+5}+\sqrt{x})=7^0=1$

$\sqrt{x+5}+\sqrt{x}=5^1=5\Rightarrow 2\sqrt{x}=0$

$\therefore\ x=0$

2. **(b)** Take any value of a,b,c such that

$a+b+c=0$ and $a\ne b\ne c$

Say $a=1,b=-1$ and c = 0

Substituting these values in

$$\frac{a^2}{2a^2+bc}+\frac{b^2}{2b^2+ac}+\frac{c^2}{2c^2+ab}=\frac{1}{2}+\frac{1}{2}+0=1$$

3. (c) $\frac{H.M.}{G.M.}=\frac{12}{13}\Rightarrow\frac{2ab}{(a+b)\sqrt{ab}}=\frac{12}{13}$

or $\frac{2\sqrt{ab}}{a+b}=\frac{12}{13}$ or $\frac{a+b}{2\sqrt{ab}}=\frac{13}{12}$

By componedo and dividendo

$$\frac{a+b+2\sqrt{ab}}{a+b-2\sqrt{ab}} = \frac{13+12}{13-12} = \frac{25}{1}$$

$$\frac{(\sqrt{a}+\sqrt{b})^2}{(\sqrt{a}-\sqrt{b})^2} = \frac{25}{1}$$

$$\frac{\sqrt{a}+\sqrt{b}}{\sqrt{a}-\sqrt{b}} = \frac{5}{1}$$

Again by compoenendo and dividendo

$$\frac{2\sqrt{a}}{2\sqrt{a}} = \frac{6}{4} \text{ or } \frac{a}{b} = \frac{9}{4} \text{ or } \frac{b}{a} = \frac{4}{9}$$

4. (a) $x^2 + Px + 12 = 0$

x = 4 will satisfy this equation

$\therefore\ 16 + 4P + 12 = 0 \Rightarrow P = -7$

Other eq. becomes $x^2 - 7x + q = 0$

Its roots are equal, so $b^2 = 4ac$

$\Rightarrow\ 49 = 4q$ or $49 = 4q$

5. **(c)** Fourth term $= 8 \Rightarrow a + 3d = 8$

sum of seven terms

$$= s_7 = \frac{7}{2}[2a = (7-1)d] = \frac{7}{2} \times 2(a+3d) = 7 \times 8 = 56$$

6. **(c)** $3m^2 - 21m + 30 < 0$

or $m^2 - 7m + 10 < 0$ or $m^2 - 5m - 2m + 10 < 0$

or $m(m-5) - 2(m-5) < 0$

or $(m-2)(m-5) < 0$

Case I : $m - 2 > 0$ and $m - 5 < 0$

$\Rightarrow\ m > 2$ and $m < 5 \Rightarrow 2 < m < 5$

Case II : $m - 2 < 0$ and $m - 5 > 0 \Rightarrow m < 2$ and m > 5

nothing common.

Hence, 2 < m < 5

7. **(a)** We know, $\dfrac{a^3+b^3}{a^2-ab+b^2} = a + b = 45 + 55 = 100$

8. **(b)** $5^6 - 1 = (125)^2 - 1 = (125-1)(125+1) = 124 \times 126 = 15624$

Which is divisible by 31.

9. **(d)** $x^2 + kx - 8 = 0$

Sum of roots $= a + b = -k = a + a^2$ ...(1)

Product of roots $= ab = -8 = a^3 \Rightarrow a = -2$

Using $a = -2$ in $(1), -k = -2 + 4 = 2$ or $x = -2$

10. **(a)** The number of stamps that were initially bought were more than one of each type. Hence the total number of stamps

= 2 (5 rupee) + 2 (2 rupee) + 3 (1 rupee) + 3 (1 rupee) =10 tickets

11. **(d)** Let the roots be m and n. The given quadratic equation can be written as $ax^2 + bx + c = 0$ where $a = 1, b = -(A-3), c = -(A-7)$.

The sum of the roots is $(m+n)=-(b/a)=A-3$
and the product of the roots is (nm)
$=(c/a)=-(A-7)$
the sum of the squares of the roots is
$[(m+n)^2-2mn]=(A-3)^2-2(-)(A-7)=0$
on solving, we get A = 5 or - 1.
None of these values are given in the options.

12. **(a)** $x^2-3x+2>0$
$\Rightarrow x^2-2x-x+2>0 \Rightarrow x(x-2)-1(x-2)>0$
$\Rightarrow (x-2)(x-1)>0$
This gives (x > 2) as one range and (x < 1) as the other.
In between these two extremes, there is no value of x which satisfies the given inequality.

13. **(c)** $\log_2[\log_7(x^2-x+37)]=1$
use $\log_p x = y \Rightarrow p^y = x$
$\therefore\ 2=\log_7(x^2-x+37)$
$\Rightarrow 49=x^2-x+37 \Rightarrow x^2-x-12=0$
$\Rightarrow (x-4)(x+3)=0 \quad \therefore\ x=4$

14. **(d)** $PQ=64=1\times 64=2\times 32=4\times 16=8\times 8$
Corresponding values of P + Q are 65, 34, 20,16.
Therefore, P + Q cannot be equal to 35.

15. **(a)** $7x_2-4x_1=47$
$x_1+x_2=2$
On solving, $11x_1=55$
$x_1=5$ and $x_2=-3$
$\therefore\ x=-15$

16. **(a)** Let turban be of cost Rs. x so, payment to the servant = 90 + x for 12 month
for 9 month $=\frac{9}{12}\times(90+x)=65+x \Rightarrow x=Rs.10$

17. **(d)** Basically, the question is of weights, so let us analyse them only 4 rubies weight as much as 3 emeralds.
4 rubies =16 crores
3 emeralds = 15 crores
All rubies, multiple of 4 allowed, is the best deal,
so $\frac{12}{0.3}=40$ rubies.

18. **(d)** $4x-17y=1$. And given that $1000\geq x$
Hence we can say that $17y+1\leq 4000$
i.e., $y\leq 235$
Further also note that every 4th value of y e.g., 3, 7,11,...... will give an intger value of x.
So number of values of y = 235/4 = 58.

19. **(c)** The lower bound is 05 and increases with $0\cdot 05$. It forms an arithmetic progression, where $0\cdot 05$ is the common difference and 05 is the first term. The term is less than 1 and hence it is $0\cdot95$. To find the number of terms in the series use the formulae on nth term i.e ; $T_n = a+(n-1)d$ where 'a' is the first term and 'd' is the common difference. Hence the value of n comes as 10. Maximum possible value of m is 10.

20. **(d)** The find the minimum number of white vessel required to empty the vessel for maximum possible value of m i.e., 10, we have to use the formulae of sum to n terms of this A.P. series. Sum to n terms is given by

$$S_n = \frac{n\times(First\,term + Last\,term)}{2}$$

where n is the number of terms in the series. For this series

$$S_n = \frac{10\times(0\cdot5+0\cdot95)}{2} = 7\cdot25$$

Hence, minimum number of white vessels that is required is 8 as the capacity of white vessel is 1 litre.

21. **(c)** From the above solution we can see that the eighth vessel is empty by 0.75 litre and hence that is the upper limit for the range. Further for the lower limit, make all the vessels equally full, which makes them all $0\cdot1$ parts empty. So, the option that satisfies the above condition is (c).

22. **(d)** nth term $T_n = \frac{1}{(4n^2-1)} = \frac{1}{2}\left[\frac{(2n+1)-(2n-1)}{(2n+1)(2n-1)}\right]$

$$= \frac{1}{2}\left[\frac{1}{(2n-1)} - \frac{1}{2n+1}\right]$$

$$S = \frac{1}{2}\left[\frac{1}{1}-\frac{1}{3}+\frac{1}{3}-\frac{1}{5}-\ldots-\frac{1}{19}+\frac{1}{19}-\frac{1}{21}\right]$$

$$= \frac{1}{2}\left[1-\frac{1}{21}\right] = \frac{10}{21}$$

23. **(d)** By putting different value of x and y we see that none of these three hold good.

24. **(b)** Let the number of direct routes from A to B be x, from A to C be z and that from C to B be y. Then the total number of routes from A to C are = xy + z = 33. Since the number of direct routes from A to B are 23, x = 23. Therefore 23y + z = 33. Then y must take value 1 and then z = 10, thus answer s = (b).

25. **(d)** Let $f(x) = x^3 - ax^3 - ax^2 + bx - a = 0$

In the given equation, there are 3 sign changes, therefore there are at most 3 positive roots. If $f(-x)$, there is no sign change. Thus there is no negative real root, i.e., if a, P and "y are the roots then they are all positive and we have $f(x) = (x-\alpha)(x-\beta)(x-\gamma) = 0$

$$x^3\overline{\alpha+\beta+\gamma}x^2 + \overline{\alpha\beta+\beta\gamma+\gamma\alpha}x - \alpha\beta\gamma$$

$$\Rightarrow\ b = \alpha\beta+\beta\gamma+\gamma\alpha \qquad \Rightarrow \qquad a = \alpha+\beta+\gamma = \alpha\beta\gamma$$

$$\Rightarrow (\alpha+\beta+\gamma)/\alpha\beta\gamma = 1 \Rightarrow 1/\alpha\beta + 1/\alpha\gamma + 1/\beta\gamma = 1$$

$$\Rightarrow\ \alpha\beta, \alpha\gamma, \beta\gamma > 1 \Rightarrow b > 3$$

Thus $b \neq 1$

26. (d) $x - y + 0\cdot2$ or $(x-y)^2 + 0\cdot04$.

Also $x^2 + y^2 = 0.1$ (since $x^2+y^2>0$)

And solving this two we get 2xy = 0.6 from this we can find value of x + y which comes out to be + 0.4 or - 0.4 and solving this two we get $|x|+|y| = 0.4$.

27. **(a)** $x, y, z > 0; x \,\&\, y$ are odd, z is even.

Note : [odd - even is odd], [odd - odd is even] [odd x odd is odd] since $(x-2)$ is odd.

$\therefore$ $(x-z)^2$ is also odd and $(x-z)^2 y$ is odd.

28. **(d)** $(x-z)^2 y$ cannot be even.
x > 5 and $y < -1 \Rightarrow 4y < -4$
(i) x > 5 and 4y < -4 so x + 4y < 1
Let x > − 4y be true ⇒ 4y < − 4 or − 4y > 4
So, x > 4, which is not true as given x > 5.
So, x > − 4y is not necessarily true.
(ii) x > 5 ⇒ − 4x < − 20 and 5y < − 5
It is not necessary that − 4x < 5y as − 4x can be greater than 5y, since 5y < − 5.
Hence none of the options is true.

29. **(a)** For total salary paid to X
= 12 × (300 + 330 +390+420 + 450 + 480+ 510 + 540+570)
$= 12 \times \frac{10}{2}$ [2× 300 + 9 ×30] [∵ sum of A.P.]
= 60 × 870 = Rs. 52,200
For total salary paid to Y
= 6 × [200 + 215 + 230 + 245 + 260 .... 20 terms]
= 6 × 10 × [2 × 200 + 19 × 15] [sum of A.P.]
= 60 × [400 + 285] = Rs. 41,000
Total sum of both = Rs. 93,300

30. **(c)** $2 < x < 3$ and $-8 < y < -7, -32 < x^3 y < -28$
While $-80 < 5xy < -70$
Hence $5xy$ is the least because xy^2 is positive.

31. **(d)** Let $y = n^3 - 7n^2 + 11n - 5$
At $n = 1, y = 0$
$\therefore$ $(n-1)(n^2 - 6n + 5) = (n-1)^2 (n-5)$
Now, $(n-1)^2$ is always positive.
Now, for n < 5, the expression gives a negative quantity.
Therefore, the least value of n will be 6. Hence m = 6.

32. **(c)** $\frac{x(x+1)}{2} = 1000 - y$
$x = 44, y = 10$

33. **(c)** $abcd = 1$
minimum value of (1 + a) (1 + b) (1 + c) (1 + d) is
$\Rightarrow 1 + a \geq 2\sqrt{a}$ [A.M. ≥G.M.]
$\therefore$ Min. value $= 2\sqrt{a} \times 2\sqrt{b} \times 2\sqrt{c} \times 2\sqrt{d} = 16\sqrt{abcd} = 16$

34. **(c)** $x_{n+1} = x_n + x_{n-1}$
$x_8 = x_7 + x_6$
$x_7^2 - x_6^2 = 517$
Taking $x_7 = 29$ and $x_6 = 18$ we have $x_8 = 47$

$\therefore\ x_9 = 47 + 29 = 76$ and $x_{10} = 76 + 47 = 123$.

35. **(c)** $\therefore\ x + y = 1$

$$\left(x+\frac{1}{x}\right)^2 + \left(y+\frac{1}{y}\right)^2 = x^2 + y^2 + \frac{1}{x^2} + \frac{1}{y^2} + 4$$

Min. value of $x^2 + y^2$ occur when $x = y$ $[\because x + y = 1]$

$\therefore$ Put $x = y = \frac{1}{2}$

Min. value $= \left(\frac{5}{2}\right)^2 + \left(\frac{5}{2}\right)^2 = \frac{25}{2} = 12\cdot5$

36. **(c)** $a = b(b-1)$

$a^2 - 2a = b^2[b^2 + 1 - 2a] - 2b(b-1)$

or $a(a-2) = b(b-1)(b^2 - b - 2)$

$= b(b-1)(b^2 - 2b + b - 2) = b(b-1)(b+1)(b-2)$

so this is divisible by 24 for $b \le 4$.

37. **(a)** $(x^2 - 7x + 12) \Rightarrow$ wrong equation $\Rightarrow$ Ujakar

(sum of roots = 7, product of roots = 12)

$x^2 - 5x + 6 \rightarrow$ wrong equation $\Rightarrow$ Keshab

(sum of roots = 5, product of roots = 6)

Hence the correct equation is $x^2 - 7x + 6$

6 and 1.

38. **(d)** $X_n = (-1)^n X_{n-1}$

put $n = 1, X_1 = (-1)^1 x_0$

$X_1 = -x$ $(x_0 = x$ given)

As x > 0 $\therefore$ X_1 is – ve

$X_2 = (-1)^2 X_1 = -x$, X_2 is- ve

$X_3 = (-1)^3 X_2 = x \Rightarrow$ X_3 is+ ve

$X_4 = (-1)^4 X_3 = x \Rightarrow$ X_4 is+ ve

therefore none of these.

39. **(c)** We know, $(x + y + z)^2 = x^2 + y^2 + z^2 + 2(xy + yz + zx)$

or $(5)^2 = x^2 + y^2 + z^2 + 2 \times 3$

For maximum value of x, y = z = 0

but both cannot be zero at the same time as $xy + yz + zx \ne 0$

So $x^2 < 19$ $\therefore$ x can be $13/3$ as $x^2 = 169/9 = 18\cdot8$

40. **(a)** $\frac{2-x}{(1-x)^3} = (2-x)(1-x)^{-3}$

Using binomial here

$= (2-x)(1 + 3x + 6x^2 + 10x^3 + \ldots \frac{(r+1)(r+2)}{2} x^2 + \ldots$

$= 2 + 5x + 9x^2 + 14x^3 + \ldots$

this is same series as given

41. **(c)** $x^2+5y^2+z^2=2y(2x+z)$

Put $x=2y$

$4y^2+5y^2+z^2=2y(4y+z)$

or $9y^2+z^2=8y^2+2yz$ ...(1)

this is not necessarily true

put $y=z$ in (1), we get

$9z^2+z^2=8z^2+2z^2$ or $10z^2=10z^2$

(1) is true for $y=z(x=2y \,\&\, x=2z \Rightarrow y=z)$

therefore only $A \,\&\, B$ satisfy the given result.

42. **(b)** Let $x_1, x_2, ..., x_{10}$ are +ve numbers

Let digits of x_{10} are interchanged.

original $x_{10}=10a+b$

after interchanging $x_{10}=10b+a$

according to question,

$$\frac{x_1+x_2+...+x_9+10b+a}{10}=1\cdot8+\frac{x_1+...+x_9+10a+b}{10}$$

$$\Rightarrow \frac{x_1+x_2+...+x_9+10b+a}{10}-\frac{x_1+x_2+...+x_9+10b+a}{10}=1\cdot8$$

$$\Rightarrow \frac{9b-9a}{10}=1\cdot8 \text{ or } (b-a)=\frac{1\cdot8\times10}{9}=2$$

43. **(d)** Since the child missed the number so actual result would be more than 575 therefore we choose n such

$\frac{n(n+1)}{2}>575$

for this least value of n is 34

$\therefore$ correct answer $=\frac{34(34+1)}{2}=595$

missing number = 595 – 575 = 20

44. **(a)** $\frac{A^2}{x}+\frac{B^2}{x-1}=1$

If only A = 0 there is only one root.

if only B = 0 there is only one root

if both A & B are not zero then there would be two roots (because quadratic equation forms)

$\therefore$ roots be 1 or 2

45. **(c)** $pqr=1$ (given)

$$\frac{1}{1+p+q^{-1}}+\frac{1}{1+q+r^{-1}}+\frac{1}{1+r+p^{-1}}$$

$$=\frac{q}{q+pq+1}+\frac{r}{r+qr+1}+\frac{p}{p+pr+1}$$

$$=\frac{q}{q+\frac{1}{r}+1}+\frac{r}{r+\frac{1}{p}+1}+\frac{p}{p+pr+1}$$

$$= \frac{qr}{qr+1+r} + \frac{pr}{pr+1+p} + \frac{p}{p+pr+1}$$

$$= \frac{qr}{\frac{1}{p}+1+r} + \frac{pr}{pr+p+1} + \frac{p}{p+pr+1}$$

$$= \frac{pqr}{1+p+pr} + \frac{pr}{1+pr+pr} + \frac{p}{1+p+pr}$$

$$= \frac{pqr}{1+p+pr} + \frac{pr}{1+p+pr} + \frac{p}{1+p+pr}$$

$$= \frac{pqr+pr+p}{1+p+pr} + \frac{1+p+pr}{1+p+pr} = 1 \quad (\because pqr = 1)$$

46. **(a)** Working from the choices, $5p-2q-r$
$= (5x+10y-15z)-(4x+12y-22z)-(x-2y+7z)=0$
For no other choices is the condition satisfied, hence (a).

47. **(c)** $T_n = a+(n-1)d$. Hence we get 3rd + 5th term $=(a+2d)+(a+4d)=2a+6d$. Similarly, 6,11 and 13th terms $=(a+5d)+(a+10d)+(a+12d)=3a+27d$. Now $2a+6d=3a+27d$, hence $a+11d=10$. This means that the 12th term is zero.

48. **(c)** It is clear that the equation $2^x-x-1=0$ is satisfied by $x=0$ and 1 only. For $x>1, f(x)=2^x-x-1$ starts increasing.

49. **(b)** Minimum value of $4m+1$ is 4 (1) +1 = 5.
Since $a+b+c+d=5$
We can have $a=b=c=1$ and $d=2$.
Then $a^2+b^2+c^2+d^2=1^2+1^2+1^2+2^2=7$

50. **(d)** This represents an A.P. with first term as 1 and common difference as 1.
Sum of terms $=\frac{n(n+1)}{2}$ which must be close to 288.
By hit and trial, we get for $n=23$, Sum $=\frac{23(24)}{2}=276$.
The 24th alphabet is x, hence the 288th term is 'x',

51. **(d)** Sum of roots, $p+q=\alpha=-2$
Product of roots, $pq=-\alpha-1$
Now $p^2+q^2=(p+q)^2-2pq=(\alpha-2)^2+2(\alpha+1)$
$=\alpha^2+4-4\alpha+2\alpha+2=(\alpha+1)^2+5$
Hence the minimum value of this will be 5.

52. **(d)** In an AP, the three terms a,b,c are related as $2b+a+c$
Hence, $2[\log_3(2^x-5)=\lg o_3 2+\log_3\left(2^x-\frac{7}{2}\right)$
$\log(2^x-5)^2=(2^{x+1}-7)$
Substitute the choices, only x = 3 satisfies the conditions.

53. **(c)** The number of balls in each layer is 1, 3, 6, 10.. ...(each term is sum of natural numbers upto 1, 2, 3, ..., n digits).

$\therefore \sum \frac{n(n+1)}{2} = 8436 \Rightarrow \sum n^2 + \sum n = 8436 \times 2$

$\frac{n(n+1)(2n+1)}{6} + \frac{n(n+1)}{2} = 8436 \times 2$

Solving, we get $n = 36$.

54. **(c)** The numbers must be reciprocals of each other.

Hence, $2 \times \frac{1}{2} = 1$ and $2 + \frac{1}{2} = 2\frac{1}{2} > 2$

Hence the sum is greater than the product of numbers.

55. **(b)** Substitute the extreme values in the inequalities :

$u = 1, u = -0 \cdot 5, z = -2$. Then $\omega = \upsilon z / u = 4$. Only (b) option gives this.

Simply substitute x = 1, y = 2 and z = 3 in the expression to get the answer.

56. **(a)** There are 2^{n-j} students who answer wrongly. For $j = 1, 2, 3, \ldots, n$, the number of students will be a GP with base 2. Hence $1 + 2 + 2^2 + ... + 2^{n-1} = 4095$.

Using the formula, we get $2^n = 4095 + 1 \Rightarrow n = 12$

57. **(c)** $S_n = 1 + \frac{4}{7} + \frac{9}{7^2} + \frac{16}{7^3}$ ...(1)

$\frac{1}{2} S_n = \frac{1}{7} + \frac{4}{7^2} + \frac{9}{7^3} + \frac{16}{7^4} + \ldots$ ....(2)

Subtracting (2) from (1),

$S_n \left(\frac{6}{7}\right) = 1 + \frac{3}{7} + \frac{5}{7^2} + \frac{5}{7^3} + \ldots$ ....(3)

$S_n \left(\frac{6}{7^2}\right) = \frac{1}{7} + \frac{3}{7^2} + \frac{5}{7^3} + \ldots$ ....(4)

Subtracting (4) from (3),

$S_n \left(\frac{36}{49}\right) = 1 + \frac{2}{7} + \frac{2}{7^2} + \ldots$

This becomes a GP with first term = 1 and common ratio =1/7

$\Rightarrow S_n \left(\frac{36}{49}\right) = 1 + \frac{2}{7} \left(\frac{1}{1 - \frac{1}{7}}\right)$ or $S_n = \frac{49}{27}$

58. **(a)** Subtract the two equations

$x^2 - 3x + 2 = 0$

$(x-1)(x-2) = 0$

the root 1 and 2 do not satisfy any of the original equation in case these was a common root, it will be the root of the subtracted equation.

So no root.

59. **(c)** $0 < \frac{1}{n} \le 1$ For positive n

$\Rightarrow 0 \le 1 - \frac{1}{n} < 1 \Rightarrow 3 < 3 + \frac{1}{2} \le 4$

$\Rightarrow 0 \le 1 - \frac{1}{n} < x \le 4 \Rightarrow 0 < x \le 4$

60. **(c)** $5x + 19y = 64$

We see that if y = 1, we get an integer solution for x = 9, now if y changes (increases or decreases) by 5, x will change (decrease or increase) by 19.

Looking at options, if x = 256 we get y = 64.

Using these values we see option 1, 2 and 4 are eliminated and also that these exists a solution for $250 < x \le 300$.

61. **(b)** $ax^2 + bx + 1 = 0$

for real roots $b^2 - 4ac \ge 0$

$b^2 - 4a \ge 0 \Rightarrow b^2 \ge 4a$

$4a = \{4, 8, 12, 16\}$

$b^2 = \{1, 4, 9, 16\}$

for $b^2 = 4$, number of solution = 1

for $b^2 = 9$, number of solution = 2

for $b^2 = 16$, number of solution = 4

Total number of solution = 4 + 2 + 1 = 7

62. **(d)** $S = \log m + \log \frac{m^2}{n} + \log \frac{m^3}{n^2} + \ldots n$ terms

$= \log\left(m \frac{m^2}{n} \frac{m^3}{n^2} \ldots \frac{m^n}{n^{n-1}}\right)$

$= \log\left(\frac{m^{\frac{n(n+1)}{2}}}{n^{\frac{n(n-1)}{2}}}\right) = \log\left(\frac{m^{(n+1)}}{n^{(n-1)}}\right)^{n/2}$

63. **(b)** $y = \frac{x+z}{2}, xyz = 4 \Rightarrow (x+z)xz = 8$

Let $x + z = a$

$\Rightarrow az(a - z) = 8 \Rightarrow az^2 - a^2z + 8 = 0$

For z to be real, $b^2 - 4ac > 1$

$\therefore a^4 - 32a > 0 \Rightarrow a^3 > 32$

$y = \frac{x+z}{2} = \frac{(32)^{1/3}}{2} = 2^{2/3}$

64. **(d)** $x = \frac{n^2 + 2\sqrt{n}(n+4) + 16}{n + 4\sqrt{n} + 4}$

Let $\sqrt{n} = t$

$\Rightarrow x = \frac{t^4 + 2t(t^2+4) + 16}{t^2 + 4t + 4} = \frac{(t+2)(t^3+8)}{(t+2)^2}$

$= \frac{t^3 + 8}{t + 2} = t^2 - 2t + 4$

For $t=6$ to $t=6\sqrt{2}$ putting in above equation)

$(40-12)<x<(72+4-12\sqrt{2})$

$\Rightarrow 28<x<76-12\sqrt{2}x$

or $28<x<60$

65. **(d)** $13x+1<2z$ and $z+3=5y^2$

$\Rightarrow 13x+1<2(5y^2-3)$

$\Rightarrow 13x+7<10y^2 \Rightarrow 10y^2>13x+7$

In the above equation, all the options (a), (b) and are possible.

66. **(b)** $b\geq 1$ or $b\leq -1x, x=-|a|b$

$a-xb=a-(-|a|b)b$

$=a+|a|b^2\geq 0$ since $b^2\geq 1$

67. **(d)** Given $a=6b=12c$

$2b=9d=12e$

So, $a=12, b=2c, d=\frac{4}{9}c, e=\frac{c}{3}$

From the options only (d) option $\left|\frac{a}{6},\frac{c}{d}\right|$ will have a fraction.

68. **(d)** Consider $T_1=\{1,2,3,4,5\}$ This does not contain a multiple.

$T_2=\{2,3,4,5,6\}$

$T_3=\{3,4,5,6,7\}$

$T_4=\{4,5,6,7,8\}$

$T_5=\{5,6,7,8,9\}$

$T_6=\{6,7,8,9,10\}$

All these do contain multiples of 6.

T_7 once again does not contain a multiple of 6, Also, one part out of every 6 taken in a sequence will not contain a multiple of 6. Therefore $\frac{96}{6}=16$ sets will not contain multiples of 6.

69. **(b)** $\frac{1}{3}\log_3 M+3\log_3 N=1+\log_{0\cdot 008}5$

$$\log_e M^{\frac{1}{3}}+\log_3 N^3=1+\frac{\log_e 5}{\log_e 0\cdot 008}$$

$$\log_e(M.N^9)^{1/3}=1+\frac{\log_e 5}{\log_e\cdot\frac{8}{1000}}$$

$$=1+\frac{\log_e 10-\log_e 2}{\log_e 8-\log_e 1000}=1+\frac{\log_e 10-\log_e 2}{3\log_e 2-3\log_e 10}$$

$$=1+\frac{\log_e 10-\log_e 2}{-3(\log_e 10-\log_e 2)}$$

$$\log_3 (MN^9)^{1/3} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$(MN^9)^{\frac{1}{3}} = 1 - \frac{1}{3} = \frac{2}{3}$$

$$MN^9 = 9$$

$$N^9 = \frac{9}{M}$$

70. (c) If $x, y \in I$

$5x + 19y = 64$

For $x = 256,$ we get that $y = -64$

Then equation stands satisfied by $y = -64$ and $x = 256$.

71. **(b)** $\log_{10} x - \log_{10} \sqrt{x} = 2\log_x 10$

$$\log_{10} x - \frac{1}{2}\log_{10} x = 2\log_x 10$$

$$\frac{1}{2}\log_{10} x = 2.\log_x 10$$

$$\log_{10} xc = 4.\log_x 10$$

$$\log_{10} x = \log_x 10^4$$

$$\log_{10} x = \log_x 10000$$

Now putting the value of x = 10

1 = 4 which is not possible.

Putting the value of $x = \frac{1}{100}$

$-2 = -2$. Thus answer is (b). { x also satisfies the equation at $x = 100$ }.

72. **(c)** By the given condition in question

Area and perimeter of $S_1 = a^2, 4a$

Area and perimeter of $S_2 = \frac{a^2}{2}, \frac{4a}{\sqrt{2}}$

Area and perimeter of $S_3 = \frac{a^2}{4}, \frac{4a}{(\sqrt{2})^2}$

Area and perimeter of $S_4 = \frac{a^2}{8}, \frac{4a}{(\sqrt{2})^3}$

Then, required ratio

$$= \frac{4a + \frac{4a}{\sqrt{2}} + \frac{4a}{(\sqrt{2})^2} + \frac{4a}{(\sqrt{2})^3} + ...}{a^2 + \frac{a^2}{2} + \frac{a^2}{4} + \frac{a^2}{8} + ...}$$

$$=\frac{4a\left|1+\frac{1}{\sqrt{2}}+\frac{1}{(\sqrt{2})^2}+\frac{1}{(\sqrt{2})^3}....\right|}{a^2\left|1+\frac{1}{2}+\frac{1}{4}+\frac{1}{8}...\right|}$$

$$=\frac{4a\left|\frac{1}{1-\frac{1}{\sqrt{2}}}+\right|}{a^2\left|\frac{1}{1-\frac{1}{2}}\right|}=\frac{4\left|\frac{\sqrt{2}}{\sqrt{2}-1}\right|}{a^2\times 2}$$

$$\Rightarrow\frac{4a\times\sqrt{2}(\sqrt{2}+1)}{2a^2}\Rightarrow\frac{2\sqrt{2}(\sqrt{2}+1)}{a}\Rightarrow\frac{2(2+\sqrt{2})}{a}$$

73. (c) Given $xy-x-y=0$

Adding 1 to both sides of the equation, we get

$xy-x-y+1=+1$

$y(x-1)-1(x-1)=1$

$(y-1)(x-1)=1$

As x and y are integers, $x-1$ and are $y-1$ integers.

Hence $x-1$ and $y-1$ must both be 1 or – 1 to satisfy equation (A) i.,e, $x=2, y=2$ or $x=0, y=0$

Hence only two integers pairs satisfy the conditions $x+y=xy$

74. **(a)** $10^{10}=10000000000$. If any one of the zeros is replaced by 1, the value of the result is between 10^{10} and 10^2. There are 10 possible number since any of the 10 zeros can be replaced by $1\cdot 2\times 10^{10}$ (2 followed by 10 zeros) also lies between 10^{10} and 10^{11}. Moreover, the sum of digits of each of the 11 numbers is two. Hence n is 11.

75. **(c)** As $\frac{a}{b+c}=\frac{b}{c+a}=\frac{c}{a+b}=\frac{a+b+c}{b+c+c+a+a+b}$

$=\frac{a+b+c}{2(a+b+c)}=r=\frac{1}{2}$. (Assuming $a+b+c\neq 0$)

If $a+b+c=0$

$\frac{a}{b+c},\frac{a}{b+c}=\frac{a}{a+b+c-a}$ (by adding and subtracting a in the denominator)

$=\frac{a}{0-a}=\frac{a}{-a}=r=-1$

$\left(Similarly\frac{b}{c+a}=\frac{c}{a+b}=r=-1\right)$

Hence r can take only $\frac{1}{2}$ or -1 as values. Choice (c)

76. **(b)** $u=(\log_2 x)^2-6(\log_2 x)+12$

let $\log_2 x=p$ ...(a)

$\Rightarrow u = p^2 - 6p + 12$

$x^u = 256 (= 2^8)$

Applying log to base 2 on both sides we get

$= u \log_2 x = \log_2 2^8$ ...(b)

Dividing (b) by (a) we get

$u = 8/p$

$\Rightarrow 8/p = p^2 - 6p + 12 \Rightarrow 8 - p^3 - 6p^2 + 12p$

or $p^3 - 6p^2 + 12p - 8 = 0$

$(p-2)^3 = 0$

$p = 2$

$\log_2 x = 2 \Rightarrow x = 2^2 = 4$

Thus we have exactly one solution.

77. **(d)** $10 < n < 1000$

Let n is two digit number

$n = 10a + b \Rightarrow P_n = a\,b, S_n = a + b$

Then, $a\,b + a + b = 10a + b$

$\Rightarrow ab = 9a \Rightarrow b = 9$

There are 9 such numbers 19, 29, 33,..., 99.

Then let n is three digit number

$\Rightarrow n = 100a + 10b + c \Rightarrow P_n = a\,b\,c, S_n = a + b + c$

$a\,b\,c + a + b + c = 100a + 10b + c$

$\Rightarrow n = 100a + 10b + c \Rightarrow 100a + 10b + c$

$\Rightarrow abc = 99a + 9b$

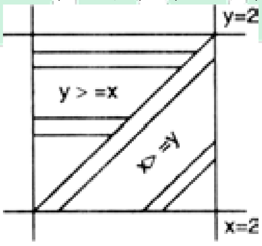
$\Rightarrow bc = 99 + 9\dfrac{b}{a}$

But the maximum value for be = 81

And RHS is more than 99, Hence, no such number is possible.

78. **(c)** Let $x \geq 0, y \geq 0$ and $x \geq y$

Then, $|x + y| + |x - y| = 4$



$\Rightarrow x + y + x - y = 4 \quad \Rightarrow \quad x = 2$

and in case $x \geq 0, y \geq 0, x \leq y$

$x + y + y - x = 4 \Rightarrow y = 2$

Area in the first quadrant is 4.

By symmetry, total area 4 × 4 = 16 unit.

79. **(d)** $P = \log_x\left(\dfrac{x}{y}\right) + \log_y\left(\dfrac{y}{x}\right)$

$= \log_x x - \log_x y - \log_y x$

$= 2 - \log_x y - \log_x y - \log_y x$

$t = \log_x y$

$\Rightarrow P = 2 - \frac{1}{t} - t = -\left(\sqrt{t} + \frac{1}{\sqrt{t}}\right)^2$

Which can never be 1.

80. **(d)** There are two equations to be formed

$40m + 50f = 1000$

$250m + 300f + 40 \times 15m + 50 \times 10 \times f = A$

$850m + 8000f = A$

m and f are the number of males and females A is amount paid by the employer.

Then, the possible values of f = 8, 9,10,11,12

If f = 8, m = 15

If f = 9,10,11, then m will not be an integer while f = 12, then m will be 10.

By putting f = 8 and m = 15, A = 18800. When f = 12 and m = 10, then A = 18100. Therefore, the number of males will be 10.

81. **(c)** There have to be 2 calls from each person to the Englishman who knows French to get all the information. So, there should be 10 calls. But when the fifth guy call he would get all the information of the previous 4 guys along with Englishman's information. Hence, 1 call can be saved. So, the total number of calls = 9.

82. **(a)** The equation forming from the data is $x + y > 41$. The values which will satisfy this equation are (1,39), (1, 38) ...(1,1),

(2,38), (2,37) ...(2,1)

•

•

(39,1)

So, the total number of cases are

39 + 38 + 37 +...+ 1

$= \frac{39 \times 40}{2} = 780$

83. **(b)** Let $A = 100x + 10y + z$

$\Rightarrow B = 100z + 10y + x$

$B - A = 99(z - x)$

For $B - A$ to be divided by 7, $z - x$ has to be divisible by 7. Only possibility is z = 9, x = 2.

∴ Biggest number A can be 299.

∴ Option (b)

84. **(c)** $a_1 = 1, a_{n+1} - 3_{an} + 2 = 4n$

$a_{n+1} = 3a_n + 4n - 2$

when $n = 2$, then $a_2 = 3 + 4 - 2 = 5$

when $n = 3$, then $a_3 = 3 \times 5 + 4 \times 2 - 2 = 2$

So, it is satisfying $3^n 2 \times n$

Hence, a $a_{100} = 3^{100} - 2 \times 100$

85. **(e)** Given equations are

$$2^{0.7x} \times 3^{-1.25y} = \frac{8\sqrt{6}}{27}$$

and $4^{0.3x} \times 9^{0.2y} = 8 \times (81)^{1/5}$

From Eq. (ii),

$4^{0.3x} \times 9^{0.2y} = 8 \times (81)^{1/5}$

$\Rightarrow (2^2)^{0.3x} \times (3^2)^{0.2y} = 8 \times (81)^{1/5}$

$\Rightarrow (2)^{0.6x} \times (3)^{0.4y} = (2)^3 \times (3)^{4/5}$

$\Rightarrow 0.6x = 3 \Rightarrow x = 5$

and $0.4y = \frac{4}{3} \Rightarrow y = 2$

If we substitute the values of x and y in Eq. (i) these values satisfy the Eq. (i). So the answer x = 5, y = 2. Hence, the correct option is (e).

86. **(b)** The given equation is $2x + y = 40$, where $x \le y$

$\Rightarrow y = (40 - 2x)$

The values of x and y that satisfy the equation are

x	1	2	3	4	…	13
y	38	36	34	32	…	14

Thus, there are 13 positive values of (x,y) which satisfy the equation such that $x \le y$.

87. **(d)** Let number of terms in an arithmetic progression be n, then

$1000 = 1 + (n-1)d$

$\Rightarrow (n-1)d = 999 = 3^3 \times 37$

Possible values of $(n-1)$ are 3, 37, 9, 111, 27, 333, 999.

Therefore, the number of possible values of n will also be 7, hence, 7 required progressions can be made.

88. **(a)** $x^{2/3} + x^{1/3} - 2 \le 0$

$\Rightarrow x^{2/3} + 2x^{1/3} - x^{1/3} - 2 \le 0$

$\Rightarrow (x^{1/3} - 1)(x^{1/3} + 2) \le 0$

$\Rightarrow -2 \le x^{1/3} \le 1$

$\Rightarrow -8 \le x \le 1$

89. **(e)** Given that $\log_y x = a \log_z y = b \log_x z = ab$

$\Rightarrow a = \frac{\log_y x}{\log_z y}$ and $b = \frac{\log_y x}{\log_x z}$

$$\Rightarrow a \times b = \frac{\log_y x}{\log_z y} \times \left(\frac{\log_y x}{\log_x z}\right) = \frac{\left(\frac{\log_k z}{\log_k y}\right)}{\left(\frac{\log_k y}{\log_k z}\right)} \times \frac{\left(\frac{\log_k x}{\log_k y}\right)}{\left(\frac{\log_k z}{\log_k x}\right)}$$

$$= \left(\frac{\log_k x}{\log_x y}\right)^3 = (\log_y x)^3 = (ab)^3$$

Therefore, $ab - a^3b^3 = 0$

$\Rightarrow ab(1 - a^2b^2) = 0 \Rightarrow ab = \pm 1$

Only option (e) does not satisfy. Hence it is the required choice.