[This question paper contains 4 printed pages.]

1461-A *Your Roll No.* ....

B.A./B.Sc. (Hons.)/I **A**

MATHEMATICS – Unit-II

(Algebra–I)

(Admissions of 2008 and before)

Time : 2 Hours *Maximum Marks : 38*

(Write your Roll No. on the top immediately on receipt of this question paper.)

Attempt any **two** *parts from each Section. Marks are indicated against each question.*

SECTION I

1. (a) (i) If $x_r = \cos\frac{\pi}{2^r} + i\sin\frac{\pi}{2^r}$, find the value of x_1, x_2, x_3 ------ upto ∞.

(ii) If α, β, γ are the roots of the equation

$$x^3 + ax^2 + bx + a = 0,$$

where a and b are real, show that

$$\tan^{-1}\alpha + \tan^{-1}\beta + \tan^{-1}\gamma = n\pi$$

except when $b = 1$. (2,2½)

(b) Sum the series

$\cos\alpha + \cos(\alpha+\beta) + \cos(\alpha+2\beta) + ---- +$ upto n term. (4½)

P.T.O.

(c) Using De-Moivre's theorem, solve the equation $z^5 + z^4 + z^3 + z^2 + z + 1 = 0$. (4½)

SECTION II

2. (a) Prove that an elementary row operation on the product of two matrices is equivalent to the same elementary row operation on the pre-factor. (5)

(b) Define the rank of a matrix. Reduce the matrix

$$A = \begin{pmatrix} 5 & 3 & 14 & 4 \\ 0 & 1 & 2 & 1 \\ 1 & -1 & 2 & 0 \end{pmatrix}$$

to normal form using elementary operations and hence determine its rank. (5)

(c) (i) State Cayley Hamilton theorem. Verify that the matrix

$$A = \begin{pmatrix} 1 & 2 & 1 \\ -1 & 0 & 3 \\ 2 & -1 & 1 \end{pmatrix}$$

satisfies its characteristic equation. (2½)

(ii) If A and P are square matrices of the same order and P is invertible, show that A and $P^{-1}AP$ have the same characteristic roots. (2½)

SECTION III

3. (a) For what values of λ does the following system of equations have a solution

$$x + y + z = 1$$

$$x + 2y + 4z = \lambda$$

$$x + 4y + 10z = \lambda^2$$

Also find the solution in each case. (5)

(b) (i) Solve the equation

$$x^4 + 2x^3 - 21x^2 - 22x + 40 = 0$$

given that the sum of two roots is equal to the sum of the other two roots. (2½)

(ii) If α, β, γ are the roots (all non-zero) of the equation

$$x^3 - px^2 + qx - r = 0$$

find the value of (i) $(\beta+\gamma)(\gamma+\alpha)(\alpha+\beta)$,

(ii) $\sum \alpha/\beta$. (2½)

(c) (i) Solve the equation

$$x^4 - 12x^3 + 49x^2 - 78x + 40 = 0$$

by removing its second term. (2½)

(ii) If $\alpha + \beta + \gamma = 1$

$\alpha^2 + \beta^2 + \gamma^2 = 2$

$\alpha^3 + \beta^3 + \gamma^3 = 3$

find the value of $\alpha^4 + \beta^4 + \gamma^4$. (2½)

SECTION IV

4. (a) (i) If p is a prime number and a, b are integers such that p divides ab, p divides a or p divides b. (2½)

(ii) If g.c.d. (a, b) = 1, show that g.c.d. $(a^n, b^n) = 1$ and g.c.d. (a+b, a–b) = 1 or 2. (2)

(b) (i) Solve the Congruence $8x \equiv 5 \pmod{27}$. (2½)

(ii) If $\sigma = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 4 & 5 & 1 & 3 \end{pmatrix}$, $\rho = \begin{pmatrix} 1 & 2 & 3 & 4 & 5 \\ 2 & 1 & 4 & 3 & 5 \end{pmatrix}$

Compute $\sigma^{-1} \rho \sigma$. (2)

(c) (i) Express $\begin{pmatrix} 1 & 2 & 3 & 4 & 5 & 6 & 7 & 8 & 9 & 10 \\ 3 & 5 & 2 & 8 & 9 & 4 & 1 & 6 & 10 & 7 \end{pmatrix}$

as a product of disjoint cycles. (2½)

(ii) Prove that in S_n, the number of distinct r-cycles $(r \le n)$ is $\dfrac{\lfloor n}{r \lfloor n-r}$. (2)