Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

May 2014 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Solution 1 : **Principle of equivalence**

Expected present value of income = Expected present value of outgo

For a life insurer, income would be premium income and outgo would be claim payments and expenses incurred.

[1 Mark]

Solution 2 :

i) Typically, insurance contracts have a level premium through the policy tenure while outgo is variable – while expenses may be highest at policy inception, the cost of paying benefits typically increases at later policy years.

It is therefore prudent that premiums not required in early years are set aside to adequately meet the cost of benefits in later years. Reserves provide a cushion to ensure the company remains solvent. **(2)**

ii) Mortality (includes longevity), expenses, investment income, bonus rates, surrenders

(2)

[Total Marks-4]

Solution 3 : ${}_{3}p_{80.5} = {}_{0.5}p_{80.5} * {}_{2}p_{81} * {}_{0.5}p_{83}$

i) Under the UDD assumption, ${}_{s}q_{x} = s * q_{x}$

${}_{0.5}p_{80.5} = p_{80} / {}_{0.5}p_{80} = (1 – q_{80}) / (1\text{-}0.5 * q_{80}) = (1 - 0.035882) / (1 – 0.5 * 0.035882) =$
0.981731

${}_{2}p_{81} = p_{81} * p_{82} = (1\text{- } q_{81}) * (1 – q_{82}) = (1 – 0.040227) * (1 – 0.044981) = 0.916601$

${}_{0.5}p_{83} = (1 - 0.5 * q_{83}) = (1 – 0.5 * 0.050166) = 0.974917$

Therefore, ${}_{3}p_{80.5} = .981731 * .916601 * .974917 = 0.877285$ **(3)**

(ii) Under the CFM assumption, ${}_{(t-s)}p_{(x+s)} = p_x^{(t-s)}$

${}_{0.5}p_{80.5} = p_{80}^{0.5} = 0.972984$

${}_{2}p_{81} = p_{81} * p_{82} = (1\text{- } q_{81}) * (1 – q_{82}) = 0.876876$

${}_{0.5}p_{83} = p_{83}^{0.5} = 0.961734$

Therefore, ${}_{3}p_{80.5} = .972984 * .876876 * .961734 = 0.820538$ **(3)**

[Total Marks-6]

<u>Solution 4</u> :

i) Let P be the monthly payment after 10 years.

The equation of value is

$2{,}000{,}000 = 12 * 5{,}000 * \ddot{a}_{\overline{5}|}^{(12)} + 12 * 10{,}000 * {}_5p_{60} * v^5 * \ddot{a}_{\overline{5}|}^{(12)} + {}_{10}p_{60} * v^{10} * (\ddot{a}_{70} - 11/24) * 12P$

$= 60{,}000 * (1-v^5)/d^{(12)} + 120{,}000 * 8821.26/9287.22 * 0.82193 * (1-v^5)/ d^{(12)} + 8054.05/9287.22 * 0.67556 * (10.375-11/24) * 12P$

$= 60{,}000 * 4.5476 + 120{,}000 * 4.5476 * .9498 * 0.82193 + .8672 * 0.67556 * (10.375 – 11/24) * 12P$

$2{,}000{,}000 = 698876 + 69.71P$

P = Rs. 18,662 per month

The equation of value is

$2{,}000{,}000 = 12 * 5{,}000 * \ddot{a}_{\overline{5}|}^{(12)} + 12 * 10{,}000 * v^5 * \ddot{a}_{\overline{5}|}^{(12)} + {}_{10}p_{60} * v^{10} * (\ddot{a}_{70} - 11/24) * 12P$

$= 60{,}000 * (1-v^5)/d^{(12)} + 120{,}000 * 0.82193 * (1-v^5)/ d^{(12)} + 8054.05/9287.22 * 0.67556 * (10.375 -11/24) * 12P$

$= 60{,}000 * 4.55 + 120{,}000 * 4.55 * 0.82193 + 0.8672 * 0.67556 * (10.375 – 11/24) * 12P$

$2{,}000{,}000 = 721{,}393 + 69.71P$

P = Rs. 18,340 per month

(4)

ii) Annuity payable would be higher if a higher valuation interest rate was used.

Effectively, we are assuming that the purchase price will accumulate by a higher interest rate over the annuity term hence the company should be able to pay a higher annuity after 10 years under the same mortality assumptions. **(1)**

[Total Marks-5]

Solution 5 :

i) **a)** The death strain at risk for a policy for year t + 1 is the excess of the sum assured (i.e. the present value at time t + 1 of all benefits payable on death during the year t + 1) over the end of year provision.

$S - {}_{t+1}V$ **(1)**

b) The expected death strain for year t + 1 is the amount that the life insurance company expects to pay in benefits in excess of the end of year provision for the policy.

$q_x * (S - {}_{t+1}V)$ **(1)**

c) The actual death strain for year t + 1 is the observed value at t+1 of the death strain random variable i.e.

ADS for year t + 1 = $(S - {}_{t+1}V)$ if the life died in the year, or

0 if the life survived to t + 1. **(1)**

(ii) **a)** Select mortality is structured on the premise that lives recently entering an insured population are likely to have lower mortality rates. However, reserves are set aside for a group of existing policies (some of which may have been on the books for a long period of time) hence reserves should be based on ultimate mortality.

(1)

b) In the working, net premium should be calculated on select mortality and reserves on ultimate mortality.

Pure endowments

$P * \ddot{a}_{[40]:\overline{20|}} = 1{,}000{,}000 * v^{20} * {}_{20}p_{[40]}$

$P * 13.930 = 1{,}000{,}000 * 0.45639 * 9287.2164/9854.3036 = 430126$

P = Rs. 30,878 p.a.

Reserve at 31 Dec 2013 = $1{,}000{,}000 * v^4 * {}_4p_{56} - 30{,}878 * \ddot{a}_{56:\overline{4|}}$

$= 1{,}000{,}000 * 9287.2164 / 9515.1040 * 0.854804 - 30{,}878 * 3.745$

= Rs. 718,683

Endowments

P * $\ddot{a}_{[40]:\overline{20|}}$ = 1,000,000 * $A_{[40]:\overline{20|}}$

P * 13.930 = 1,000,000 * 0.46423

P = Rs. 33,326p.a.

Reserve at 31 Dec 2013 = 1,000,000 * $A_{56:\overline{4|}}$ – 33326 * $\ddot{a}_{56:\overline{4|}}$

= 1,000,000 * 0.85595 – 33326 * 3.745

= Rs. 731,144

Death strain at risk = $(S_{PE} - {}_{15}V_{PE}) + (S_E - {}_{15}V_E)$ = (0 – 718,683) + (1,000,000 – 731,144) = - 449,827

Expected death strain = 100 * q_{55} * (-449,827) = 100*0.004469 * (-449,827) = -201,027

Actual death strain = 2 * -449,827 = -899,654

Mortality profit = Rs. 698,627

Pure endowments

P * $\ddot{a}_{[40]:\overline{20|}}$ = 1,000,000 * v^{20} * ${}_{20}p_{[40]}$

P * 13.930 = 1,000,000 * 0.45639 * 9287.2164/9854.3036 = 430126

P = Rs. 30,878 p.a.

Reserve at 31 Dec 2013 = 1,000,000 * v^4 * ${}_{4}p_{[56]}$ – 30,878 * $\ddot{a}_{[56]:\overline{4|}}$

= 1,000,000 * 9287.2164 / 9501.4839 * 0.854804 – 30,878 * 3.749

= Rs. 719,766

Endowments

P * $\ddot{a}_{[40]:\overline{20|}}$ = 1,000,000 * $A_{[40]:\overline{20|}}$

P * 13.930 = 1,000,000 * 0.46423

P = Rs33,326 p.a.

Reserve at 31 Dec 2013 = 1,000,000 * $A_{[56]:\overline{4}|}$ – 33326 * $\ddot{a}_{[56]:\overline{4}|}$

= 1,000,000 * 0.85580 – 33326 * 3.749

= Rs730,860

Death strain at risk = $(S_{PE} - {}_{15}V_{PE}) + (S_E - {}_{15}V_E)$ = (0 – 719,766) + (1,000,000 – 730,860) = - 450,626

Expected death strain = 100 * $q_{[55]}$ * (-450,626) = 100*0.003358 * (-450,626) = -151,320

Actual death strain = 2 * -450,626 = -901,252

Mortality profit = Rs. 749,932

[8]

c) Actual deaths were more than expected.

In the case of endowments, death benefit is payable hence the company has paid out more than expected hence there is a mortality loss.

In the case of pure endowments, no death claim is paid and more deaths than expected means the release of reserves is greater than expected and the company therefore makes a profit.

Overall, the mortality profit on pure endowment contracts outweighs the loss on endowment contracts as the company now needs to pay no benefit at all on the exiting policies. On endowments, higher deaths than expected in year 15 only means that the payment is made sooner than expected (as the sum assured is payable at maturity). **[4]**

d) Even in the case of annuity contracts, we would expect mortality profit if deaths were higher than expected. Annuity contracts pay survival benefits and higher deaths than expected would lead to a higher release of reserves than expected.

[1]

[Total Marks-17]

<u>Solution 6</u> :

(i) $\ddot{a}_{x:\overline{n|}} = (1 - A_{x:\overline{n|}}) / d$ **[1]**

(ii) $\ddot{a}_{50:\overline{30|}} = (1 - A_{50:\overline{30|}}) / d$

$A_{50:\overline{30|}} = A_{50} + (1 - A_{80}) * v^{30} * {}_{30}p_{50}$

= 0.32907 + (1-0.73775) * 5266.4604 / 9712.0728 * 1/1.04^30 = 0.372912

$\ddot{a}_{50:\overline{30|}}$ = (1 - 0.372912) / .038462 = 16.3043

[3]
[Total Marks-4]

<u>Solution 7</u> :

$({}_tV + OP + e_t) * (1+i) = q_{x+t} * S + p_{x+t} * ({}_{t+1}V)$, where

${}_tV$ gross premium provision at time t

OP = office premium

e_t expenses incurred at time t

i = interest rate in premium/valuation basis

S = sum assured

p_{x+t} is the probability that a life aged x + t survives one year on the premium/valuation mortality basis

q_{x+t} is the probability that a life aged x + t dies within one year on the premium/valuation mortality basis

[2 Marks]

<u>Solution 8</u> : **(i)** Let P be the monthly premium.

PV of premiums = PV benefits + PV expenses

$12P * \ddot{a}^{(12)}_{[30]:\overline{30|}} = (3{,}000{,}000 + 500) * A_{[30]} + 120{,}000 * IA_{[30]} + 0.1 * 12P * \ddot{a}^{(12)}_{[30]:\overline{30|}} -$
$0.05 * 12P * \ddot{a}^{(12)}_{[30]:\overline{1|}} - 0.05P + 0.5 * 12P + 3{,}000$

$12P * 0.9 * \ddot{a}^{(12)}_{[30]:\overline{30|}} - 5.95P = 3{,}000{,}500 * A_{[30]} + 120{,}000 * IA_{[30]} + 3000 - 0.05 * 12P$
$* \ddot{a}^{(12)}_{[30]:\overline{1|}}$

$\ddot{a}^{(12)}_{[30]:\overline{30|}} = \ddot{a}_{[30]:\overline{30|}} - 11/24 * (1- v^{30} * {}_{30}p_{[30]})$

$= 17.759 - 11/24 * (1 - 0.30832 * 9287.2164 / 9923.7497) = 17.432$

$\ddot{a}^{(12)}_{[30]:\overline{1|}} = \ddot{a}_{[30]:\overline{1|}} - 11/24 * (1- v * {}_{1}p_{[30]}) = 1 - 11/24 * (1 - 0.96153 * (1-.000476)) =$
0.98216

Therefore,

$12P * 0.9 * 17.432 - 5.95P + 0.5893P = 3{,}000{,}500 * 0.16011 + 120{,}000 * 6.91644 +$
3000

$182.9049P = 1{,}313{,}382$

P = Rs7181

Also allow credit if bonus is not assumed to be payable for year of death.

$12P * \ddot{a}^{(12)}_{[30]:\overline{30|}} = (2{,}880{,}000 + 500) * A_{[30]} + 120{,}000 * IA_{[30]} + 0.1 * 12P * \ddot{a}^{(12)}_{[30]:\overline{30|}} -$
$0.05 * 12P * \ddot{a}^{(12)}_{[30]:\overline{1|}} - 0.05P + 0.5 * 12P + 3{,}000$

$12P * 0.9 * \ddot{a}^{(12)}_{[30]:\overline{30|}} - 5.95P = 3{,}000{,}500 * A_{[30]} + 120{,}000 * IA_{[30]} + 3000 - 0.05 * 12P$
$* \ddot{a}^{(12)}_{[30]:\overline{1|}}$

$\ddot{a}^{(12)}_{[30]:\overline{30|}} = \ddot{a}_{[30]:\overline{30|}} - 11/24 * (1- v^{30} * {}_{30}p_{[30]})$

$= 17.759 - 11/24 * (1 - 0.30832 * 9287.2164 / 9923.7497) = 17.432$

$\ddot{a}^{(12)}_{[30]:\overline{1}|} = \ddot{a}_{[30]:\overline{1}|} - 11/24 * (1- v * {}_1p_{[30]}) = 1 - 11/24 * (1 - 0.96153 * (1-.000476)) =$ 0.9821

Therefore,

12P * 0.9 * 17.432 – 5.95P + 0.5893P = 2,880,500 * 0.16011 + 120,000 * 6.91644 + 3000

182.9049P = 1,294,170

P = Rs7076

[4]

(ii) The revised equation of value is

$12P * \ddot{a}^{(12)}_{[30]:\overline{30}|} = 3{,}000{,}000 * A_{[30]}$ @0% + $500 * A_{[30]}$ @4% + $0.1 * 12P * \ddot{a}^{(12)}_{[30]:\overline{30}|}$ - $0.05 * 12P * \ddot{a}^{(12)}_{[30]:\overline{1}|} - 0.05P + 0.5 * 12P + 3{,}000$

182.9049P = 3,000,000 + 500 * 0.16011 + 3,000 = 3,003,080

P = 16,419

Also allow credit if bonus is not assumed to be payable for year of death.

$12P * \ddot{a}^{(12)}_{[30]:\overline{30}|} = 3{,}000{,}000/1.04 * A_{[31]}$ @0% + $500 * A_{[30]}$ @4% + $0.1 * 12P * \ddot{a}^{(12)}_{[30]:\overline{30}|}$ - $0.05 * 12P * \ddot{a}^{(12)}_{[30]:\overline{1}|} - 0.05P + 0.5 * 12P + 3{,}000$

182.9049P = 2,884,615 * 1 + 500 * 0.16011 + 3,000 = 2,887,695

P = 15,788

[2]

(iii) Compound bonus deferred surplus distribution more hence granting more investment freedom.

Simple bonus may be used if it is common market practice or customer preference.

[2]
[Total Marks-8]

Solution 9 :

(i) The notation describes an annuity of 1/m payable m times a year in advance to a life aged x for a period of n years contingent on survival till the end of each monthly period.

[1]

(ii) $\ddot{a}_{50:\overline{5}|} = a_{50:\overline{5}|} + 1 - {}_5p_{50} * v^5 = 4.18 + 1 - 0.7354 = 4.44$

Therefore, $\ddot{a}^{(12)}_{50:\overline{5}|} = \ddot{a}_{50:\overline{5}|}$ - 11/24 (1 – D_{55} / D_{50}) = 4.44 – 11/24 * (1 - ${}_5p_{50}$ * v^5)

= 4.44 – 11/24 * 0.2646 = 4.32 **[2]**

[Total Marks-3]

Solution 10 :

Spurious selection occurs when mortality differences ascribed to groups are formed by factors which are not the true causes of these differences.

For example, mortality differences by region may be put down to the actual class structure of the region itself, whereas a differing varying mix of occupations region by region could be having a major effect. So region is spurious and being confounded with occupation.

[2 Marks]

Solution 11 :

$$100{,}000\left[n\frac{M_x^r}{D_x}+\frac{\overline{R}_x^r}{D_x}\right]$$

where $D_x = v^x l_x$

$$C_x^r = v^{x+\frac{1}{2}} r_x \text{ for } x < 60$$

$$C_{60}^r = v^{60} r_{60}$$

$$M_x^r = \sum_{t=0}^{60-x} C_{x+t}^r$$

$$\overline{M}_x^r = M_x^r - \tfrac{1}{2} C_x^r \text{ for } x < 60$$

$$\overline{R}_x^r = \sum_{t=0}^{60-x} \overline{M}_{x+t}^r$$

[5 Marks]

Solution 12 :

Occupation – either because of environmental or lifestyle factors mortality may be directly affected. Occupations may also have health barriers to entry, e.g. airline pilot
Nutrition – poor quality nutrition increases morbidity and hence mortality
Housing – low standard of housing (reflecting poverty) increases morbidity
Climate – climate can influence morbidity and may also be linked to natural disaster
Education – linked to occupation but better education can reduce morbidity, e.g. by reducing smoking
Genetics – there is genetic evidence of a predisposition to contracting certain illnesses, even if this has no predictive capability

[6 Marks]

Solution 13 :

Let P be the single premium.

EPV of annuity benefit = $12 \times 100{,}000 \times \ddot{a}^{(12)}_{60} + 12 \times 50{,}000 \times \ddot{a}^{(12)}_{60|55}$

EPV of death benefit = $P \times \bar{A}_{\overline{60:55}}$

EPV of expenses = $0.02 \times P + 12 \times 1{,}000 \times \left[\ddot{a}^{(12)}_{60} + \ddot{a}^{(12)}_{60|55}\right]$

$\ddot{a}^{(12)}_{60} = \ddot{a}_{60} - \frac{11}{24} = 15.632 - 0.458 = 15.174$

$\ddot{a}^{(12)}_{60|55} = \ddot{a}^{(12)}_{55} - \ddot{a}^{(12)}_{60:55} = \left(\ddot{a}_{55} - \frac{11}{24}\right) - \left(\ddot{a}_{60:55} - \frac{11}{24}\right) = (18.210 - 0.458) - (14.756 - 0.458)$

$= 3.454$

$\bar{A}_{\overline{60:55}} \approx (1 + i/2) A_{\overline{60:55}}$

$= 1.02(1 - d\ddot{a}_{\overline{60:55}})$

$= 1.02\left[1 - \frac{0.04}{1.04} \times \left(\ddot{a}_{60} + \ddot{a}_{55} - \ddot{a}_{60:55}\right)\right]$

$= 1.02\,[1 - 0.038 \times (15.632 + 18.210 - 14.756)] = 0.28$

The equation of value is given by:

$P = 1{,}200{,}000 \times 15.174 + 600{,}000 \times 3.454 + 0.28\,P + 0.02\,P + 12{,}000 \times (15.174 + 3.454)$

$P = 18{,}208{,}800 + 2{,}072{,}400 + 0.3\,P + 223{,}536$ => P = Rs. 29,292,480

[7 Marks]

Solution 14 :

The values of the transition probabilities, not provided explicitly, are as follows:

t	p_{58+t}^{HH}	$p_{58+t}^{C_1C_1}$	$p_{58+t}^{C_2C_2}$	p_{58+t}^{DD}
0	0.89	0.70	1.00	1.00
1	0.82	0.55	1.00	1.00

Transition (1)	**Probability (2)**	**PV Benefit (3)**	**EPV Benefit (4) = (2) × (3)**
HH, HH	0.89 × 0.82 = 0.7298	0	0
HH, HC_1	0.89 × 0.09 = 0.0801	$500{,}000v^2 = 436{,}719$	34,981.19
HH, HC_2	0.89 × 0.06 = 0.0534	$1{,}000{,}000v^2 = 873{,}439$	46,641.64
HH, HD	0.89 × 0.03 = 0.0267	0	0
HC_1, C_1C_1	0.06 × 0.55 = 0.0330	$500{,}000v = 467{,}290$	15,420.57
HC_1, C_1C_2	0.06 × 0.15 = 0.0090	$500{,}000v + 500{,}000v^2 = 904{,}009$	8,136.08
HC_1, C_1D	0.06 × 0.30 = 0.0180	$500{,}000v = 467{,}290$	8,411.22
HC_2, C_2C_2	0.03 × 1.00 = 0.0300	$1{,}000{,}000v^2 = 934{,}579$	28,037.37
HD, DD	0.02 × 1.00 = 0.0200	0	0
Total	**1.0000**		**141,628.07**

[10 Marks]

Solution 15 :

(i) Multiple decrement table

Age	56	57	58	59
AM92 Select Mortality	0.0037420	0.0055070	0.0063520	0.0071400
q_x^d	0.0044904	0.0066084	0.0076224	0.0085680
q_x^s	0.1000000	0.0500000	0.0500000	0.0500000
$(aq)_x^d$	0.0042659	0.0064432	0.0074318	0.0083538
$(aq)_x^s$	0.0997755	0.0498348	0.0498094	0.0497858
$(ap)_x$	0.8959586	0.9437220	0.9427588	0.9418604
${}_{t-1}(ap)_x$	1.0000000	0.8959586	0.8455358	0.7971363

Unit fund (per policy at start of year)

Year	1	2	3	4
Value of units at start	0.000	73,458.000	182,026.825	295,958.951
Allocation	70,000.000	100,000.000	100,000.000	100,000.000
Interest	4,200.000	10,407.480	16,921.610	23,757.537
Management charge	742.000	1,838.655	2,989.484	4,197.165
Value of units at end	73,458.000	182,026.825	295,958.951	415,519.323

Non- unit fund cash flows (per policy at start of year)

Year	1	2	3	4
Unallocated Premium	30,000.000	0.000	0.000	0.000
Expenses	C + 10,500.000	2,200.000	2,200.000	2,200.000
Interest	780.000 – 0.04 C	-88.000	-88.000	-88.000
Management charge	742.000	1,838.655	2,989.484	4,197.165
Extra death benefit	1,392.996	1,404.445	773.212	0.000
End of year cash flow	19,629.004 – 1.04 C	-1,853.790	-71.728	1,909.165

where C is the initial commission.

Non-unit reserves (end of year)

$$_2V = \frac{0-(-71.728)}{1.04} = 68.969$$

$$_1V = \frac{(ap)_{57} \times {_2V} - (-1,853.790)}{1.04} = 1,845.075$$

Based on the above non-unit reserves, the cash flows for years 2 and 3 will be zeroised, that of year 4 will remain unchanged while the cash flow for year 1 will be:

$$19,629.004 - 1.04\,C - {_1V} \times (ap)_{56} = 17,975.893 - 1.04\,C$$

Profit Margin

Year	1	2	3	4
Net end of year cash flow	17,975.893 – 1.04 C	0.000	0.000	1,909.165
Inforce probability	1.0000000	0.8959586	0.8455358	0.7971363
Discount factor	0.9216590	0.8494553	0.7829081	0.7215743
Discounted cash flow	16,567.644 – 0.958525 C	0.000	0.000	1,098.138

NPV Profit = 17,665.782 – 0.958525 C
=> 10,000 = 17,665.782 – 0.958525 C
=> C = 7,997.48

[18]

(ii) If the withdrawal rates are higher in the first year, then there would be fewer deaths and hence the extra death benefit paid would be lower in the first year.

Due to higher withdrawal rates in the first year, the number of inforce policies at the end of the year would be lower and hence lower non-unit reserves would be required at the end of first year i.e. transfer to reserves would be lower in the first year.

The other elements of first year profit would not be impacted due to higher withdrawal rates in the first year.

Therefore, everything else being equal, if withdrawal rates are higher than assumed in the first year, then the first year profit would increase.

[2]
[Total Marks-20]

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