Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

May 2011 Examinations

INDICATIVE SOLUTIONS

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasonable.

Q.1)

$${}_{n|}A_x = v^n A_{x+n}$$ is not correct.

The RHS should also include an *npx* factor, since the life will only receive cover from the deferred assurance on the LHS if he or she is still alive at age *x* +*n*.

$$A_{x:\overline{n}|} = A_{x:\overline{t}|} + \frac{D_{x+t}}{D_x} A_{x+t:\overline{n-t}|}$$ is not correct.

The *Ax*:*t*¬ on the RHS is an endowment assurance factor, which provides a payment on survival to age *t*,as well as life cover up to that point. This factor should be replaced with A^1*x*:*t*¬.

[Total Marks – 2]

Q.2)

(i)

$${}_{1.75}p_{45.5} = {}_{0.5}p_{45.5} * p_{46} * {}_{0.25}p_{47}$$

$$= \frac{1-q_{45}}{1-0.5q_{45}} * (1-q_{46}) * (1-0.25q_{47})$$

$$= \frac{1-0.001465}{1-0.5*0.001465} * (1-0.001622) * (1-0.25*0.001802)$$

$$= 0.999267 * 0.998378 * 0.99955 = 0.997197$$

(ii) Occupation can have several direct effects on mortality and morbidity. Occupation determines a person's environment for 40 or more hours each week. The environment may be rural or urban, the occupation may involve exposure to harmful substances e.g. chemicals, or to potentially dangerous situations e.g. working at heights. Much of this is moderated by health and safety at work regulations.

Some occupations are healthier by their very nature e.g. bus drivers have a sedentary and stressful occupation while bus conductors are relatively more active and less stressed. Some work environments e.g. pubs, give exposure to a less healthy lifestyle. Some occupations by their very nature attract more healthy workers. This may be accentuated by health checks made on appointment or by the need to pass regular health checks e.g. airline pilots. Some occupations can attract less healthy workers, for example, former miners who have left the mining industry as a result of ill health and then chosen to sell newspapers. This will inflate the mortality rates of newspaper sellers.

A person's occupation largely determines their income, which permits them to adopt a particular lifestyle e.g. content and pattern of diet, quality of housing. This effect can be positive or negative e.g. over-indulgence.

[Total Marks – 6]

Q.3)

(i) EPV of annuity

The expected present value of this annuity is:

$50000\, \bar{a}_{60:\overline{5}|} + 60000\, v^5 \; {}_5p_{60}\, \bar{a}_{65:\overline{5}|}$

Since the force of mortality is constant between age 60 and age 65:

$$v^5 \; {}_5p_{60} = e^{-5\delta} e^{-5\mu} = e^{-5(0.05+0.03)} = e^{-0.4} = 0.67032$$

Also:

$$\bar{a}_{60:\overline{5}|} = \int_0^5 v^t \; {}_tp_x \, dt$$

$$= \int_0^5 e^{-(\delta+\mu)t} \; dt$$

$$= \frac{1}{\delta + \mu}\left[1 - e^{-5(\delta+\mu)}\right]$$

$$= \frac{1}{0.08}\left(1 - e^{-0.4}\right) = 4.12100$$

and similarly :

$$\bar{a}_{65:\overline{5}|} = \frac{1}{0.05 + 0.04}\left(1 - e^{-0.45}\right) = 4.02635$$

So the expected present value of the annuity is:
(50,000 × 4.12100) + (60,000 × 0.67032 × 4.02635) = Rs. 3,67,986.58

(ii)

(a) EPV of term assurance

The expected present value of this term assurance is:

$2{,}50{,}000 \; \bar{A}^{1}_{60:\overline{10}|}$ = $2{,}50{,}000 \left(\bar{A}^{1}_{60:\overline{5}|} + v^5 \; {}_5p_{60} \; \bar{A}^{1}_{65:\overline{5}|}\right)$

Since the force of mortality is constant between age 60 and age 65:

$$\bar{A}^{\,1}_{60:\overline{5}|} = \int_0^5 v^t \;{}_t p_x \;\mu_{x+t}\, dt$$

$$= \mu\, \bar{a}_{60:\overline{5}|}$$

$$= 0.03 \times 4.12100$$

$$= 0.12363$$

and similarly:

$$\bar{A}^{\,1}_{65:\overline{5}|} = 0.04 \times 4.02635 = 0.16105$$

So the expected present value is:

2,50,000 * (0.12363+0.67032×0.16105)= Rs. 57896.26

(b) EPV of endowment assurance

The EPV of the maturity benefit is 2,50,000 $A_{60:\overline{10}|}^{\;\;\;\;1}$, where:

$$A_{60:\overline{10}|}^{\;\;\;\;1} = v^{10}\;{}_5p_{60}\;{}_5p_{65} = e^{-10\times 0.05} \times e^{-5\times 0.03} \times e^{-5\times 0.04} = e^{-0.85} = 0.42741$$

2,50,000 * (0.12363 + 0.67032 × 0.16105 + 0.42741) = Rs. 1,64,748.76

[Total Marks – 9]

Q.4)

(i) $X = \begin{cases} v^n & K_x \ge n \\ 0 & K_x < n \end{cases} \qquad Y = \begin{cases} 0 & K_x \ge n \\ v^{K_x+1} & K_x < n \end{cases}$

$$\Rightarrow XY = 0 \text{ for all } K_x$$

$$COV(X,Y) = E[XY] - E[X]E[Y] = 0 - (A_{x:\overline{n}|}^{\;\;\;1})(A^1_{x:\overline{n}|})$$

(ii) $VAR(X+Y) = VAR(X) + VAR(Y) + 2COV(X,Y)$

$$= {}^2A_{x:\overline{n|}}^{\;\;1} - (A_{x:\overline{n|}}^{\;\;1})^2 + {}^2A_{x:\overline{n|}}^{1} - (A_{x:\overline{n|}}^{1})^2 - 2(A_{x:\overline{n|}}^{\;\;1})(A_{x:\overline{n|}}^{1})$$
$$= \{{}^2A_{x:\overline{n|}}^{\;\;1} + {}^2A_{x:\overline{n|}}^{1}\} - \{(A_{x:\overline{n|}}^{\;\;1})^2 + (A_{x:\overline{n|}}^{1})^2 + 2(A_{x:\overline{n|}}^{\;\;1})(A_{x:\overline{n|}}^{1})\}$$
$$= \{{}^2A_{x:\overline{n|}}^{\;\;1} + {}^2A_{x:\overline{n|}}^{1}\} - \{(A_{x:\overline{n|}}^{\;\;1}) + (A_{x:\overline{n|}}^{1})\}^2$$
$$= {}^2A_{x:\overline{n|}} - (A_{x:\overline{n|}})^2$$

[Total Marks – 7]

Q.5)

$$(aq)_x^{\alpha} = \int_0^1 {}_t(ap)_x\,(a\mu)_{x+t}^{\alpha}dt = \int_0^1 {}_tp_x^{\alpha}\;{}_tp_x^{\beta}\mu_{x+t}^{\alpha}dt = \int_0^1 (\,{}_tp_x^{\alpha}\mu_{x+t}^{\alpha})\;{}_tp_x^{\beta}dt$$

$$\text{But }\;{}_tp_x^{\alpha}\mu_{x+t}^{\alpha} = -\frac{d(\,{}_tp_x^{\alpha})}{dt} = -\frac{d}{dt}\left(\frac{x}{x+t}\right) = \frac{x}{(x+t)^2}$$

$$(aq)_x^{\alpha} = \int_0^1 \frac{x}{(x+t)^2}\cdot\frac{x^2}{(x+t)^2}dt = x^3\int_0^1 \frac{1}{(x+t)^4}dt$$

$$= \frac{-x^3}{3(x+t)^3}\bigg|_0^1 = \frac{1}{3}\left[1 - \frac{x^3}{(x+1)^3}\right]$$

[Total Marks – 5]

Q.6)

(a) $\bar{a}_{xy} = \int_0^{\infty} {}_tp_{xy}v^t\,dt = \int_0^{\infty} {}_tp_x\;{}_tp_y\,v^t\,dt$

$$= \int_0^{\infty} e^{-2\mu t}e^{-\delta t}dt = \frac{e^{-(2\mu+\delta)t}}{-(2\mu+\delta)}\bigg|_0^{\infty} = \frac{1}{\delta+2\mu}$$

(b)

$$\bar{A}_{xy} = 1 - \delta\bar{a}_{xy} = \frac{2\mu}{\delta+2\mu}$$

Using the above results, $\bar{a}_x = \frac{1}{\delta+\mu}$ and $\bar{A}_x = \frac{\mu}{\delta+\mu}$

The actuarial present value of the annuity part of the benefit is

$100{,}000\bar{a}_{30:30} + 60{,}000(\bar{a}_{30} - \bar{a}_{30:30}) + 60{,}000(\bar{a}_{30} - \bar{a}_{30:30})$

$$= 100{,}000\frac{1}{0.05+2(0.04)} + 2*60{,}000\left(\frac{1}{0.05+0.04} - \frac{1}{0.05+2(0.04)}\right)$$
$$= 1{,}179{,}500$$

The actuarial present value of the insurance part of the benefit is

$$100{,}000\bar{A}_{30:30} + 80{,}000\bar{A}_{\overline{30:30}} = 100{,}000\bar{A}_{30:30} + 80{,}000\left(\bar{A}_{30} + \bar{A}_{30} - \bar{A}_{30:30}\right)$$

$$= 100{,}000\frac{2(0.04)}{0.05+2(0.04)} + 80{,}000\left(2*\frac{0.04}{0.05+0.04} - \frac{2(0.04)}{0.05+2(0.04)}\right) = 83{,}200$$

Total present value of the special annuity = 1179500 + 83200 = 1,262,700

[Total Marks – 8]

Q.7)

$$\text{EPV} = 5{,}000\int_0^{20} e^{-\delta t}\ \ p_{35,t}^{hh}dt \qquad \text{(premiums)}$$

$$-200{,}000\int_0^{20} e^{-\delta t}\ \ p_{35,t}^{hh}\mu_{35+t}dt \qquad \text{(death from healthy)}$$

$$-300{,}000\int_0^{20} e^{-\delta t}\ \ p_{35,t}^{hs}\nu_{35+t}dt \qquad \text{(death from sick)}$$

$$-30{,}000\int_0^{20} e^{-\delta t}\ \ p_{35,t}^{hs}dt \qquad \text{(sickness income)}$$

[Total Marks – 4]

Q.8)

(a)

Year (t)	Premium	Allocated Premium	Fund at start of year	Policy charge	Mortality Charge	Guarantee Charge
	(1)	(2)	(3)	(4)	(5)	(6)
1	10,000	8,000.0	8,000.0	600.0	101.9	65.7
2	10,000	9,800.0	17,513.4	630.0	182.9	150.3
3	10,000	9,800.0	27,450.8	661.5	241.6	238.9

Year (t)	Fund after charges	Fund after growth	FMC	Fund at end of year w/o guarantee	Fund at end of year with guarantee
	(7)	(8)	(9)	(10)	(11)
1	7,232.5	7,811.1	97.6	7,713.4	7,713.4
2	16,550.3	17,874.3	223.4	17,650.8	17,650.8
3	26,308.8	28,413.5	355.2	28,058.4	28,558.4

$(2)_t = (1)_t * [1 – \text{Allocation Charge}_t]$

$(3)_t = (2)_t + (11)_{t-1}$

$(4)_t = 600 * 1.05^{t-1}$

$(5)_t = 110\% * \text{Standard Table mortality rate}_t * \max [SA – \{(3)_t – (4)_t\}, 0]$

$(6)_t = [(3)_t – (4)_t – (5)_t] * 0.9\%$

$(7)_t = (3)_t – (4)_t – (5)_t – (6)_t$

$(8)_t = (7)_t * 1.08$

$(9)_t = (8)_t * 0.0125$

$(10)_t = (8)_t – (9)_t$

$(11)_3 = (10)_3 + 5\% * 10,000$

Accumulated value of Guarantee charge at end of 3 years is:

$65.7 * 1.06^3 + 150.3 * 1.06^2 + 238.9 * 1.06 = 500.4$

Since the accumulated value of the guarantee charge is greater than 5% of the annual premium, 0.9% pa is hence sufficient to meet the guarantee cost.

(b)

Year (t)	Unit Price at start of year	Allocated Units	Units at start of year	Policy charge Units	Mortality charge Units	Guarantee charge Units	Remaining Units	Unit Price at end of year
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
1	10.0000	800.0	800.0	60.0	10.2	6.6	723.2	10.6650
2	10.6650	918.9	1,642.1	59.1	17.1	14.1	1,551.8	11.3742
3	11.3742	861.6	2,413.4	58.2	21.2	21.0	2,313.0	12.1306

$(1)_t = (8)_{t-1}$

$(2)_t$ = Allocated Premium$_t$ / $(1)_t$

$(3)_t = (2)_t + (7)_{t-1}$

$(4)_t$ = Policy Charge$_t$ / $(1)_t$

$(5)_t$ = Mortality Charge$_t$ / $(1)_t$

$(6)_t$ = Guarantee Charge$_t$ / $(1)_t$

$(7)_t = (3)_t - (4)_t - (5)_t - (6)_t$

$(8)_t$ = Unit Fund at end of year wo guarantee$_t$ / $(7)_t$

[Total Marks – 13]

Q.9)

(a)

Year	Best estimate mortality	Reserving mortality (q)
	(1)	(1) * 115%
1	0.0015	0.0017
2	0.0020	0.0023
3	0.0025	0.0029
4	0.0030	0.0035

Year	Best estimate expense	Reserving expense (E)
	(1)	(1) * 115%
1	1000	1150.0
2	200	230.0
3	210	241.5
4	220	253.0

Recursive formula to calculate the reserve is:

$V_t * (1 + i) = q_{t+1} * SA + E_{t+1} * (1 + i) + (B_{t+1} + V_{t+1}) * (1 - q_{t+1})$

Where,

V_t = reserve at end of year t

i = valuation rate of 5.0%

B = maturity benefit of 62,500

$V_4 = 0$, since no reserves are needed at end of policy term

Using above formula,

$V_3 = 59,777$

$V_2 = 57,179$

$V_1 = 54,698$

(b)

For endowment assurance:

Year	Policies at start of year	Number of deaths	Number of surrenders	Policies at end of year
	(1)	(2)	(3)	(4)
1	10.00	0.015	0.50	9.49
2	9.49	0.019	0.24	9.23
3	9.23	0.023	0.14	9.07
4	9.07	0.027	0.00	9.04

$(1)_t = (4)_{t-1}$

$(2)_t = (1)_t$ * mortality rate_t

$(3)_t = [(1)_t - (2)_t]$ * surrender $rate_t$

$(4)_t = (1)_t - (2)_t - (3)_t$

Year	Reserves per policy	Reserves in force
	(1)	(2)
1	54,698	518,852
2	57,179	527,771
3	59,777	542,112
4	-	-

$(2)_t = (1)_t$ * policies at end of year

Year	Surrender Penalty	Surrender Value per policy
	(1)	(2)
1	5,000	45,000
2	2,500	47,500
3	1,250	48,750
4	-	-

$(1)_t$ = Surrender penalty $percentage_t$ * Single premium

$(2)_t$ = Single premium – $(1)_t$

Year	Premium Income	Interest	Expense	Death Outgo	Surrender Outgo	Maturity Outgo	Reserves in force	Increase in Reserves	Profit
	(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)	(9)
1	500,000	31,850	10,000	938	22,466	-	518,852	518,852	(20,406)
2	-	33,602	1,897	1,186	11,242	-	527,771	8,918	10,359
3	-	34,179	1,938	1,442	6,733	-	542,112	14,341	9,725
4	-	35,108	1,995	1,700	-	565,108	-	(542,112)	8,416

$(1)_t$ = Single premium * policies at start of year

$(2)_t = [(1)_t + (7)_{t-1} - (3)_t]$ * 6.5%

$(3)_t$ = $Expense_t$ * policies at start of year

$(4)_t$ = SA * number of $deaths_t$

$(5)_t$ = Surrender value per $policy_t$ * number of $surrenders_t$

$(6)_4$ = SA * policies at end of year 4

$(8)_t = (7)_t – (7)_{t-1}$

$(9)_t = (1)_t + (2)_t – (3)_t – (4)_t – (5)_t – (6)_t – (8)_t$

For term assurance:

We only need to calculate the profit for third policy year as the profit for fourth and fifth policy year is already provided

Number of deaths in the third policy year= 0.002 * 25 = 0.05 and policies remaining at end of year is 25 – 0.05 = 24.95

Recursive formula to calculate the reserve per policy is:

$(V_t + P_t) * (1 + i) = q_{t+1} * SA + E_{t+1} * (1 + i) + V_{t+1} * (1 – q_{t+1})$

Substituting the values, we get

$(231 + 500) * 1.05 = 0.002 * 1.15 * 125000 + 200 * 1.15 * 1.05 + V_3 * (1 – 0.002 * 1.15)$

$V_3 = 239.1$

Therefore, increase in reserves for the third policy year is:

239.1 * 24.95 – 231 * 25 = 191

Profit for third policy year:

Premium Income	Interest	Expense	Death Outgo	Surrender Outgo	Reserves in force	Increase in Reserves	Profit
(1)	(2)	(3)	(4)	(5)	(6)	(7)	(8)
					5775		
12500	863	5000	6250	0	5966	191	1922

$(1)_t$ = Annual premium * policies at start of year

$(2)_t = [(1)_t + (6)_{t-1} – (3)_t] * 6.5\%$

$(3)_t$ = $Expense_t$ * policies at start of year

$(4)_t$ = SA * number of $deaths_t$

$(5)_t = 0$, since term assurance

$(8)_t = (1)_t + (2)_t - (3)_t - (4)_t - (7)_t$

For both blocks of policies:

Profit of endowment assurance	Profit of term assurance	Combined Profit
(20,406)	1922	(18,484)
10,359	1800	12,159
9,725	1700	11,425
8,416	0	8,416

Net Assets at start of year = 25000 – 5775 = 19,225

Net Assets at end of year = 19225 – 18484 = 741

Discounting rate v = 1/1.13 = 0.8850

Net present value at end of year = $8416v^3 + 11425v^2 + 12159v = 25{,}540$

Embedded Value at end of year = 741 + 25540 = 26,282

(c) Net Assets would reduce as the reserves would increase.

Net present value would increase as more reserves would get released in future.

Since the risk discount rate is greater than the investment rate, overall the embedded value would reduce.

[Total Marks – 19]

Q.10)

(a) Advantage: Pooling the data will give rise to more credible estimates of underlying mortality rates because greater exposure means lower variance.

Disadvantage: There may be heterogeneity in the data from the two companies due to

i. differing geographical coverage,
ii. differing underwriting standards,
iii. different distribution,

iv. differing target market
v. etc.

(b) If its age/sex profile is such that if it experienced the same age/sex specific mortality rates as the country, then its crude death rate would be twice that of the country, i.e. the region has a much older age structure than the country.

[Total Marks – 4]

Q.11)

(i) $P\ddot{a}_{[40]:\overline{20}|} = 75,000A_{[40]:\overline{20}|}^{\ \ 1} = 75,000v^{20}\ {}_{20}p_{[40]}$

$\Rightarrow P(13.930) - (75,000)(0.45639)(0.94245)$

$\Rightarrow P = 32,259.45/13.93 = 2,315.83$

Mortality profit = Expected Death Strain – Actual Death Strain

$$DSAR = 0 - {}_{15}V = -(75,000A_{55:\overline{5}|}^{\ \ 1} - P\ddot{a}_{55:\overline{5}|})$$
$$= -(75,000v^5\ {}_5p_{55} - P\ddot{a}_{55:\overline{5}|})$$
$$= -\{(75,000)(0.82193)(0.97169) - (2,315.83)(4.585)\} = -49,281.51$$

$$EDS = (q_{54})(500)(-49,281.51) = (0.003976)(500)(-49,281.51) = -97,971.64$$
$$ADS = (3)(-49,281.51) = -147,844.53$$

Mortality Profit = -97,971.64 - (-147,844.53) = 49,872.89 profit

(ii) Expected number of deaths = $500q_{54}$ = 1.988

Actual deaths were 3.

With pure endowments, the death strain is negative because no death claim is paid and there is a release of reserves to the company on death. In this case, more deaths than expected means this release of reserves is greater than required by the equation of equilibrium and the company therefore makes a profit.

[Total Marks – 9]

Q.12)

(i) ***Annual premium***

If the annual premium is *P,* then:

$$\text{EPV premiums} = P\,\ddot{a}_{60:\overline{5}|}$$
$$= P\left(\ddot{a}_{60} - v^5 \frac{l_{65}}{l_{60}} \ddot{a}_{65}\right)$$
$$= \left(16.652 - 1.04^{-5} \times \frac{9,703.708}{9,848.431} \times 14.871\right)$$
$$= 4.6087P$$

EPV of Benefits = 35,000 $\frac{D_{65}}{D_{60}}\left(\ddot{a}_{\overline{5}|}^{4\%} + \frac{D_{70}}{D_{65}}\ddot{a}_{70}\right)$

= 35,000x

$$1.04^{-5} \times \frac{9,703.708}{9,848.431}$$
$$\left(4.6299 + 1.04^{-5} \times \frac{9,392.621}{9,703.708} \times 12.934\right)$$

= Rs. 4,22,900

So the premium equation is:

4.6087*P* = 422900

P= Rs. 91,760

(ii) ***Prospective and retrospective reserves***

Prospective reserve after 5 years

This is equal to the present value of future benefits, *ie*:

35000 x $\left(\frac{D_{70}}{D_{65}} \times \ddot{a}_{70} + \ddot{a}_{\overline{5}|}^{4\%}\right)$ = 35000 (0.79558 x12.934 + 4.6299) = 5,22,198

Retrospective reserve after 5 years

This is equal to the accumulated premiums allowing for mortality, *ie*:

91760 x $\ddot{s}_{60:\overline{5}|}$ =91760 x $\frac{D_{60}}{D_{65}}\ddot{a}_{60:\overline{5}|}$ =91760 x $1.04^5\,\frac{l_{60}}{l_{65}}\ddot{a}_{60:\overline{5}|}$

= 91760 x 1.2348 x 4.6087

= 522190

Prospective reserve after 10 years
Prospective reserve is equal to the value of future benefits.
Hence the reserve is equal to 35000 × $\ddot{a}_{70}$ = 35000 × 12.934 = 4,52,690

Retrospective reserve after 10 years
The retrospective reserve is equal to the accumulated reserve less the accumulated payments, adjusted to allow for deaths between 65 and 70.

The accumulated payments equal:

35000 x $\ddot{s}_{\overline{5}|}^{4\%}$ = 35000 × 5.6330 = 197155

Therefore the reserve per surviving policy is:
522190 × 1.04^5 −197155 = 438169
Adjusting by the survival function gives the reserve as:

438169 x $\frac{l_{65}}{l_{70}}$ = 438169 x 9,703.708 / 9,392.621 = 452681

(Here prospective reserve 452690 and retrospective reserve 452681 are almost similar and the difference is due to rounding off).

[Total Marks – 14]

[Total Marks – 100]

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