Institute of Actuaries of India

Subject CT5 – General Insurance, Life and Health Contingencies

October 2014 Examination

INDICATIVE SOLUTION

Introduction

The indicative solution has been written by the Examiners with the aim of helping candidates. The solutions given are only indicative. It is realized that there could be other points as valid answers and examiner have given credit for any alternative approach or interpretation which they consider to be reasona

Solution 1:-

Overhead expenses are those expenses that in the short term do not vary with the amount of business.

An example of an overhead expense is the cost of the company's premises (as the sale of an extra policy now will have no impact on these costs).

Direct expenses are those that do vary with the amount of business.

An example of a direct expense is commission payment to a direct salesman (as the sale of an extra policy now will have an impact on these costs).

[4]

Solution 2:-

2. The following are three types of guaranteed reversionary bonuses. The bonuses are usually allocated annually in arrears, following a valuation.

Simple: The rate of bonus each year is a percentage of the initial (basic) sum assured under the policy. The effect is that the sum assured increases linearly over the term of the policy.

Compound: The rate of bonus each year is a percentage of the initial (basic) sum assured and the bonuses previously added. The effect is that the sum assured increases exponentially over the term of the policy.

Super compound: Two compound bonus rates are declared each year. The first rate (usually the lower) is applied to the initial (basic) sum assured. The second rate is applied to bonuses previously added.

The effect is that the sum assured increases exponentially over the term of the policy. The sum assured usually increases more slowly than under a compound allocation in the earlier years and faster in the later years.

(Note: Credit should be given if special reversionary bonus is mentioned)

[4]

Solution 3:-

(i) Let P be the monthly premium. Then:

EPV of premiums valued at rate i where i=0.06 is:

$$12P\ddot{a}^{(12)}_{[40]:\overline{25|}} = 12P[\ddot{a}_{[40]:\overline{25|}} - \frac{11}{24}(1-v^{25}\frac{l_{65}}{l_{[40]}})]$$

$$= 12P[13.290 - \frac{11}{24}(1-0.94340^{25}\frac{8,821.2612}{9,854.3036})]$$

$$= 12P[13.290 - \frac{11}{24}(1- 0.20859)] = 12P\times 12.92727 = 155.12724P$$

EPV of benefits valued at rate i where i=0.06 is:

$$100,000A_{[40]:\overline{25}|} = 100,000 * 0.24774 = 24,774$$

EPV of expenses not subject to inflation and hence valued at rate i where i=0.06 is:

$$1,000 + 0.5 \times 12P + 0.05 \times 12P\ddot{a}^{(12)}_{[40]:\overline{25}|} - 0.05P$$

$$= 1,000 + 13.706362P$$

EPV of expenses subject to inflation and hence valued at rate j where 1+j=(1+i)/1.0192038=1.04 is:

$$500(\ddot{a}_{[40]:\overline{25}|} - 1) + 2,000A_{[40]:\overline{25}|}$$

$$= 500(15.887 - 1) + 2,000 \times 0.38896 = 8,221.42$$

Equating EPV of premiums with EPV of benefits and expenses, we get:

$$155.12724P = 24,774 + 1,000 + 13.706362P + 8,221.42$$

$$\Rightarrow P = 240.38$$

[7]

(ii) The gross retrospective policy value is given by:

$$V^{retrospective}$$

$$= (1+i)^{20}\frac{l_{[40]}}{l_{60}}\left[\begin{array}{c} 12 \times 0.95P\ddot{a}^{(12)}_{[40]:\overline{20}|} @\, i + 0.05P - 12 \times 0.5P - 1,000 - 100,000A^{1}_{[40]:\overline{20}|} @\, i \\ -500(\ddot{a}_{[40]:\overline{20}|} @\, j - 1) - 2,000A^{1}_{[40]:\overline{20}|} @\, j \end{array}\right]$$

where,

$$(1+i)^{20}\frac{l_{[40]}}{l_{60}} = (1.06)^{20}\frac{9,854.3036}{9,287.2164} = 3.40297$$

and at rate i=0.06:

$$\ddot{a}^{(12)}_{[40]:\overline{20}|} = \ddot{a}_{[40]:\overline{20}|} - \frac{11}{24}\left(1 - v^{20}\frac{l_{60}}{l_{[40]}}\right) = 12.000 - \frac{11}{24}(1 - 0.29389) = 11.67637$$

$$A^{1}_{[40]:\overline{20}|} = A_{[40]:\overline{20}|} - v^{20}\frac{l_{60}}{l_{[40]}} = 0.32076 - 0.29389 = 0.02687$$

and at rate j=0.04:

$$\ddot{a}_{[40]:\overline{20}|} = 13.930$$

$$A^{1}_{[40]:\overline{20}|} = A_{[40]:\overline{20}|} - v^{20}\frac{l_{60}}{l_{[40]}} = 0.46423 - 0.43013 = 0.03410$$

$$\Rightarrow V^{retrospective}$$

$$= 3.40297(31{,}946.5483 + 12 - 1{,}440 - 1{,}000 - 2{,}687 - 6{,}465 - 68.2)$$

$$= 69{,}074.67$$

[7]

[14 Marks]

<u>Solution 4</u>:-

(i) Let *t* be the median of the future lifetime. Then, by definition:

$${}_{t}p_{50} = 0.5 \Rightarrow \frac{l_{50+t}}{l_{50}} = 0.5$$

$$l_{50} = 9{,}712.0728 \Rightarrow l_{50+t} = 4{,}856.0364$$

$l_{81} = 4{,}901.4789, l_{82} = 4{,}527.4960 \Rightarrow$ Median future lifetime (nearest integer) = 81 – 50 = 31

[3]

(ii) The loss random variable is given by:

$$L = v^{K+1} - P\ddot{a}_{\overline{K+1}|}$$

For *K* = *k* and for a given premium *P*, the present value of the financial loss at policy issuance is given by:

$$l(k) = v^{k+1} - P\ddot{a}_{\overline{k+1}|}; k = 0,1,2,...,99$$

Since *l(k)* is a decreasing function of *k*, the least value of *P* for which the probability of financial loss is at most 0.05, is given by:

$$l(5) = 0 \Rightarrow v^{6} - P\ddot{a}_{\overline{6}|} = 0$$

Then the financial loss is positive only if *K*<5 which has probability 0.05.

$$\therefore P = \frac{v^{6}}{\ddot{a}_{\overline{6}|}} = \frac{0.63019}{4.99271} = 0.12622$$

[5]
[8 Marks]

Solution 5:-

(i) $P_{x:\overline{n}|}\ddot{a}_{x:\overline{n}|} = A_{x:\overline{n}|} = A^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1}$

$${}_nP_x\ddot{a}_{x:\overline{n}|} = A_x = A^{1}_{x:\overline{n}|} + A_{x:\overline{n}|}^{\ \ 1}A_{x+n}$$

Subtracting the second equation from the first, we get:

$$(P_{x:\overline{n}|} - {}_nP_x)\ddot{a}_{x:\overline{n}|} = A_{x:\overline{n}|}^{\ \ 1}(1 - A_{x+n})$$
$$\Rightarrow P_{x:\overline{n}|} = {}_nP_x + P_{x:\overline{n}|}^{\ \ 1}(1 - A_{x+n})$$

[4]

(ii) Both $P_{x:\overline{n}|}$ and ${}_nP_x$ are payable during the survival of *(x)* to a maximum of *n* years. During this period, both insurances provide a death benefit of 1 payable at the end of the year of death of *(x)*.

If *(x)* survives the *n* years, $P_{x:\overline{n}|}$ provides a maturity benefit of 1, while ${}_nP_x$ provides a whole life insurance without further premiums i.e. a benefit with expected present value of A_{x+n}. Hence, the difference, $P_{x:\overline{n}|} - {}_nP_x$ is the level annual premium for *n*-year pure endowment of $1 - A_{x+n}$.

[2]
[6 Marks]

Solution 6:-

(i) Let *P* be the annual benefit premium. Then

$$P\ddot{a}_{30:\overline{30}|} = 10,000,000A^{1}_{30:\overline{30}|} + P(IA)^{1}_{30:\overline{30}|}$$

$$\Rightarrow P = \frac{10,000,000A^{1}_{30:\overline{30}|}}{\ddot{a}_{30:\overline{30}|} - (IA)^{1}_{30:\overline{30}|}}$$

$$A^{1}_{30:\overline{30}|} = A_{30:\overline{30}|} - \frac{D_{60}}{D_{30}} = 0.31706 - \frac{882.85}{3,060.13} = 0.02856$$

$$(IA)^{1}_{30:\overline{30}|} = (IA)_{30} - \frac{D_{60}}{D_{30}}[(IA)_{60} + 30A_{60}]$$

$$\Rightarrow (IA)^{1}_{30:\overline{30}|} = 6.91559 - \frac{882.85}{3,060.13}(8.36234 + 30 \times 0.45640) = 0.55289$$

$$\therefore P = \frac{10,000,000 \times 0.02856}{17.756 - 0.55289} = 16,601.65$$

[3]

ii) We first calculate the expected present value for an *n*-year term insurance on (*x*) for which the benefit payable at the end of the year, in case death occurs in year *k*+1, is $\ddot{s}_{\overline{k+1}|}$ (where the present value and the accumulation are both calculated at the same interest rate). The present value random variable of this benefit at policy issuance is given by:

$$Z = \begin{cases} v^{K+1}\ddot{s}_{\overline{K+1}|} = v^{K+1}\dfrac{[(1+i)^{K+1}-1]}{d} = \dfrac{1}{d}(1-v^{K+1}) & ;0 \le K < n \\ 0 & ;K \ge n \end{cases}$$

$$\Rightarrow E(Z) = \frac{P(K<n) - A^{1}_{x:\overline{n}|}}{d} = \frac{{}_{n}q_x - A^{1}_{x:\overline{n}|}}{d}$$

$$\Rightarrow E(Z) = \frac{1 - {}_{n}p_x - A_{x:\overline{n}|} + v^n\,{}_{n}p_x}{d} = \ddot{a}_{x:\overline{n}|} - {}_{n}p_x\ddot{a}_{\overline{n}|}$$

Now, let *P* be the annual benefit premium. Then

$$P\ddot{a}_{30:\overline{30}|} = 10{,}000{,}000A^{1}_{30:\overline{30}|} + P(\ddot{a}_{30:\overline{30}|} - {}_{30}p_{30}\ddot{a}_{\overline{30}|})$$

$$\Rightarrow P = \frac{10{,}000{,}000A^{1}_{30:\overline{30}|}}{{}_{30}p_{30}\ddot{a}_{\overline{30}|}}$$

$${}_{30}p_{30} = \frac{l_{60}}{l_{30}} = \frac{9{,}287.2164}{9{,}925.2094} = 0.93572$$

$$\ddot{a}_{\overline{30}|} = \frac{1-v^{30}}{1-v} = \frac{1-0.96154^{30}}{1-0.96154} = 17.98414$$

$$\therefore P = \frac{10{,}000{,}000 \times 0.02856}{0.93572 \times 17.98414} = 16{,}971.59$$

[5]
[8 Marks]

Solution 7:-

Let *P* be the annual benefit premium. The benefit reserves at times *t* and *t*+1 are related by the formula:

$({}_{t}V_x + P)(1+i) = q_{x+t}S_{t+1} + p_{x+t}\,{}_{t+1}V_x$ where S_t is the death benefit payable at time t.

Rearranging this equation, we get:

$v\,{}_{t+1}V_x - {}_{t}V_x = P - (S_{t+1} - {}_{t+1}V_x)vq_{x+t}$

Here $S_{t+1} = 1 + {}_{t+1}V_x$ for *t* = 0,1,2,…,*n*-1 and hence

$$v\,{}_{t+1}V_x - {}_tV_x = P - vq_{x+t}$$

Multiplying both sides by v^t, we get:

$$v^{t+1}\,{}_{t+1}V_x - v^t\,{}_tV_x = v^tP - v^{t+1}q_{x+t}$$

Summing over t = 0,1,2,…,*n*-1 we get:

$$v^n\,{}_nV_x = P\ddot{a}_{\overline{n}|} - \sum_{t=0}^{n-1} v^{t+1}q_{x+t}$$

As the maturity benefit is 1 i.e. ${}_nV_x = 1$, we get:

$$P = \frac{v^n + \sum_{k=0}^{n-1} v^{k+1}q_{x+k}}{\ddot{a}_{\overline{n}|}}$$

[6 Marks]

Solution 8:-

(i) EPV past service benefits:

100,000 * 20/80 *[($z\,M_{55}^{ia}$ + $z\,M_{55}^{ra}$) / ($s_{54}D_{55}$)] = 100000 * 20/80 * (34,048+128,026)/(9.745*1389) = 299,343

EPV future service benefits:
100,000/80 *[($z\,\bar{R}_{55}^{ia}$ + $z\,\bar{R}_{55}^{ra}$) / $s_{54}\,D_{55}$] = 100000/80 * (163,063 + 963,869)/(9.745*1389) = 104,070

EPV total pension benefits = 299,343 + 104,070 = 403,413

[3]

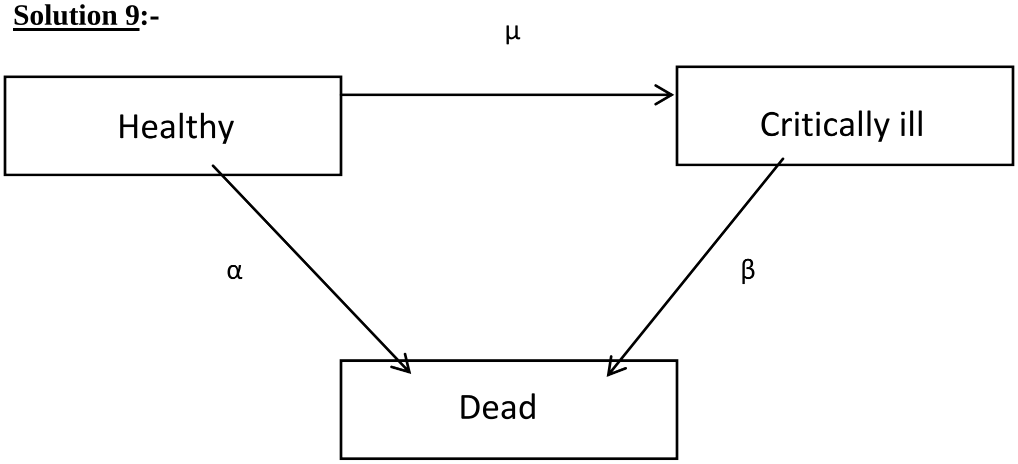
(ii) k * 100,000 * $s\bar{N}_{55}$ / $s_{54}\,D_{55}$ = 104,070

k * 100,000 * 88,615 / (9.745*1,389) = 104,070

k = 15.9% per annum **[2]**

[5 Marks]

Solution 9:-



EPV = 200,000 * $\int_0^{20} e^{-\delta t} * \; {}_tp_{40}^{hh} * \mu_{40+t}$ dt = 200,000 * $\int_0^{20} e^{-\delta t} * \; {}_tp_{40}^{\overline{hh}} * \mu_{40+t}$ dt

= 200,000 * $\int_0^{20} e^{-\delta t} * \; e^{-\int_0^t (\alpha_{40+r} + \mu_{40+r})\, dr} * \mu_{40+t}$ dt

where,

δ is the force of interest

${}_tp_{40}^{\overline{hh}}$ is the probability of a healthy life aged 40 being healthy at all points until age 40+t

μ_{40+t} is the probability that a healthy life becomes critically ill at age 40 + t

[5 Marks]

Solution 10:-

(i) Joint lifetime of x and y (time until first death) $T_{xy} = \min(T_x, T_y)$
Last survivor of x and y (time until last death) $T_{\overline{xy}} = \max(T_x, T_y)$

Joint lifetime distribution and density functions

$F_{T_{xy}}(t)$= P $[T_{xy} \leq t]$ = 1 - ${}_tp_{xy}$ = P [min(T_x, T_y) $\leq t$] = 1 – P [$T_x > t$ and $T_y > t$]

= 1 - P [$T_x > t$] P [$T_y > t$] (since T_x, T_y are independent)

= 1 - ${}_tp_x \, {}_tp_y$

$f_{T_{xy}}(t)$ = d/dt [1 - ${}_tp_x \, {}_tp_y$]
= -${}_tp_x$ (-${}_tp_y \; \mu_{y+t}$) - ${}_tp_y$ (-${}_tp_x \; \mu_{x+t}$)
= ${}_tp_x \; {}_tp_y \; (\mu_{x+t} + \mu_{y+t})$

Last survivor distribution and density functions

$F_{T_{\overline{xy}}}(t)$ = P [$T_{\overline{xy}} \le t$] = P [max(T_x, T_y) $\le t$] = P [$T_x \le t$ and $T_y \le t$]

= P [$T_x \le t$] P [$T_y \le t$] (since T_x, T_y are independent)

= $(1 - {}_tp_x)\,(1 - {}_tp_y)$
= $(1 - {}_tp_x - {}_tp_y + {}_tp_x\,{}_tp_y)$
= $(1 - {}_tp_x) + (1 - {}_tp_y) - (1 - {}_tp_x\,{}_tp_y)$
= $F_{T_x}(t) + F_{T_y}(t) - F_{T_{xy}}(t)$

$$\boldsymbol{F_{T_{\overline{xy}}}(t) = F_{T_x}(t) + F_{T_y}(t) - F_{T_{xy}}(t)}$$

$f_{T_{\overline{xy}}}(t)$ = d/dt $(1 - {}_tp_x - {}_tp_y + {}_tp_x\,{}_tp_y)$

= ${}_tp_x\,\mu_{x+t} + {}_tp_y\,\mu_{y+t} - {}_tp_x\,{}_tp_y(\mu_{x+t} + \mu_{y+t})$

= $f_{T_x}(t) + f_{T_y}(t) - f_{T_{xy}}(t)$

$$\boldsymbol{f_{T_{\overline{xy}}}(t) = f_{T_x}(t) + f_{T_y}(t) - f_{T_{xy}}(t)}$$

[5]

(ii) PV premiums = PV expenses + PV benefits

Let P be annual premium.

P $\ddot{a}_{\overline{60:60}}$ = 5,000 + 0.05 * P * $\ddot{a}_{\overline{60:60}}$ + 1,000,000 * $\bar{A}_{\overline{60:60}}$

0.95P $\ddot{a}_{\overline{60:60}}$ = 5,000 + 1,000,000 * $\bar{A}_{\overline{60:60}}$

$\ddot{a}_{\overline{60:60}}$ = $\ddot{a}_{60(m)}$ + $\ddot{a}_{60(f)}$ - $\ddot{a}_{60:60}$ = 15.632 + 16.652 – 14.090 = 18.194

$\bar{A}_{\overline{60:60}}$ = 1.04 ^ 0.5 * $A_{\overline{60:60}}$ = 1.04 ^ 0.5 * (1 – d * $\ddot{a}_{\overline{60:60}}$) = 1.0198 * (1 – 0.038462 * 18.194) = 0.306177

Therefore,

0.95 * 18.194 P = 5,000 + 1,000,000 * 0.306177 = 311,176
P = 18,003 p.a.

Also give credit if renewal expenses are assumed to be payable second year onwards.

[5]
[10 Marks]

Solution 11:- **(i) Decrements table**

Age	Independent probability of death	Independent probability of surrender	Dependent probability of surrender	Probability of survival at end	Probability of survival at start
45	0.00120	0.12000	0.11986	0.87894	1.00000
46	0.00156	0.06000	0.05991	0.93854	0.87894
47	0.00180	-	-	0.99820	0.82492

Unit fund

Year	1	2	3
Premium	20,000	20,000	20,000
Fund at start	-	18,569	38,706
Add: Allocated premium	19,000	20,000	21,000
Less: Bid offer spread	*950*	*1,000*	*1,050*
Less: Policy fee	*50*	*50*	*50*
Fund before interest	18,000	37,519	58,606
Add: Interest	900	1,876	2,930
Fund after interest	18,900	39,395	61,536
Less: FMC	*331*	*689*	*1,077*
Fund at end	18,569	38,706	60,459

Non-unit fund

Unallocated premium	1,000	-	(1,000)
Add: Bid offer spread	950	1,000	1,050
Add: Policy fee	50	50	50
Less: Commission	*3,000*	*400*	*400*
Less: Expenses	*200*	*51*	*52*
Fund before interest	(1,200)	599	(352)
Add: Interest	(48)	24	(14)
Add: Surrender penalty	445	232	0
Add: FMC	331	689	1,077
Less: Extra death benefit	*5.58*	*15.07*	*27.24*
Non-unit cash flow	(478)	1,529	684

Profit margin

Year	1	2	3	
Profit	(477.70)	1,529.18	683.54	
Probability of survival at start	1	0.87894	0.824920129	
Discount factor profit	0.934579439	0.873438728	0.816297877	
Present value of profit	(446.45)	1,173.96	460.29	**1,187.80**
Discount factor premium	1.00	0.93	0.87	
Present premium	20,000.00	16,428.84	14,410.34	**50,839.19**
Profit margin	**2.34%**			

[13]

(ii) Reduce premium allocation / increase bid offer spread
Reduce commission
Increase surrender penalty
Invest in higher yielding assets
Increase fund management charge
Increase policy fee

Marketability needs to be considered as competitors may offer better charging structure.
Distributors may not promote the product if commissions are low.
Investment portfolio will be based on the fund options provided to customers.

[3]

(iii) Discount rate is broadly based on the cost of capital (rate at which funds can be borrowed or rate which funds would earn if diverted from alternative investment opportunities). To this, a margin is added to reflect risks and uncertainties associated with the cashflows.

[1]

(iv) Can't say. It could increase, decrease or remain unchanged as the impact depends on the interaction between PV premiums and PV profits.

[1]

(v) It is a principle of prudent financial management that once sold and funded at outset a product should be self-supporting. Many products produce profit signatures that usually have a single financing phase. However, some products, particularly those with substantial expected outgo at later policy durations, can give profit signatures which have more than one financing phase. In such cases these later negative cashflows should be reduced to zero by establishing reserves in the non-unit fund at earlier durations. These reserves are funded by reducing earlier positive cashflows.

[2]
[20 Marks]

Solution 12:-

(i) (a) Crude Mortality Rate the ratio of the total number of deaths in a category to the total exposed to risk in the same category.

(b) Directly Standardised Mortality Rate the mortality rate of a category weighted according to a standard population.

(c) Indirectly Standardised Mortality Rate an approximation to the directly standardised mortality rate being the crude rate for the standard population multiplied by the ratio of actual to expected deaths for the region.

This is the same as the crude rate for the local population multiplied by the Area Comparability Factor. **[3]**

(ii) Area compatibility factor = $\frac{\sum_x sE^c_{x,t} sm_{x,t}}{\sum_x sE^c_{x,t}} \;/\; \frac{\sum_x E^c_{x,t} m_{x,t}}{\sum_x E^c_{x,t}}$

Age group	Population	Deaths	Population - Province A	Calculated	
	$sE^c_{x,t}$	$sE^c_{x,t}\ sm_{x,t}$	$E^c_{x,t}$	$sm_{x,t}$	$E^c_{x,t}\ sm_{x,t}$
0-19	3,000,000	580	800,000	0.0001933	155
20-39	3,500,000	2,450	1,000,000	0.0007	700
40-69	2,500,000	20,300	700,000	0.00812	5,684
70 and over	500,000	49,000	300,000	0.098	29,400
Total	9,500,000	72,330	2,800,000	0.0076137	35,939

Area Compatibility Factor = (72,330 / 9,500,000) / (35,939 / 2,800,000)

= 0.593186

Indirectly standardised mortality rate = (ACF)*(Province crude rate)
0.593186 * 21,453 / 2,800,000 = 0.004544

[5]
[8 Marks]

Solution 13:-

(i) Adverse selection against competitors as smokers will take insurance from competitors while non-smokers will prefer to take insurance from this company.

This implies that the company's mortality experience may be better than expected (as higher non-smokers) while competitors may have worse mortality experience than expected (as higher smokers).

(ii) No single dominant selection. Some degree of class, time and temporary initial selection could be observed.

[2]

****