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10MAT11

First Semester B.E. Degree Examination, January 2013

Engineering Mathematics – I

Time: 3 hrs. Max. Marks:100

Note: 1. ***Answer any FIVE full questions, choosing at least two from each part.***
2. Answer all objective type questions only on OMR sheet page 5 of the answer booklet.
3. Answer to objective type questions on sheets other than OMR will not be valued.

Important Note : 1. On completing your answers, compulsorily draw diagonal cross lines on the remaining blank pages.
2. Any revealing of identification, appeal to evaluator and /or equations written eg, 42+8 = 50, will be treated as malpractice.

PART – A

1 a. Choose correct answers for the following : **(04 Marks)**

i) If $y = 3^{2x}$ then y_n = ____: A) $2^{3x}(2\log 3)^n$ B) $3^{2x}(\log 3)^n$ C) $3^{2\log x}$ D) $3^{2x}(2\log 3)^n$

ii) If $y = \log(1-x)$ the y_n = ____: A) $\frac{(-1)^{n-1}n!}{(1-x)n}$ B) $\frac{(-1)^{2n-1}(n-1)!}{(1+x)^n}$ C) $\frac{(-1)^{2n-1}(n-1)!}{(1-x)^n}$ D) $\frac{(-1)^{2n+1}(n-1)!}{(1-x)^{n+1}}$

iii) By Rolle's theorem the number C = _____ when $f(x) = x^2 - 4x + 8$ in [1, 3]: A) 1 B) 2 C) 3 D) 4

iv) By Maclaurins series, the expansion $x - \frac{x^3}{3!} + \frac{x^5}{5!}$ ------- is equal to ____: A) e^x B) cosx C) sinx D) x cosx

b. Find the n^{th} derivative of $x^2 \sin 3x$. **(04 Marks)**

c. Show that $\frac{b-a}{1+b^2} < \tan^{-1} b - \tan^{-1} a < \frac{b-a}{1+a^2}$, if $0 < a < b$ and deduce that $\frac{\pi}{4} + \frac{3}{25} < \tan^{-1}\frac{4}{3} < \frac{\pi}{4} + \frac{1}{6}$. **(06 Marks)**

d. Expand $\tan^{-1}x$ in powers of x -1 upto the term containing fourth degree. **(06 Marks)**

2 a. Choose correct answers for the following : **(04 Marks)**

i) $\underset{x \to 0}{\text{Limit}}\left[\frac{\log \sin ax}{\log \sin bx}\right]$ = ________ : A) 1 B) a/b C) b/a D) ab

ii) The angle between the radius vector and the tangent of the curve $r = \sin\theta + \cos\theta$ is _____
A) $\pi/2 + \theta$ B) $\pi/4 + \theta$ C) $\pi/3 + \theta$ D) $\pi/6 + \theta$

iii) Derivative of arc length for polar curve, the value $ds/d\theta$ = _____

A) $\sqrt{r^2 + \frac{d^2r}{d\theta^2}}$ B) $\sqrt{r + \left(\frac{dr}{d\theta}\right)^2}$ C) $\sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}$ D) $\sqrt{r^2 + \left(\frac{d\theta}{dr}\right)^2}$

iv) Radius of curvature of $y = x^2$ at x = 1 is = _____ : A) $5\sqrt{5}$ B) $\frac{4\sqrt{5}}{2}$ C) $\frac{3\sqrt{5}}{2}$ D) $\frac{5\sqrt{5}}{2}$

b. Evaluate $\underset{x \to 0}{\text{Limit}}\left[\frac{a^x + b^x + c^x + d^x}{4}\right]^{1/x}$. **(04 Marks)**

c. Find the angle of intersection between the curves $r^2 \sin 2\theta = 4$ and $r^2 = 16 \sin 2\theta$. **(06 Marks)**

d. Find the radius of curvature at any point t of the curve $x = a(\cos t + \log \tan t/2)$, $y = a \sin t$. **(06 Marks)**

3 a. Choose correct answers for the following : **(04 Marks)**

i) If $F(u) = \sin u = \frac{x^2y^2}{x^2+y^2}$ the value of $x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y}$ = _____ :A) cot u B) tan u C) 2 tanu D) 3 tan u

ii) Jacobian for $x = r\cos\theta$, $y = r\sin\theta$ is ____: A) r B) $1/r^2$ C) 1/r D) r^2

iii) The necessary condition for u = f(x, y) have maxima or minima is
A) $\partial u/\partial x \neq 0$, $\partial u/\partial y \neq 0$ B) $\partial u/\partial x = 0$, $\partial u/\partial y = 0$ C) $\partial u/\partial x > 0$, $\partial u/\partial y > 0$ D) $\partial u/\partial x < 0$, $\partial u/\partial y < 0$

iv) The percentage error in the area of the rectangle when an error of 1.0% is made in measuring the sides x and y is _____ : A) 4 B) 3 C) 2 D) 1

b. If $x = r\sin\theta\cos\phi$, $y = r\sin\theta\sin\phi$, $z = r\cos\theta$, find the Jacobian of (x, y, z) with respect to r, θ, φ. **(04 Marks)**

c. Find the percentage error in computing resistance r of two resistances r_1 and r_2 connected in parallel of both r_1 and r_2 are in error by 2%. **(06 Marks)**

d. Find the extreme values of the function $f(x, y) = x^3y^2(1 - x - y)$. **(06 Marks)**

4 a. Choose correct answers for the following : **(04 Marks)**

i) If $\vec{R}$ is a position vector of any point P(x, y, z) then $\nabla \cdot \vec{R}$ is ____: A) 0 B) 1 C) 2 D) 3

ii) Any motion in which the curl of the velocity vector is zero, then the vector $\vec{V}$ is said to be
A) solenoidal B) Vector C) Constant D) Irrotational

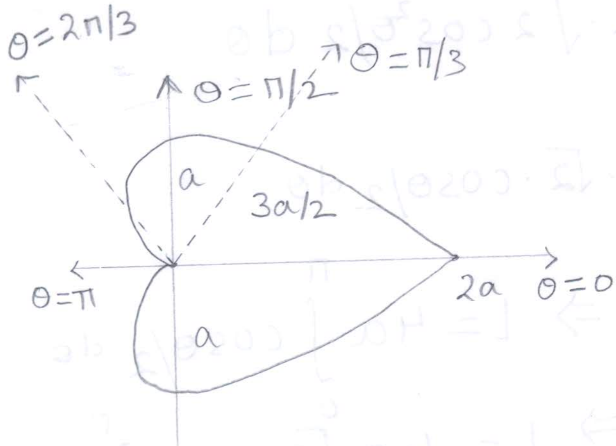
iii) If φ is the scalar point function then the value of curl (grad φ) = ____: A) > 0 B) < 0 C) 0 D) ∞

iv) In orthogonal curvilinear coordinates the value of $\frac{\partial(x,y,z)}{\partial(u,v,w)}$ is = _____
A) $h_1 h_2 h_3$ B) $1/h_1h_2h_3$ C) h_1/h_2h_3 D) h_1h_2/h_3

287

b. Show that the vector field $F = (x^2 - yz)i + (y^2 - zx)j + (z^2 - xy)k$ is irrotational and find its scalar potential. **(04 Marks)**

c. Prove that $\nabla\left(\phi\vec{A}\right) = (\nabla\phi)\cdot\vec{A} + \phi\left(\nabla\cdot\vec{A}\right)$ where φ is a scalar field. **(06 Marks)** 295 d.)



$$r = a(1+\cos\theta)$$

$$\frac{dr}{d\theta} = -a\sin\theta$$

Total length of cardiode —

$$l = 2\int_0^{\pi} \sqrt{r^2 + \left(\frac{dr}{d\theta}\right)^2}\, d\theta$$

$$\Rightarrow l = 2\int_0^{\pi} \sqrt{a^2(1+\cos\theta)^2 + a^2\sin^2\theta}\, d\theta$$

$$\Rightarrow l = 2\int_0^{\pi} \sqrt{a^2(1+\cos^2\theta+2\cos\theta) + a^2\sin^2\theta}\, d\theta$$

$$\Rightarrow l = 2\int_0^{\pi} \sqrt{a^2 + a^2\cos^2\theta + 2a^2\cos\theta + a^2\sin^2\theta}\, d\theta$$

$$\Rightarrow l = 2\int_0^{\pi} \sqrt{a^2 + 2a^2\cos\theta + a^2(\cos^2\theta + \sin^2\theta)}\, d\theta$$

$$\Rightarrow l = 2\int_0^{\pi} \sqrt{a^2 + 2a^2\cos\theta + a^2}\, d\theta$$

$$\Rightarrow l = 2\int_0^{\pi} \sqrt{2a^2 + 2a^2\cos\theta}\, d\theta$$

$$\Rightarrow l = 2\int_0^{\pi} \sqrt{2a^2(1+\cos\theta)}\, d\theta$$

$\because \frac{dy}{dx} = \frac{2x+3y+4}{4x+6y+5}$

$\Rightarrow \frac{dY}{dX} = \frac{2(X+h)+3(Y+k)+4}{4(X+h)+6(Y+k)+5}$

$\Rightarrow \frac{dY}{dX} = \frac{2X+3Y+2h+3k+4}{4X+6Y+4h+6k+5}$

Let us take

$2h+3k+4=0$

$\Rightarrow 2h+3k=-4$

$4h+6k+5=0$

$\Rightarrow 4h+6k=-5$

$\Rightarrow \frac{dY}{dX} = \frac{2X+3Y}{4X+6Y}$

$\Rightarrow \frac{dY}{dX} = \frac{\cancel{2X+3Y}}{2(\cancel{2X+3Y})}$

$\Rightarrow dY = \frac{dX}{2}$

$\Rightarrow \int dY = \int \frac{dX}{2} \Rightarrow Y = \frac{X}{2}$

~~$\Rightarrow y-k = x-k$~~

$\Rightarrow y-k = \frac{x-h}{2}$

7.) a.) (i) D) $R(A) = R(A:B)$

(ii) $A = \begin{bmatrix} 1 & 3 & -2 \\ 2 & -1 & 4 \\ 1 & -11 & 14 \end{bmatrix}$

$R_2 \to -2R_1 + R_2$

$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 1 & -11 & 14 \end{bmatrix}$

$R_3 \to -R_1 + R_3$

$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & -14 & 16 \end{bmatrix}$

$R_3 \to 2R_2 - R_3$

$A = \begin{bmatrix} 1 & 3 & -2 \\ 0 & -7 & 8 \\ 0 & 0 & 0 \end{bmatrix}$

$\therefore \rho(A) = 2$

Correct Answer is C) 2

(iii) A) $a_{ij} = a_{ji}$

(iv) D) Upper Triangular Matrix

b.) $A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 2 & 5 & -4 & 6 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

$R_2 \to -2R_1 + R_2$

$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ -1 & -3 & 2 & -2 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

$R_3 \to R_1 + R_3$

$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 2 & 4 & -1 & 6 \end{bmatrix}$

$R_4 \to -2R_1 + R_4$

$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & -1 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

$R_3 \to R_2 + R_3$

$A = \begin{bmatrix} 1 & 2 & -2 & 3 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

$C_3 \to C_3 + C_2$

$A = \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 1 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 3 & 0 \end{bmatrix}$

$\therefore \rho(A) = 4$

$[A : B]$ = Augmented Matrix = $\begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 7 & 3 & -2 & : & 8 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$

$R_2 \to -7R_1 + 2R_2$

$$[A : B] \sim \begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 0 & -15 & -39 & : & -47 \\ 2 & 3 & \lambda & : & \mu \end{bmatrix}$$

$R_3 \to -R_1 + R_2$

$$[A : B] \sim \begin{bmatrix} 2 & 3 & 5 & : & 9 \\ 0 & -15 & -39 & : & -47 \\ 0 & 0 & \lambda-5 & : & \mu-9 \end{bmatrix}$$

(i) Unique solution

For a unique solution $R(A) = R(A : B)$

$\therefore \boxed{\lambda \neq 5 \ , \mu = 9}$

(ii) No solution

For a no solution $R(A) \neq R(A : B)$

$\therefore \boxed{\lambda = 5 \quad \mu \neq 9}$

(iii) Infinite solution

For an infinite solution $R(A) = R(A ; B) <$ no. of variables

$\therefore \boxed{\lambda \neq 5 \quad \mu = 9}$

8.) a) (i)

(ii) C) $A \cdot A' = I$

(iii) D) (1, 6)

(iv) A) Canonical form

b.) $y_1 = 2x_1 + x_2 + x_3$ $\quad y_2 = x_1 + x_2 + 2x_3$

$y_3 = x_1 - 2x_3$

$$\begin{bmatrix} y_1 \\ y_2 \\ y_3 \end{bmatrix} = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix} \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$$

$\Rightarrow Y = AX \quad \Rightarrow X = A^{-1}Y$

$$A = \begin{bmatrix} 2 & 1 & 1 \\ 1 & 1 & 2 \\ 1 & 0 & -2 \end{bmatrix}$$

$$|A| = 2(-2) - 1(-2-2) + 1(-1) = -4 + 4 - 1 = -1$$

$\because |A| \neq 0$, The transformation is regular

$$\therefore \operatorname{adj} A = \begin{bmatrix} -2 & 4 & -1 \\ 2 & -5 & 1 \\ 1 & -3 & 1 \end{bmatrix}^T$$

$$\Rightarrow \operatorname{adj} A = \begin{bmatrix} -2 & 2 & 1 \\ 4 & -5 & -3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$\therefore A^{-1} = \frac{1}{|A|} \operatorname{adj} A$$

$$= \frac{1}{-1} \begin{bmatrix} -2 & 2 & 1 \\ 4 & -5 & -3 \\ -1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 2 & -2 & -1 \\ -4 & 5 & 3 \\ 1 & -1 & -1 \end{bmatrix}$$

Now

$X = A^{-1}Y$

$$\cancel{\lambda=6} \quad \lambda+2 \,\big)\, \lambda^3-7\lambda^2+36 \,\big(\, \lambda^2-9\lambda+18$$

$$\begin{array}{r}
\lambda^3+2\lambda^2 \\
(-)\quad(-) \\
\hline
-9\lambda^2+36 \\
-9\lambda^2 \quad -18\lambda \\
(+)\quad\quad (+) \\
\hline
18\lambda+36 \\
+18\lambda+36 \\
(-)\quad\quad(-) \\
\hline
0
\end{array}$$

$$\Rightarrow (\lambda+2)(\lambda^2-6\lambda-3\lambda+18)=0$$

$$\Rightarrow (\lambda+2)(\lambda(\lambda-6)-3(\lambda-6))=0$$

$$\Rightarrow (\lambda+2)(\lambda-6)(\lambda-3)=0$$

$$\Rightarrow \lambda=-2,6,3$$

$\therefore$ $-2, 6, 3$ are the eigen values of matrix $A=\begin{bmatrix}1&1&3\\1&5&1\\3&1&1\end{bmatrix}$

Now

$$[A-\lambda I][X]=0$$

$$\Rightarrow \begin{bmatrix}1-\lambda & 1 & 3\\ 1 & 5-\lambda & 1\\ 3 & 1 & 1-\lambda\end{bmatrix}\begin{bmatrix}x\\y\\z\end{bmatrix}=0$$

$$\Rightarrow (1-\lambda)x+y+3z=0$$

$$x+(5-\lambda)y+z=0$$

$$3x+y+(1-\lambda)z=0$$

Case (i) When $\lambda=-2$

$\therefore$

$$3x+y+3z=0 \longrightarrow (1)$$

$$x+7y+z=0 \longrightarrow (2)$$

$$3x+y+3z=0 \longrightarrow (3)$$

Case -(iii) When $\lambda = 6$

$$\therefore \quad -5x + y + 3z = 0 \longrightarrow (7)$$
$$x - y + z = 0 \longrightarrow (8)$$
$$3x + y - 5z = 0 \longrightarrow (9)$$

Solving eq. (7) & eq. (9)

$$\frac{x}{\begin{vmatrix} 1 & 3 \\ 1 & -5 \end{vmatrix}} = \frac{-y}{\begin{vmatrix} -5 & 3 \\ 3 & -5 \end{vmatrix}} = \frac{z}{\begin{vmatrix} -5 & 1 \\ 3 & 1 \end{vmatrix}}$$

$$\Rightarrow \frac{x}{|-5-3|} = \frac{-y}{|25-9|} = \frac{z}{|-5-3|}$$

$$\Rightarrow \frac{x}{-8} = \frac{-y}{16} = \frac{z}{-8}$$

$$\Rightarrow \frac{x}{1} = \frac{y}{2} = \frac{z}{1}$$

$\therefore$ $X_3 = [x, y, z] = [1, 2, 1]$ is the eigen vector corresponding to $\lambda = 6$

d.) $x^2 + 5y^2 + z^2 + 2yz + 6xz + 2xy$

$$A = \begin{bmatrix} 1 & 1 & 3 \\ 1 & 5 & 1 \\ 3 & 1 & 1 \end{bmatrix}$$

$$A = I A I$$

$R_2 \to R_2/4 \qquad C_3 \to C_3/4$

$$\Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & -9 \end{bmatrix} = \begin{bmatrix} 1 & 0 & 0 \\ -1/4 & 1/4 & 0 \\ -7 & 1 & 2 \end{bmatrix} A \begin{bmatrix} 1 & - & \\ 0 & & \\ 0 & & \end{bmatrix}$$

$$\Rightarrow D = \mathrm{Diag}(1, 4, -36) = P'AP$$

$$P' = \begin{bmatrix} 1 & 0 & 0 \\ -1 & 1 & 0 \\ -7 & 1 & 2 \end{bmatrix} \qquad P = \begin{bmatrix} 1 & -1 & -7 \\ 0 & 1 & 1 \\ 0 & 0 & 2 \end{bmatrix}$$

$\therefore$ Canonical form is $x_1^2 + 4x_2^2 - 36x_3^2$

4) a) i) D) 3 ii) D) Irrotational

iii) c) 0 iv) A) $h_1 h_2 h_3$

b) Given
$\vec{F} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$

We have to show that $\nabla \times \vec{F} = \vec{0}$

$$\therefore \nabla \times \vec{F} = \begin{vmatrix} \hat{i} & \hat{j} & \hat{k} \\ \frac{\partial}{\partial x} & \frac{\partial}{\partial y} & \frac{\partial}{\partial z} \\ (x^2 - yz) & (y^2 - zx) & (z^2 - xy) \end{vmatrix}$$

$\nabla \times \vec{F} = \hat{i}(-x + x) - \hat{j}(-y + y) + \hat{k}(-z + z)$

$\nabla \times \vec{F} = 0$

$\therefore \vec{F}$ is conservative.

$\therefore \vec{F}$ is irrotational.

Now we need to find ϕ such that $\nabla \phi = \vec{F}$

i.e $\frac{\partial \phi}{\partial x}\hat{i} + \frac{\partial \phi}{\partial y}\hat{j} + \frac{\partial \phi}{\partial z}\hat{k} = (x^2 - yz)\hat{i} + (y^2 - zx)\hat{j} + (z^2 - xy)\hat{k}$

$\Rightarrow \frac{\partial \phi}{\partial x} = x^2 - yz \qquad \therefore \phi = \frac{x^3}{3} - yzx + f_1(y, z)$

$\frac{\partial \phi}{\partial y} = y^2 - zx \qquad \therefore \phi = \frac{y^3}{3} - zxy + f_2(x, z)$

$\frac{\partial \phi}{\partial z} = z^2 - xy \qquad \therefore \phi = \frac{z^3}{3} - xyz + f_3(x, y)$

Let us choose $f_1(y, z) = f_2(x, z) = f_3(x, y) = 0$

$$\therefore \boxed{\phi = \frac{x^3}{3} + \frac{y^3}{3} + \frac{z^3}{3} - xyz}$$

$\Rightarrow$

$$\nabla\times(F_1 e_1) = \nabla\times(F_1 h_1 \nabla u_1) \quad (\text{by } ②)$$

$$= \nabla\times(\phi\vec{a}) \quad \text{where } \phi = F_1 h_1,\ \vec{a} = \nabla u_1$$

$$= \phi(\nabla\times\vec{a}) + \nabla\phi \times \vec{a}$$

$$= F_1 h_1\{\nabla\times(\nabla u_1)\} + \nabla(F_1 h_1)\times\nabla u_1$$

$$= 0 + \nabla(F_1 h_1)\times\nabla u_1 \quad \because \nabla\times\nabla\phi = 0$$

$\therefore$

$$\nabla\times(F_1 e_1) = \left\{\frac{1}{h_1}\frac{\partial}{\partial u_1}(F_1 h_1)e_1 + \frac{1}{h_2}\frac{\partial}{\partial u_2}(F_1 h_1)e_2 + \frac{1}{h_3}\frac{\partial}{\partial u_3}(F_1 h_1)e_3\right\}\times\left\{\frac{\hat{e}_1}{h_1}\right\}$$

By $\hat{e}_1\times\hat{e}_1 = 0$, $\hat{e}_2\times\hat{e}_1 = -\hat{e}_3$, $\hat{e}_3\times\hat{e}_1 = \hat{e}_2$
we have

$$\nabla\times(F_1\hat{e}_1) = \frac{-\hat{e}_3}{h_1h_2}\frac{\partial}{\partial u_2}(F_1h_1) + \frac{\hat{e}_2}{h_1h_3}\frac{\partial}{\partial u_3}(F_1h_1)$$

Similarly,

$$\nabla\times(F_2 e_2) = \frac{-\hat{e}_1}{h_2h_3}\frac{\partial}{\partial u_3}(F_2h_2) + \frac{\hat{e}_3}{h_2h_1}\frac{\partial}{\partial u_1}(F_2h_2)$$

$$\nabla\times(F_3 e_3) = \frac{-\hat{e}_2}{h_3h_1}\frac{\partial}{\partial u_1}(F_3h_3) + \frac{\hat{e}_1}{h_3h_2}\frac{\partial}{\partial u_2}(F_3h_3)$$

Adding these Results L.H.S becomes $\nabla\times\vec{F}$ according to ① and R.H.S can be put in the determinant form as follows.