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# Target IIT-JEE 2015

# Physics

# Gravitation

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### Introduction

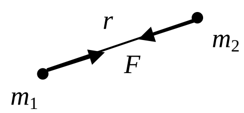
- Earth attracts all bodies towards its centre. Newton generalized the law by saying that not only the earth but all materials bodies in the universe attract each other with some force. This is known as Newton's Universal Law of Gravitation.

### Newton's Universal Law of Gravitation

- According to Newton's Universal Law of Gravitation, **the *force of attraction F* between two *point masses* $m_1$ and $m_2$ is directly proportional to the product of their masses and inversely proportional to the square of the distance *r* between them and is directed along the line joining them.**
- Therefore,

$F \propto m_1 m_2$

$F \propto \frac{1}{r^2}$




Combining we get, $F \propto \frac{m_1 m_2}{r^2}$

$$\boxed{F = \frac{G m_1 m_2}{r^2} \quad \text{(Newton's Universal Law of Gravitation)}}$$

where $G$ = constant of proportionality known as **Universal Gravitational Constant.**

$$G = 6.67 \times 10^{-11} \frac{Nm^2}{kg^2} \quad \text{(Universal Gravitational Constant)}$$

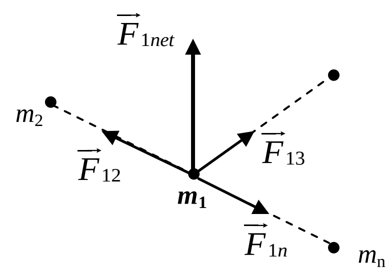
### Important Points about Newton's Universal Law of Gravitation

1. Newton's Universal Law of Gravitation states that $F = \frac{G m_1 m_2}{r^2}$
   This expression is valid for **point masses** (**particles**) only.
1. Gravitational force $F$ acts along the line joining the two particles and is **attractive** in nature.
2. We can treat large bodies as point masses, if the distance $r$ between them is very large compared to their sizes.
   For example, we can treat Earth and Sun as particles because the distance between them is quite large compared to their radii.
3. ***A uniform sphere or a spherical shell attracts a particle that is outside it as if all the mass were concentrated at its centre.***
   Thus, a uniform spherical body or a uniform hollow spherical body act as a point mass at its centre when another particle is outside.
4. ***A uniform shell of matter exerts no net gravitational force on a particle located inside it.***

### Principle of Superposition

- **Definition:** *The force on any mass due to a number of other masses is the vector sum of all the forces on that mass due to the other masses, taken one at a time. The individual forces are unaffected due to the presence of other masses.*
- Suppose there are $n$ particles of masses $m_1, m_2, \ldots, m_n$ in the space. What is the total gravitational force acting on particle $m_1$ due to rest of the particles?
- Just find the individual attractive gravitational forces acting on particle $m_1$ due to rest of the particles. Let the magnitude of these attractive forces be $\boldsymbol{F_{12}}, \boldsymbol{F_{13}}, \ldots \boldsymbol{F_{1n}}$ respectively. Here $\boldsymbol{F_{1n}}$ = attractive gravitational force on particle 1 because of the nth particle.
- NOTE: Gravitational force $\boldsymbol{F_{1n}}$ is a vector whose direction is along the line joining $m_1$ & $m_n$ and pointing towards point mass $m_n$. Similar notation for others.
  Therefore net gravitational force $\boldsymbol{F_{1net}}$ acting on particle 1 due to rest of the particles is given by vector sum of $\boldsymbol{F_{12}}$, $\boldsymbol{F_{13}}, \ldots, \boldsymbol{F_{1n}}$.

$$\boxed{\vec{F}_{1net} = \vec{F}_{12} + \vec{F}_{13} + \ldots + \vec{F}_{1n}}$$




### Gravitational force on a particle from an extended body

- We have to find gravitational force acting on a particle of mass $m_1$ from an extended body of mass $m$. Here we use the concept of superposition.
- We divide the body into infinitesimally small segments, such that each segments behaves as a particle.
- Let us consider an infinitesimally small segment of mass $dm$ at a distance $r$ from the particle $m_1$.
- Hence, net gravitational force acting on particle from extended body

$$\boxed{F = \int dF = \int G m_1 \frac{dm}{r^2}}$$

## Acceleration due to Gravity

- **Earth attracts every body with an acceleration which is known as acceleration due to gravity.** It is denoted by $\boldsymbol{g}$. Its value depends on many factors and *does not remain constant.* We use $g = 9.8$ m/s$^2$ near the Earth's surface to solve most of the problems.
- **NOTE:** Acceleration due to gravity $g$ (small $g$) is not same as Universal gravitational constant $G$ (capital $G$). Value of $G$ is constant anywhere in the Universe but the value of $g$ changes from place to place.

## Acceleration due to Gravity Near the Earth's surface

- Let us consider Earth to be a **uniform sphere** of mass $M$.
- As we have assumed earth to be a uniform sphere, it will attract a body outside its surface as though all the mass were concentrated at its centre.
- At a particular instant when the distance of particle from Earth centre is $r$, gravitational force is given by $F = \frac{GmM}{r^2}$
- Using Newton's Second Law of Motion ($F = m\,a = mg$) and Newton's Law of Gravitation $F = \frac{GmM}{r^2}$, we get

$$F = \frac{GmM}{r^2} = mg$$

$$g = \frac{GM}{r^2}$$

- Note that this acceleration is independent of the mass of the body itself.
- If the body is near the Earth's surface, we can replace $r$ with radius $R$ of the Earth. Therefore, acceleration due to gravity near the Earth's surface is

$$\boxed{g = \frac{GM}{R^2} \qquad \text{(accl}^{\text{n}}\text{ due to gravity near Earth's surface)}}$$

- The value of $g$ is typically taken to be 9.8 m/s$^2$ near the Earth's surface.

## Important Points to note about acceleration due to Gravity

- The expression $g = \frac{GM}{R^2}$ for acceleration due to gravity is deduced assuming Earth to be a *uniform sphere* and *neglecting effect of Earth's rotation*.
- We are going to study the variation in acceleration due to gravity because of following effects:

1. Altitude (height) above the earth's surface
2. Depth below the Earth's surface
3. Rotation of Earth (Variation with latitude).

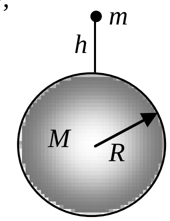
## Variation of acceleration due to gravity with Altitude above Earth's surface

- Let us consider a body of mass $m$ at a height $h$ above the surface of the Earth. Let us consider Earth to be a *uniform sphere* of mass $M$ and radius $R$. Clearly, the body is at a distance ($R + h$) from the center of the Earth. Therefore, gravitational force is

$$F = \frac{GMm}{(R+h)^2}$$

- If $g_h$ is the value of acceleration due to gravity at that point, then from Newton's 2$^{\text{nd}}$ law,

$F = ma = mg_h$




- Hence, $F = \frac{GMm}{(R+h)^2} = mg_h$

$$\boxed{g_h = \frac{GM}{(R+h)^2} \qquad \text{...(i)}}$$

- We want to write this expression in terms of acceleration due to gravity near Earth's surface, i.e., in terms of $g = \frac{GM}{R^2}$. Therefore, we multiply & divide denominator of expression (i) by R$^2$. Thus, we get,

$$g_h = \frac{GM}{R^2\left(1+\frac{h}{R}\right)^2} = \left(\frac{GM}{R^2}\right)\left(1+\frac{h}{R}\right)^{-2}$$

- As, $g = \frac{GM}{R^2}$ = acceleration due to gravity at Earth's surface. Therefore,

$$\boxed{g_h = g\left(1+\frac{h}{R}\right)^{-2}}$$
(acceleration due to gravity at a height h from surface)

- In general, we are interested in those bodies which are not very far away from the surface. Therefore, for the case, when $h$ is much smaller than $R$, i.e., $\boldsymbol{h << R}$, we have from **Binomial Expansion**,

$$\left(1+\frac{h}{R}\right)^{-2} = 1 - \frac{2h}{R}$$

Hence,

$$\boxed{g_h = g\left(1-\frac{2h}{R}\right) \qquad \text{(for } h << R\text{)}}$$
(acceleration due to gravity at a height $h$ from surface)

- Thus, the value of gravitational acceleration *decreases linearly* as one moves away from the surface of the Earth.

**Funda Session:** The main idea here is that *the body will be attracted by the mass of the Earth which is enclosed in a sphere of radius (R – h). There will be no net attraction from the mass of the Earth which lies outside the sphere of radius (R – h).*

***Reason:*** *A uniform shell of matter exerts no net gravitational force on a particle located inside it and earth can be considered as a nest of such shells. Hence shells outside this body produces no net attraction*

**Variation of acceleration due to gravity with Depth below Earth's surface**

- Consider a body of mass $m$ taken to a **depth *h*** *inside the Earth's surface*. Let us consider Earth to be a ***uniform sphere*** of mass $M$ and radius $R$.
- Let us find the mass of this portion of the sphere which lies inside radius $(R - h)$.
- Mass per unit volume of the earth = $\dfrac{M}{V} = \dfrac{M}{\frac{4}{3}\pi R^3}$
- Hence, mass of the Earth inside the radius (R – h),

M' = (Mass per unit volume)

× (Volume the sphere of radius (R – h))

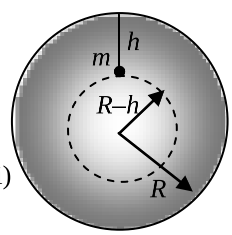
$$\Rightarrow M' = \frac{M}{\frac{4}{3}\pi R^3} \times \frac{4}{3}\pi (R-h)^3$$

$$\Rightarrow M' = \frac{M(R-h)^3}{R^3} \qquad \ldots \text{(i)}$$

- Thus, gravitational force acting on the body at a depth $h$ below the Earth's surface is

$$F = \frac{GM'm}{(R-h)^2}$$

$$\Rightarrow F = \frac{GMm(R-h)}{R^3} \qquad \ldots \text{(ii)}$$




(Using (i))

- Let us denote acceleration due to gravity at this point by $g'_h$. Therefore, from Newton's 2nd Law we have,

$F = ma \Leftrightarrow F = m g'_h$ …(iii)

- Using (ii) & (iii) we get:

$$mg'_h = \frac{GMm(R-h)}{R^3} \Rightarrow g'_h = \frac{GM(R-h)}{R^3} = \left(\frac{GM}{R^2}\right)\left(\frac{R-h}{R}\right)$$

$$\boxed{\Rightarrow \; g'_h = g\left(1 - \frac{h}{R}\right)}$$

(acceleration due to gravity at a depth $h$ from surface)

- Thus, the value of gravitational acceleration decreases if one moves towards the center of the Earth, e.g., in mines.

**Discussion**

- Acceleration due to gravity **near the Earth's surface**

$$g = \frac{GM}{R^2}$$

- Acceleration due to gravity at a **altitude (height) *h* above** the Earth's surface

$$g_h = g\left(\frac{1+h}{R}\right)^{-2}$$

For the case when Body is not very far away from the surface, i.e., for $h << R$

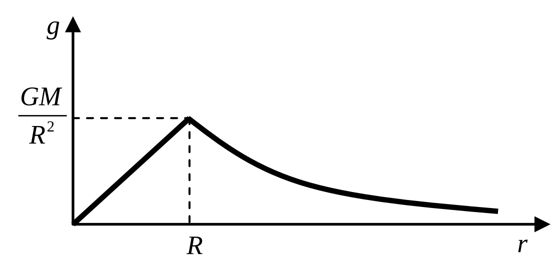
$$g_h = g\left(1 - \frac{2h}{R}\right)$$

Clearly, $\boldsymbol{g_h}$ decreases with increase in height $h$ from the Earth's surface.

- Acceleration due to gravity at a **depth *h* below** the Earth's surface

$$g_h = g\left(1 - \frac{2h}{R}\right)$$

- Clearly, $g'_h$ decreases with decrease in depth $h$ from the Earth's surface.
- Therefore, *value of* acceleration *due to gravity is maximum at the surface of the Earth. It decreases with both height as well as depth from the Earth's surface.*
- Graphically, we can plot variation of acceleration due to gravity with $r$ (distance from the centre of the earth as):




**Variation of *g* with Rotation and Latitude**

- Now, we are going to study about variation of $g$ with ***axial rotation*** of earth and with ***latitude***.
- Let us first see what do we mean by latitude. Earth rotates about an axis passing through its north and south poles. This axis is sometimes called **polar axis**.
- A plane passing through the centre of the Earth and perpendicular to the axis of rotation of the Earth is called **Equatorial Plane**.
- **The latitude *of a point P* is defined as the angle *ø* which the radial line OP makes with the equatorial plane.**
- Therefore, latitude of a point A on the equator $= 0^{\circ}$
- And latitude of a point N on the north pole = $90^{\circ}$

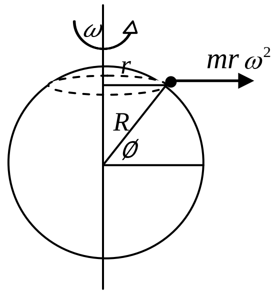
### Variation of $g$ with Rotation and Latitude

- Let us consider Earth is rotating with an angular speed $\omega$. Let us consider a particle of mass $m$ at rest wrt the Earth at latitude $\phi$.
- Then the pseudo force acting on the particle is $mr\omega^2$ in outward direction. The true acceleration 'g' is acting towards the centre O of the earth. Thus, the effective acceleration 'g' is the resultant of g and $r\omega^2$ or,

$$g' = \sqrt{g^2 + \left(r\omega^2\right)^2 + 2g\left(r\omega^2\right)\cos\left(180-\phi\right)}$$

$$g' = \sqrt{g^2 + \left(r\omega^2\right)^2 + 2g\left(r\omega^2\right)\cos\phi}$$

- Here, the term $\sqrt{r^2\omega^4}$ comes out to be too small as $\omega = \dfrac{2\pi}{T} = \dfrac{2\pi}{24\times3600}$ rad/s is small. Hence, this term can be ignored . Also $r = R\cos\phi$. Therefore , Eq. (i) can be written as




$$g' = \left(g^2 - 2gR\omega^2\cos^2\phi\right)^{\frac{1}{2}}$$

$$= g\left(1 - \frac{2R\omega^2\cos^2\phi}{g}\right)$$

$$\approx g\left(1 - \frac{R\omega^2\cos^2\phi}{g}\right)$$

Thus, $g' = g - \omega^2 R\cos^2\phi$

- Following conclusions can be drawn from the above discussion:
  - (i) The effective value of g is not truly vertical.
  - (ii) The effect of centrifugal force due to rotation of earth is to reduce the effective value of g.
  - (iii) At equators $\phi = 0^0$
    Therefore g'= $g - R\omega^2$
    And at poles $\theta = 90^0$
    Therefore, g' = g
    Thus, at equation g' is minimum while at poles g' is maximum.

### Gravitational Potential Energy

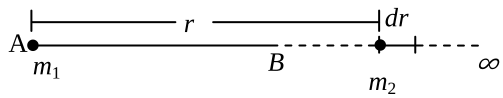
- **Gravitational Potential Energy (U):** *The work done by external force (or negative of work done by gravitational force) in bringing point masses from infinity to their respective positions without acceleration is called the gravitational potential energy of the system.*
- It is represented usually by $U$ and is given mathematically as

$$U = -W = -\frac{Gm_1m_2}{r}$$

- The gravitational potential energy at infinity is assumed to be zero.
- Gravitational Potential Energy is a scalar quantity.
- The gravitational potential energy is a property of system of two or more particles rather than of either particle alone.

### Derivation of Gravitational Potential Energy

- Consider a system of two masses $m_1$and $m_2$. Suppose, the mass $m_1$ is fixed at a point $A$ and the mass $m_2$ is taken from a point $B$ to infinity (∞) along the line $AB$.




- Consider a small displacement of the charges $m_2$ in which its distance from $m_1$ changes from $r$ to $r + dr$. The gravitational force on the mass $m_2$ is

$$F = G\frac{m_1m_2}{r^2} \quad \text{towards } \overrightarrow{BA}$$

- The work done by this force in the small displacement $dr$ is

$$dW = -G\frac{m_1m_2}{r^2}dr$$

(negative sign as force is opposite to displacement)

- The total work done as the mass $m_2$ moves from $B$ to $C$ is

$$W = -\int_\infty^r G\frac{m_1m_2}{r^2}dr = -Gm_1m_2\int_\infty^r \frac{1}{r^2}dr = -Gm_1m_2\left(-\frac{1}{r_1}\right)_\infty^r$$

$$\Rightarrow W = -Gm_1m_2\left(-\frac{1}{r} - \left(-\frac{1}{\infty}\right)\right)$$

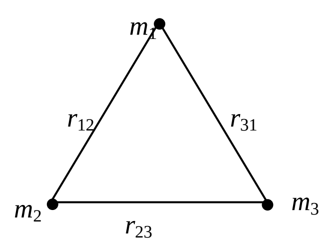
$$\Rightarrow W = \frac{Gm_1m_2}{r}$$

- Thus the potential energy is, therefore,

$$U = -W = -\frac{Gm_1m_2}{r} \qquad \ldots\text{(i)}$$

- We choose the potential energy of the two-mass system to be zero when they have infinite separation (that means when they are widely separated). This means $U(\infty) = 0$.

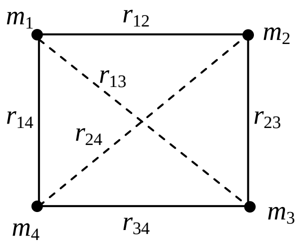
### Gravitational Potential Energy of three mass system




- $$U = -\left(\frac{Gm_1m_2}{r_{12}} + \frac{Gm_2m_3}{r_{23}} + \frac{Gm_3m_1}{r_{31}}\right)$$

(Derivation is discussed in class)

**Gravitational Potential Energy of four mass system**




$$\boxed{U=-\left(\frac{Gm_1m_2}{r_{12}}+\frac{Gm_2m_3}{r_{23}}+\frac{Gm_3m_1}{r_{31}}+\frac{Gm_1m_4}{r_{14}}+\frac{Gm_2m_4}{r_{24}}+\frac{Gm_3m_4}{r_{34}}\right)}$$

## Kepler's Laws

Kepler formulated the laws of planetary motion. The three laws are:

1. **All planets move in elliptical orbits with the sun at one of the focus.**
   This law is also known as **The Law of Orbits.**

2. **The radius vector from the sun to the planet sweeps equal areas in equal time.**
   This law is also known as **The Law of Areas**.

- We can say it in a different way as, ***the rate dA/dt at which a planet sweeps out an area A is constant.***
  dA/dt is also known as areal velocity meaning area swept out per unit time by the radius vector from the Sun to the planet. Thus, we can state this law as: **The areal velocity of planet is constant.**

3. **The square of the time period of a planet is proportional to the cube of the semi-major axis of the ellipse.**
   This law is also known as **The Law of Periods**.

- Let the time period of the planet be $T$. Also assume the length of the semi-major axis be $a$. Then according to Kepler's third law we have:

  $$T^2 \propto a^3$$

  For two planets we have,

  $$\boxed{\left(\frac{T_1}{T_2}\right)^2=\left(\frac{a_1}{a_2}\right)^3}$$

- Time period is also given by formula: $T^2=\left(\frac{4\pi}{GM}\right)a^3$
  where M = mass of the Sun which is constant.

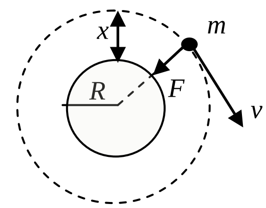
**Orbital Velocity and time period of a satellite**

a) **Orbital velocity:**

- *Orbital velocity of a satellite is the velocity required to put the satellite into its orbit around the earth.*
- Suppose that a satellite of mass m has to be put into circular orbit around the earth at a height $x$ above its surface. Consider that earth is a sphere of mass M and radius $R$. Then , the radius of the orbit of the satellite will be $R+x$. Suppose that $v$ is the required orbital velocity for the satellite.
- The gravitational force of attraction between the satellite and the earth will provide the necessary centripetal force to the satellite to move around the earth in the circular orbit i.e.

$$\frac{GMm}{\left(R+x^2\right)}=\frac{mv^2}{R+x}$$

$$\Rightarrow v=\sqrt{\frac{GM}{R+x}} \qquad \ldots\text{(i)}$$




- The acceleration due to gravity, $g$, on the surface of the earth is given by $g=\frac{GM}{R^2}$ so that $GM=gR^2$
  Putting the value of $GM$ in equation (i), we have
  $$v=\sqrt{\frac{gR^2}{R+x}} \qquad \ldots\text{(ii)}$$
- The equations (i) and (ii) are used to find the orbital velocity for a satellite orbiting at a height $x$ above the surface of earth.
- It may be pointed out that the orbit of the satellite may not be circular in shape. However , the orbits are assumed to be circular ones, only for the sake of simplicity.
- **Special case:** When the satellite is orbiting very close to the surface of the earth : In such a case, $x \approx 0$. From equation (i) and (ii) , we have
  $$v=\sqrt{\frac{GM}{R}}=\sqrt{gR} \qquad \ldots\text{(iii)}$$
  By substituting $g$ = 9.8 ms$^{-2}$ and $R$ =6.4×10$^6$m in equation (iii), it comes out that a satellite requires a velocity of about **7.92 km s$^{-1}$** to revolve in a orbit just near the surface of earth.

**(b) Time period of satellite:**

- The time period of a satellite is the time taken by it to go once around the earth. Therefore,
  $$T=\frac{\text{circumference of the orbit}}{\text{orbital velocity}}$$
  $$T=\frac{2\pi(R+x)}{v} \qquad \ldots\text{(iv)}$$
- Substituting the value of $v$ given by equation (i) in equation (iv), we have
  $$T=\frac{2\pi(R+x)}{\sqrt{\frac{GM}{R+x}}}$$

$$T = 2\pi\sqrt{\frac{(R+x)^3}{GM}} \quad \ldots(v)$$

- Setting $GM = gr^2$ in equation (v), we have

$$T = \frac{2\pi}{R}\sqrt{\frac{(R+x)^3}{g}} \quad \ldots(vi)$$

Equations (v), (vi) give the time period of a satellite revolving at a height x above the surface of the earth.

- **Special case:** When the satellite revolves very close to the surface of the earth: In such a case, $x \approx 0$.
  Setting $x = 0$, equations (v), (vi) gives

$$T = 2\pi\sqrt{\frac{R^3}{GM}} = 2\pi\sqrt{\frac{R}{g}} \quad \ldots(vii)$$

By substituting g = 9.8 m $s^{-2}$ and R = 6.4 × $10^6$ m in the equation (vii) , it comes out that the time period of a satellite revolving around the earth just close to its surface is about **84.6 minutes**.

### Quick exercise

Q. Find the total mechanical energy of an orbiting satellite.

Ans: $E = -\frac{GMm}{2(R+x)}$

**Explanation:**
When a satellite revolves around a planet in its orbit, it possesses both potential energy of the satellite in the orbit at a height $x$ above the surface of earth is given by

$$U = -\frac{GMm}{R+x} \quad \ldots(i)$$

Since the satellite revolves in its orbit around the planet under the effect of its gravitational pull, we have

$$\frac{mv^2}{R+x} = \frac{GMm}{(R+x)^2}$$

or $mv^2 = \frac{GMm}{R+x}$

Therefore, the kinetic energy of the satellite in its orbit,

$$K = \frac{1}{2}mv^2 = \frac{GMm}{2(R+x)} \quad \ldots(ii)$$

Hence, total energy of a satellite in its orbit,

$$ME = U + K = -\frac{GMm}{R+x} + \frac{GMm}{2(R+x)}$$

$$ME = -\frac{GMm}{2(R+x)} \quad \ldots(iii)$$

It follows that energy of an orbiting satellite is negative. For this reason, the planet and the orbiting satellite are said to from a bound system.

### Escape Velocity

- *Escape velocity is the initial speed required to go from an initial point in a gravitational potential field to infinity with a residual velocity of zero, with all speeds and velocities measured with respect to the field.*
- We can also define escape velocity as the minimum velocity an object must have in order to escape the gravitational field of the earth, that is, escape the earth without ever falling back.
- From the surface of the Earth, escape velocity (ignoring air friction) is about 11.2 km/s relative to Earth. Given that initial speed, an object needs no additional force applied to completely escape Earth's gravity.
- The simplest way of deriving the formula for escape velocity is to use conservation of energy.
- Suppose a particle of mass $m$ is at a distance $r$ from the center of mass of the planet , whose mass is $M$.
- Its initial speed is equal to its escape velocity, $v_e$.
- At its final state, it will be an infinite distance away from the planet, and its speed will be negligibly small and assumed to be 0.
- Kinetic energy $K$ and gravitational potential energy $U$ are the only types of energy that we will deal with, so by the conservation of mechanical energy for the planet-mass system, we have
  $K_1 + U_1 = K_2 + U_2$

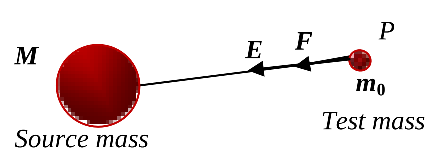
$$\Leftrightarrow \frac{1}{2}mv_e^2 - \frac{GMm}{r} = 0 + 0$$

($K_2$ = 0 because final velocity is zero, and $U_2$ = 0 because its final distance is infinity)

$$\Leftrightarrow v_e = \sqrt{\left(\frac{2GM}{r}\right)}$$

### Gravitational field Intensity

- Suppose we are interested to find gravitational field intensity due to a mass $M$ at a point $P$. This mass $M$ is known as **source mass $M$**.
- We need a **small mass $m_o$** at point P. This second kind of mass is very small compared to that of source mass $M$ and is known as **test mass $m_o$**.




- In the next step, we find gravitational force $\vec{F}$ acting on test mass $m_o$ because of source charge $M$. Gravitational field $\vec{E}$ because of source mass $M$ at point P is nothing but the gravitational force $\vec{F}$ per unit small positive test mass $m_o$ placed at that point P. Thus,

$$\boxed{\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{m_0} \qquad \text{(Gravitational field intensity)}}$$

- Clearly, the direction of $\vec{E}$ is along $\vec{F}$.

- **Definition of Gravitational Field Intensity**

  The gravitational field intensity or simply electric field $\vec{E}$ due to a source mass $M$ at a point is defined as the gravitational force $\vec{F}$ per unit small mass charge $m_o$ acting on $m_o$ due to $M$ placed at that point.

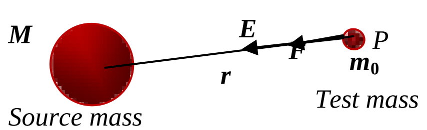
  $\vec{E} = \lim_{m_0 \to 0} \frac{\vec{F}}{m_0}$ (Gravitational field intensity)

- **Important points about electric field**

1. In the LHS of the expression $\vec{E} = \lim_{q_0 \to 0} \frac{\vec{F}}{m_0}$, $\vec{E}$ is the gravitational field due to source mass $M$ but in the RHS, test mass $m_o$ appears. Be careful of this.
2. $\vec{F}$ is the gravitational force acting on the test mass $m_o$ because of source mass $M$.
3. Gravitational field intensity is a **vector quantity**.
4. The direction of $\vec{E}$ is along $\vec{F}$.
5. **SI unit of gravitational field**: **N/kg** = newton per kg

### Gravitational Field due to a Point Mass

- We wish to find out the gravitational field intensity $E$ due to a point mass $M$ at a point $P$ which is $r$ distance from it. For this, we have to place another small test mass $m_o$ at point P.




- Now magnitude of gravitational force acting on a particle of mass m placed at point $P$ is

$$F = \frac{GMm_0}{r^2}$$

- Therefore, magnitude of Gravitational field at that point is

$$E = \frac{F}{m_0}$$

$$\Rightarrow \boxed{E = \frac{GM}{r^2}}$$

(Gravitational field due to a point mass $M$ at a distance $r$)

- The direction of $E$ is along $F$.

### Force on a point mass placed in a Gravitational Field

- Suppose a particle of mass $m$ is placed in a gravitational field at a point where intensity of gravitational field is $\vec{E}$. Then particle experiences a force $\vec{F}$ in the direction of $\vec{E}$, which is given by

$$\vec{F} = m\vec{E}$$

### Gravitational field due to a Uniform Solid Sphere

- **Field at an external point at distance r from centre:**
  *A* uniform spherical body acts as a point mass at its centre when another particle is outside it. Therefore,

$$E = \frac{GM}{r^2}$$

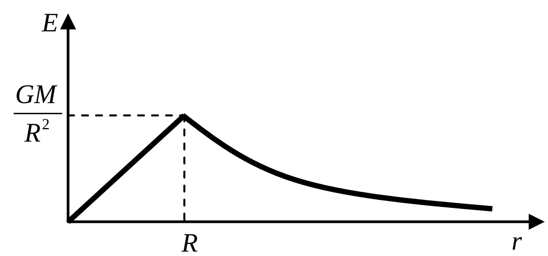
- **Field at an internal point at a distance r from centre:**

  Use the concepts from topic "Variation of acceleration due to gravity with depth" and find the force, subsequently field. You will get,

$$E = \frac{GMr}{R^3}$$

  Here $R$ is the radius of the sphere.

- Here, we observe that gravitational field magnitude is maximum at surface.
- Graphically, we can plot variation of field with $r$ (distance from the centre of the sphere) as:



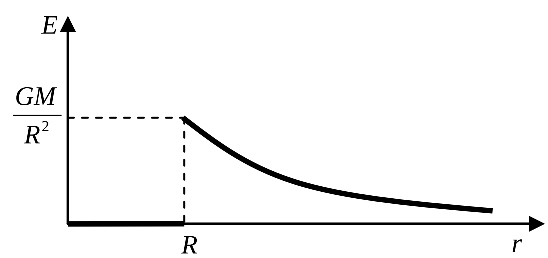

### Gravitational field due to a Uniform Hollow Sphere

- **Field at an external point at distance r from centre:**
  *A* hollow sphere acts as a point mass at its centre when another particle is outside it. Therefore,

$$E = \frac{GM}{r^2}$$

- **Field at an internal point at a distance r from centre:**
  We know that a uniform shell of matter exerts no net gravitational force on a particle located inside it. Thus, electric field inside a uniform hollow sphere is zero.

$$E = 0$$



### Gravitation Potential

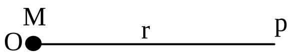
The work done in bringing a unit mass from infinity to a point in the gravitational field is called the gravitational potential at that point.

$$V = \frac{W_{ext}}{m_0} = -\frac{W_g}{m_0} = \frac{U}{m_0}$$

**(i) Potential due to a point Mass**

Suppose a point mass M is situated at a point O. We want to find the gravitational potential due to this mass at a point P a distance r from O. For this let us find work done in taking the unit mass from P to infinity. This will be,

$$W = \int_r^\infty F dr = \int_r^\infty \frac{GM}{r^2}.dr = \frac{GM}{r}$$




Hence, the work done in bringing units from infinity to P will be $-\frac{GM}{r}$. Thus , the gravitational potential at P will be,

$$V = -\frac{GM}{r}$$

**(ii) Potential due to a Uniform Solid Sphere**

Potential at an External point

The gravitational potential due to a uniform sphere at an external point is same as that due to a single particle of same mass placed at its centre. Thus,

$$V(r) = -\frac{GM}{r} \quad r \geq R$$

*At* the surface, $r = R$ and $V = -\frac{GM}{R}$

**Potential at Internal Point**

At some internal point, potential at a distance r from the centre is given by,

$$V(r) = -\frac{GM}{R^3}\left(\frac{3}{2}R^2 - \frac{1}{2}r^2\right) \quad r \leq R$$

At $r = R,\ V = -\frac{GM}{R}$

while at $r = 0,\ V = -\frac{1.5GM}{R}$

i.e., at the centre of the sphere the potential is 1.5 times the potential at surface.

**(iii) Potential due to a Uniform Thin Spherical Shell**

Potential at an External point

To calculate the potential at an external point , a uniform spherical shell may be treated as a point mass of same magnitude at its centre. Thus, potential at a distance r is given by,

$$V(r) = -\frac{GM}{r}, \quad r \geq R$$

$$V = -\frac{GM}{R}, \quad \text{at } r = R,$$

**Potential at an Internal point**

The potential due to a uniform spherical shell is constant throughout at any point inside the shell and this is equal to $-\frac{GM}{R}$.

### Relation between gravitation field and potential

Gravitational potential is a field function . It depends on the position of the point where potential is desired. Gravitation potential are related by the following relation.

$$\vec{E} = -\left[\frac{\partial V}{\partial x}\hat{i} + \frac{\partial V}{\partial y}\hat{j} + \frac{\partial V}{\partial z}\hat{k}\right]$$

Here, $\frac{\partial V}{\partial x}$ = partial derivative of potential function $V$ w.r.t. $x$, i.e., differentiate $V$ w.r.t. $x$ assuming $y$ and $z$ to be constant

Eq.(i) can be written in following different forms.

i. $E = -\frac{dv}{dx}$, if gravitational field is along x-direction only

ii. $dV = -\vec{E}.d\vec{r}$,

Here,

$$d\vec{r} = dx\hat{i} + dy\hat{j} + dz\hat{k}$$

$$and \quad \vec{E} = E_x\hat{i} + E_Y\hat{j} + E_z\hat{k}$$

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## Physics Classes by Pranjal Sir

## (Admission Notice for XI & XII - 2014-15)

### Batches for Std XIIth

**Batch 1** (Board + JEE Main + Advanced): (Rs. 16000)
**Batch 2** (Board + JEE Main): (Rs. 13000)
**Batch 3** (Board): (Rs. 10000)
**Batch 4** (Doubt Clearing batch): Rs. 8000

### About P. K. Bharti Sir (Pranjal Sir)

- **B. Tech., IIT Kharagpur (2009 Batch)**
- H.O.D. Physics, Concept Bokaro Centre
- Visiting faculty at D. P. S. Bokaro
- Produced AIR 113, AIR 475, AIR 1013 in JEE - Advanced
- Produced AIR 07 in AIEEE (JEE Main)

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## Physics Class Schedule for Std XIIth (Session 2014-15) by Pranjal Sir

| Sl. No. | Main Chapter | Topics | Board level | JEE Main Level | JEE Adv Level |
|---|---|---|---|---|---|
| **Basics from XIth** | | Vectors, FBD, Work, Energy, Rotation, SHM | $3^{rd}$ Mar to $4^{th}$ Apr 14 | | |
| 1. | Electric Charges and Fields | Coulomb's Law | $5^{th}$ & $6^{th}$ Apr | $5^{th}$ & $6^{th}$ Apr | $5^{th}$ & $6^{th}$ Apr |
| | | Electric Field | $10^{th}$ & $12^{th}$ Apr | $10^{th}$ & $12^{th}$ Apr | $10^{th}$ & $12^{th}$ Apr |
| | | Gauss's Law | $13^{th}$ & $15^{th}$ Apr | $13^{th}$ & $15^{th}$ Apr | $13^{th}$ & $15^{th}$ Apr |
| | | **Competition Level** | NA | $17^{th}$ & $19^{th}$ Apr | $17^{th}$ & $19^{th}$ Apr |
| 2. | Electrostatic Potential and Capacitance | Electric Potential | $20^{th}$ & $22^{nd}$ Apr | $20^{th}$ & $22^{nd}$ Apr | $20^{th}$ & $22^{nd}$ Apr |
| | | Capacitors | $24^{th}$ & $26^{th}$ Apr | $24^{th}$ & $26^{th}$ Apr | $24^{th}$ & $26^{th}$ Apr |
| | | **Competition Level** | NA | $27^{th}$ & $29^{th}$ Apr | $27^{th}$ & $29^{th}$ Apr, $1^{st}$, $3^{rd}$ & $4^{th}$ May |
| **PART TEST 1** | | **Unit 1 & 2** | **$4^{th}$ May** | **NA** | **NA** |
| | | | **NA** | **$11^{th}$ May** | **$11^{th}$ May** |
| 3. | Current Electricity | Basic Concepts, Drift speed, Ohm's Law, Cells, Kirchhoff's Laws, Wheatstone bridge, Ammeter, Voltmeter, Meter Bridge, Potentiometer etc. | $6^{th}$, $8^{th}$, $10^{th}$, $13^{th}$ May | $6^{th}$, $8^{th}$, $10^{th}$, $13^{th}$ May | $6^{th}$, $8^{th}$, $10^{th}$, $13^{th}$ May |
| | | **Competition Level** | NA | $15^{th}$ & $16^{th}$ May | $15^{th}$, $16^{th}$, $17^{th}$, $18^{th}$ & $19^{th}$ May |
| **PART TEST 2** | | **Unit 3** | **$18^{th}$ May** | **NA** | **NA** |
| | | | **NA** | **$20^{th}$ May** | **$20^{th}$ May** |
| **SUMMER BREAK** | | **$21^{st}$ May 2013 to $30^{th}$ May 2013** | | | |
| 4. | Moving charges and Magnetism | Force on a charged particle (Lorentz force), Force on a current carrying wire, Cyclotron, Torque on a current carrying loop in magnetic field, magnetic moment | $31^{st}$ May, $1^{st}$ & $3^{rd}$ Jun | $31^{st}$ May, $1^{st}$ & $3^{rd}$ Jun | $31^{st}$ May, $1^{st}$ & $3^{rd}$ Jun |
| | | Biot Savart Law, Magnetic field due to a circular wire, Ampere circuital law, Solenoid, Toroid | $5^{th}$, $7^{th}$ & $8^{th}$ Jun | $5^{th}$, $7^{th}$ & $8^{th}$ Jun | $5^{th}$, $7^{th}$ & $8^{th}$ Jun |
| | | **Competition Level** | NA | $10^{th}$ & $12^{th}$ Jun | $10^{th}$, $12^{th}$, $14^{th}$ & $15^{th}$ Jun |
| **PART TEST 3** | | **Unit 4** | **$15^{th}$ Jun** | **NA** | **NA** |
| | | | **NA** | **$22^{nd}$ Jun** | **$22^{nd}$ Jun** |

| | | | | | |
|---|---|---|---|---|---|
| 5. | Magnetism and Matter | | 17th, 19th & 21st Jun | 17th, 19th & 21st Jun | Not in JEE Advanced Syllabus |
| 6. | Electromagnetic Induction | Faraday's Laws, Lenz's Laws, A.C. Generator, Motional Emf, Induced Emf, Eddy Currents, Self Induction, Mutual Induction | 24th, 26th & 28th Jun | 24th, 26th & 28th Jun | 24th, 26th & 28th Jun |
| | | **Competition Level** | NA | 29th Jun & 1st Jul | 29th Jun, 1st, 3rd & 5th Jul |
| **PART TEST 4** | | **Unit 5 & 6** | **6th Jul** | **NA** | **NA** |
| | | | **NA** | **13th Jul** | **13th Jul** |
| 7. | Alternating current | AC, AC circuit, Phasor, transformer, resonance, | 8th, 10th & 12th Jul | 8th, 10th & 12th Jul | 8th, 10th & 12th Jul |
| | | **Competition Level** | NA | 15th July | 15th & 17th July |
| 8. | Electromagnetic Waves | | 19th & 20th July | 19th & 20th July | Not in JEE Advanced Syllabus |
| **PART TEST 5** | | **Unit 7 & 8** | **27th Jul** | **27th Jul** | **27th Jul** |
| **Revision Week** | | **Upto unit 8** | 31st Jul & 2nd Aug | 31st Jul & 2nd Aug | 31st Jul & 2nd Aug |
| **Grand Test 1** | | **Upto Unit 8** | **3rd Aug** | **3rd Aug** | **3rd Aug** |
| 9. | Ray Optics | Reflection | 5th & 7th Aug | 5th & 7th Aug | 5th & 7th Aug |
| | | Refraction | 9th & 12th Aug | 9th & 12th Aug | 9th & 12th Aug |
| | | Prism | 14th Aug | 14th Aug | 14th Aug |
| | | Optical Instruments | 16th Aug | 16th Aug | Not in JEE Adv Syllabus |
| | | **Competition Level** | NA | 19th & 21st Aug | 19th, 21st, 23rd, 24th Aug |
| 10. | Wave Optics | Huygens Principle | 26th Aug | 26th Aug | 26th Aug |
| | | Interference | 28th & 30th Aug | 28th & 30th Aug | 28th & 30th Aug |
| | | Diffraction | 31st Aug | 31st Aug | 31st Aug |
| | | Polarization | 2nd Sep | 2nd Sep | 2nd Sep |
| | | **Competition Level** | NA | 4th & 6th Sep | 4th, 6th, 7th, 9th, 11th Sep |
| **PART TEST 6** | | **Unit 9 & 10** | **14th Sep** | **14th Sep** | **14th Sep** |
| **REVISION ROUND 1 (For JEE Main & JEE Advanced Level): 13th Sep to 27th Sep** | | | | | |
| **Grand Test 2** | | **Upto Unit 10** | **28th Sep** | **28th Sep** | **28th Sep** |
| **DUSSEHRA & d-ul-Zuha Holidays: 29th Sep to 8th Oct** | | | | | |
| 11. | Dual Nature of Radiation and Matter | Photoelectric effect etc | 9th & 11th Oct | 9th & 11th Oct | 9th & 11th Oct |
| **Grand Test 3** | | **Upto Unit 10** | **12th Oct** | **12th Oct** | **12th Oct** |
| 12. | Atoms | | 14th & 16th Oct | 14th & 16th Oct | 14th & 16th Oct |
| 13. | Nuclei | | 18th & 19th Oct | 18th & 19th Oct | 18th & 19th Oct |
| X-Rays | | | NA | 21st Oct | 21st & 25th Oct |
| **PART TEST 7** | | **Unit 11, 12 & 13** | 26th Oct | NA | NA |
| 14. | Semiconductors | Basic Concepts and Diodes, transistors, logic gates | 26th, 28th, 30th Oct & 1st Nov | 26th, 28th, 30th Oct & 1st Nov | Not in JEE Adv Syllabus |
| 15. | Communication System | | 2nd & 4th Nov | 2nd & 4th Nov | Not in JEE Adv Syllabus |
| **PART TEST 8** | | **Unit 14 & 15** | 9th Nov | 9th Nov | NA |
| Unit 11, 12 & 13 | | **Competition Level** | NA | 8th, 9th & 11th Nov | 8th, 9th, 11th, 13th & 15th Nov |
| **PART TEST 9** | | **Unit 11, 12, 13, X-Rays** | **NA** | **16th Nov** | **16th Nov** |
| **Revision Round 2 (Board Level)** | | **Mind Maps & Back up classes for late registered students** | **18th Nov to Board Exams** | **18th Nov to Board Exams** | **18th Nov to Board Exams** |
| **Revision Round 3 (XIth portion for JEE)** | | | **18th Nov to JEE** | **18th Nov to JEE** | **18th Nov to JEE** |
| | **30 Full Test Series** | **Complete Syllabus** | **Date will be published after Oct 2014** | | |