

# Department of Pure Mathematics

# Course Structure for M.Sc. Semester-wise distribution of Courses (4 semester in total)

Semester	Course ID	Group	Name of the Courses	Full	Credit	Classes
			Page Number	Marks	Point	per week
I	PM1/01		Algebra-I 4	50	4	6 hr
	PM1/02	GrA	Real Analysis-I 5	25	4	6 hr
		GrB	Measure Theory-I 6	25		
	PM1/03	GrA	Complex Analysis-I 7	25	4	6 hr
		GrB	Differential Geometry 8	25		
	PM1/04	GrA	General Topology-I 9	30	4	6 hr
		GrB	Ordinary Differential Equation-I 10	20		
			Total	200	16	24 hr
	PM2/05	GrA	Linear Algebra 11	25	4	5 hr
	,	GrB	Complex Analysis-II 12	25		
	PM2/06	GrA	Real Analysis-II 13	25	4	5 hr
	,	GrB	Measure Theory-II 14	25		
II	PM2/07		General Topology-II 15	50	4	5 hr
	PM2/08	GrA	Functional Analysis-I 17	30	4	5 hr
		GrB	Discrete Mathematics-I 18	20		
	PM2/09	GrA	Multivariate Calculus-I 19	20	4	5 hr
	,	GrB	Theory of Manifolds 20	30		
			Total	250	20	25 hr
	PM3/10	GrA	Algebra-II 21	25	4	5 hr
	,	GrB	Partial Differential Equation 22	25		
	PM3/11	GrA	Ordinary Differential Equation-II 23	30	4	5 hr
III	,	GrB	Lie Groups and Lie Algebras 24	20		
	PM3/12	GrA	Algebraic Topology-I 25	30	4	5 hr
		GrB	Functional Analysis-II 26	20		
	PM3/E1/101-110		Elective-I 2	50	4	4 hr
	PM3/E2/201-210		Elective-II 3	50	4	4 hr
			Total	250	20	23 hr
		GrA	Algebraic Topology-II 27	20		
	PM4/13	GrB	Calculus of Variations and 28	30	4	5 hr
			Integral Equations			
	PM4/14/Th		Computational Mathematics(Theory) 29	30	2.4	3 hr
	PM4/14/Pr		Computational Mathematics(Practical)30	20	1.6	2 hr
	, ,	GrA	Discrete Mathematics-II 31	25		
IV			(OP1)* Mathematical Logic 32	25		
			(OP2)* Number Theory 33	25		
	PM4/15	GrB	(OP3)* Algebraic Geometry 34	25	4	5 hr
			(OP4)* Multivariate Calculus-II 35	25		
			(OP5)* Automata Theory 36	25		
	PM4/E1/101-110		Elective-I 2	50	4	4 hr
	PM4/E2/201-210		Elective-II 3	50	4	4 hr
	PM4/16		Dissertation, Internal Assessment, Seminar & Grand Viva	50	4	
			Total	300	24	23 hr
			Grand Total	1000	80	

<sup>\*</sup>N.B.: For the Course PM4/15 Gr.-B, a student has to opt (subject to availability) for  $\underline{any\ one}$  of the subjects from (OP1), (OP2), (OP3), (OP4) and (OP5).

# Details of Code for Elective - I Courses\*\*

Index Elective I Elective II

Serial	Course	Subject	Name of the Courses in Elective - I	Full
No.	ID	$\mathbf{Code}$	Page Number	Marks
1	PM3/E1	101	Abstract Harmonic Analysis -I 37	50
	PM4/E1		Abstract Harmonic Analysis -II 38	50
2	PM3/E1	102	Algebraic Aspects of Cryptology -I (Theory & Practical) 39	50 (40+10)
	PM4/E1		Algebraic Aspects of Cryptology -II (Theory & Practical)41	50 (40+10)
3	PM3/E1	103	Advanced Real Analysis -I 43	50
	PM4/E1		Advanced Real Analysis -II 44	50
4	PM3/E1	104	Rings of Continuous functions -I 45	50
	PM4/E1		Rings of Continuous functions -II 46	50
5	PM3/E1	105	Structures on Manifolds -I 47	50
	PM4/E1		Structures on Manifolds -II 48	50
6	PM3/E1	106	Advanced Algebraic Topology -I 49	50
	PM4/E1		Advanced Algebraic Topology -II 50	50
7	PM3/E1	107	Universal Algebra, Category theory & Lattice theory -I 51	50
	PM4/E1		Universal Algebra, Category theory & Lattice theory -II 52	50
8	PM3/E1	108	Advanced Graph Theory -I 53	50
	PM4/E1		Advanced Graph Theory -II 54	50
9	PM3/E1	109	Theory of Linear Operators -I 55	50
	PM4/E1		Theory of Linear Operators -II 56	50
10	PM3/E1	110	Banach Algebra -I 57	50
	PM4/E1		Banach Algebra -II 58	50

\*\*N.B. : A student has to opt (subject to availability) for  $\underline{any\ one}$  of the subjects from above list.

# Details of Code for Elective - II Courses\*\*\*

Index Elective I Elective II

Serial	Course	Subject	Name of the Courses in Elective -II	Full
No.	ID	$\mathbf{Code}$	Page Number	Marks
1	PM3/E2	201	Module theory and non-commutative rings -I 59	50
	PM4/E2		Module theory and non-commutative rings -II 60	50
2	PM3/E2	202	Advanced Functional Analysis -I 61	50
	PM4/E2		Advanced Functional Analysis -II 62	50
3	PM3/E2	203	Advanced Riemannian Manifold -I 63	50
	PM4/E2		Advanced Riemannian Manifold -II 64	50
4	PM3/E2	204	Advanced Number Theory -I 65	50
	PM4/E2		Advanced Number Theory -II 66	50
5	PM3/E2	205	Dynamical System and Integral Equations -I 67	50
	PM4/E2		Dynamical System and Integral Equations -II 68	50
6	PM3/E2	206	Advanced General Topology -I 69	50
	PM4/E2		Advanced General Topology -II 70	50
7	PM3/E2	207	Advanced Complex Analysis -I 71	50
	PM4/E2		Advanced Complex Analysis -II 72	50
8	PM3/E2	208	Algebraic Coding Theory -I 73	50
	PM4/E2		Algebraic Coding Theory -II 74	50
9	PM3/E2	209	Advanced Measure theory & Integration -I 75	50
	PM4/E2		Advanced Measure theory & Integration -II 76	50
10	PM3/E2	210	Differential Topology -I 77	50
	PM4/E2		Differential Topology -II 78	50

\*\*\*N.B. : A student has to opt (subject to availability) for  $\underline{any\ one}$  of the subjects from above list.

# Algebra-I

Semester: I
Course ID: PM1/01 Full Marks: 50

Minimum number of classes required: 60

Index Elective I Elective II

- Group Theory: External direct product and internal direct product of groups. Direct product of cyclic groups. Group actions, extended Cayley's theorem, Burnside theorem. Conjugacy classes, class-equation. Cauchy's theorem on finite groups, p-group, Centre of p-groups. Sylow's theorems, some applications of Sylow's theorems, Simple groups, characterizations of commutative simple groups, non simplicity of groups of order  $p^n(n > 1)$ , pq,  $p^2q$ ,  $p^2q^2(p, q)$  are primes), determination of all simple groups of order  $\leq 60$ , nonsimplicity of  $A_n$  ( $n \geq 5$ ). Finite groups, classification of all groups of order 6, the groups  $D_4$  and  $Q_8$ , classification of all groups of order 8. Structure theorem for finite Abelian groups. Normal and subnormal series, composition series, Jordan Holder theorem, solvable groups and nilpotent groups.
- Ring Theory: Ideal, Quotient ring, Ring embeddings, direct sum of rings, Euclidean domain, principal ideal domain, prime elements and irreducible elements, maximal ideals, prime ideals, primary ideals, chain condition, polynomial ring and factorization of polynomials over a commutative ring with identity, polynomials over rational field, the division algorithm in K[x] where K is a field, K[x] as Euclidean domain, unique factorization domain, If D is UFD, then so are D[x] and  $D[x_1, x_2, \ldots, x_n]$ , Application of unique factorization domain to the introductory algebraic number theory, Eisenstein's criterion of irreducibility, Noetherian and Artinian rings, Hilbert Basis Theorem. Algebraic approach to Fermat's Theorem, Euler's Theorem, Willson's Theorem, Chinese Remainder Theorem, primitive roots.

### References

- [1] Malik, Mordeson and Sen; Fundamentals of Abstract Algebra; McGraw-Hill, 1997.
- [2] T. H. Hungerford; Algebra; Holt, Reinhart and Winston. 1974.
- [3] I. N. Herstein; Topics in Algebra; Wiley Eastern Ltd. New Delhi, 1975.
- [4] Joseph J. Rotman; An introduction to the theory of groups; Springer-Verlag, 1990.
- [5] S. Lang; Algebra (2nd ed.); Addition-Wesley.
- [6] D. S. Dummit, R. M. Foote; Abstract Algebra, 2nd edition; Wiley Student edition.
- [7] Michael Artin; Algebra; PHI. (Eastern Economy Edition) Prentice Hall.
- [8] Saban Alaca, Kenneth S. Williams; Introduction to Algebraic Number Theory; Cambridge University Press.

# Real Analysis – I

Semester: I Group: A
Course ID: PM1/02 Full Marks: 25

Minimum number of classes required: 30

Index Elective I Elective II

- Cardinal Number: Concept of Cardinal number of an infinite set, arithmetic of Cardinal numbers, order relation of Cardinal numbers, Schröder-Bernstein theorem, the set 2<sup>A</sup>, Axiom of choice, Cardinality of Cantor set, continuum hypothesis.
- Continuity of a function in  $\mathbb{R}$ : Weierstrass Approximation theorem, Equicontinuity, Absolute Continuity and bounded variation, Luzin (N) property of an absolutely continuous function, semi-continuity.
- Derivatives: The Vitali-covering theorem and its applications to the derivative of an absolutely continuous function, Dini's derivates and its simple properties, everywhere continuous but nowhere differentiable functions, continuity and differentiability of convex function.
- Double Sequences and Series : Double sequences, Double series, Stolz's theorem, Double series of positive terms, Absolute convergence of double series.

#### References

- [1] T. M. Apostol: Mathematical Analysis; Addison-Wesley Publishing Co. 1957.
- [2] A. Bruckner, J. Bruckner & B. Thomson: Real Analysis; Prentice Hall, 1997.
- [3] T. J. I' A. Bromwitch: Infinite Series; MacMillan, London, 1949.
- [4] C. Goffman: Real Functions; Holt, Rinehart and Winston, N.Y, 1953.
- [5] J. F. Randolph: Basic Real and Abstract Analysis; Academic Press, N.Y, 1968.
- [6] W. Ruddin: Principles of Mathematical Analysis; McGraw-Hill, N.Y, 1964.
- [7] E. Hewitt and K. Stromberg: Real and Abstract Analysis; John Wiley, N.Y., 1965.

### Measure Theory - I

Semester: I Group: B
Course ID: PM1/02 Full Marks: 25

Minimum number of classes required: 30

Index | Elective I | Elective II

- Lebesgue outer measure, Lebesgue measurable sets, Borel sets, Approximations of Lebesgue measurable sets by topologically nice sets, Non measurable sets, Cantor sets.
- Abstract measurable spaces, extended real valued measurable functions on these spaces, their algebraic and lattice structures, limits of sequences of measurable functions, simple functions, measurable functions as point-wise limits of sequences of simple functions, Approximation of Lebesgue measurable functions by continuous functions, Luzin's theorems.
- Almost uniform convergence, convergence in measure, Egoroff's theorem, Riesz's theorem interrelating convergence in measure and point-wise convergence.
- Derived numbers of a function at a point, Dini derivatives, Growth theorems concerning derived numbers, Vitali covering theorem. Functions of bounded variation admitting of finite derivatives almost everywhere on their domain, Lebesgue-Young theorem.

### References

- [1] G. De. Barra; Measure Theory & Integration; Wiley Eastern Limited, 1987.
- [2] Charles Schwartz; Measure, Integration & Function Spaces; World Scientific, 1994.
- [3] Inder Kumar Rana; An Introduction to measure & Integration; Narosa Publishing House, 1997.
- [4] P. R. Halmos; Measure Theory; D. Van Nostrand Co. inc. London, 1962.
- [5] P. K. Jain & V. P. Gupta; Lebesgue Measure & Integration; New Age International(P)limited Publishing Co, New Delhi, 1986.
- [6] H. L. Royden; Real Analysis; Macmillan Pub.Co.inc 4th Edition, New York, 1993.
- [7] Walter Rudin; Real and Complex Analysis; Tata McGraw Hill Publishing Co limited, New Delhi, 1966.
- [8] Bruckner, Bruckner and Thomson; Real Analysis; Prentice Hall International, Inc, 1997.

# Complex Analysis - I

Semester: I Group: A
Course ID: PM1/03 Full Marks: 25

Minimum number of classes required: 30

Index | Elective I | Elective II |

- Analytic functions: Definition of analytic function, functions represented by power series as natural examples of analytic functions, the functions  $\sin z$ ,  $\cos z$ ,  $\exp z$  and  $\log z$ , branches of logarithmic functions; Möbius transformations as special example of analytic functions and some properties of Möbius transformation.
- Complex Integration: Line integral of complex functions and its basic properties, weaker form of Cauchy's integral formula, power series representation of analytic function, zeros of an analytic function, maximum modulus theorem and its applications, Liouville's theorem, fundamental theorem of algebra, winding number of a closed rectifiable curve about points in  $\mathbb{C}$ , Cauchy's integral formula, Morera's theorem, Schwarz lemma, Phragmen-Lindelöf theorem, open mapping theorem, interior uniqueness theorem, Cauchy-Goursat theorem.

### References

- [1] J. B. Conway; Functions of One Complex Variable; Narosa Publishing; New Delhi, 1973.
- [2] R. P. Agarwal, K. Perera and S. Pinelas; An Introduction To Complex Analysis; Springer-Verlag.
- [3] S. Lang; Complex Analysis, Fourth edition; Springer-Verlag, 1999.
- [4] L. V. Ahlfors; Complex Analysis: an introduction to the theory of analytic functions of one complex variable; McGraw-Hill; New York, 1966.
- [5] A. I. Markushivich; Theory of Functions of Complex Variables, Vol-I, II; Prentice-Hall, 1965.
- [6] S. Ponnusamy; Foundations of Complex Analysis; Narosa Publishing; New Delhi, 1973.
- [7] R. V. Churchill and J. W. Brown; Complex Variables and Applications; McGraw-Hill; New York, 1996.

# Differential Geometry

Semester: I Group: B
Course ID: PM1/03 Full Marks: 25

Minimum number of classes required: 30

Index | Elective I | Elective II

• Tensors: Different transformation laws, Properties of tensors, Metric tensor, Riemannian space, Covariant Differentiation, Einstein space. Curves in Space: Intrinsic Differentiation, Parallel Vector Fields, Serret-Frenet formulii.

- Surface: First fundamental form, Angle between two intersecting curves on a surface, Geodesic, Geodesic curvature, Gaussian Curvature, Developable Surface.
- Surface in Space: Tangent and Normal Vector on a Surface, Second Fundamental Form, Gauss's Formula, Weingarten Formula, Third Fundamental Form, Gauss and Codazzi Equations, Principal Curvature, Lines of Curvature, Asymptotic lines.

# References

- [1] I. S. Sokolnikoff: Tensor Analysis, Theory and Applications to Geometry and Mechanics of Continua, 2nd Edition, John Wiley and Sons., 1964.
- [2] B. Spain: Tensor Calculus, John Wiley and Sons, 1960.
- [3] M. Spivak: A Comprehensive Introduction to Differential Geometry, Vols I-V, Publish or Perish, Inc. Boston, 1979.

# General Topology - I

Semester: I Group: A
Course ID: PM1/04 Full Marks: 30

Minimum number of classes required: 35

Index Elective I Elective II

- Definition and examples of topological spaces, closed sets, closure, dense subsets, Neighbourhood, interior, exterior and boundary, accumulation point, derived set. Bases and subbases, Subspace topology, finite product of topological spaces, alternative methods for defining a topology in terms of Kuratowski closure operator and neighbourhood system.
- Open, closed and continuous functions and homeomorphism, Topological invariants, Isometry and metric invariants.
- Countability Axioms: First and second countability, separability and Lindelöf property.
- Separation Axioms:  $T_i$ -property ( $i = 0, 1, 2, 3, 3\frac{1}{2}, 4, 5$ ), regularity, complete regularity, normality and complete normality; their characterizations and basic properties. Urysohn's lemma, Tietze's extension theorem,  $T_5$ -property of a metric space.

#### References

- [1] N. Bourbaki; General Topology Part-I (Transl.); Addison Wesley, Reading(1966).
- [2] J. Dugundji; Topology; Allyn and Bacon, Boston, 1966(Reprinted in India by Prentice Hall of India Pvt. Ltd.).
- [3] R. Engelking; General Topology; Polish Scientific Publishers, Warsaw (1977).
- [4] J. G. Hocking and C. S. Young; Topology; Addison-Wesley, Reading (1961).
- [5] S. T. Hu; Elements of General Topology; Holden-Day, San Francisco (1964).
- [6] K. D. Joshi; Introduction to Topology; Wiley Eastern Ltd. (1983).
- [7] J. L. Kelley; General Topology; Van Nostrand, Princeton (1955).
- [8] M. J. Mansfield; Introduction to Topology; D-van Nostrand Co. Inc, Princeton N.Y. (1963).
- [9] B. Mendelson; Introduction to Topology; Allyn and Becon Inc, Boston (1962).
- [10] James R. Munkress; Topology (2nd edit.); Pearson Education (2004).
- [11] W. J. Pervin; Foundations of General Topology; Academic Press, N.Y. (1964).
- [12] George F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill, N.Y.(1963).
- [13] L. Steen and J. Seebach; Counterexamples in Topology; Holt, Rinechart and Winston, N.Y. (1970).
- [14] W. J. Thron; Topological Structures; Holt, Rinehart and Winston, N.Y. (1966).
- [15] Stephen Willard; General Topology; Addison-Wesley, Reading (1970).

# Ordinary Differential Equation - I

Semester: I Group: B
Course ID: PM1/04 Full Marks: 20

Minimum number of classes required: 25

Index | Elective I | Elective II

• Existence and Uniqueness of Solutions: Introduction, Lipschitz condition and Gronwall's inequality, Successive approximations and Picard's theorem, Dependence of solutions on the initial conditions, Dependence of solutions on the functions, Continuation of the solutions, Non local existence of solutions.

• The Theory of Linear Differential Equations: Basic theory of the nth order Homogeneous Linear Differential Equation, Wronskian and its properties, a formula for the Wronskians, The *n*-th order non-homogeneous linear equation, Sturm theory.

# References

- [1] S. L. Ross; Introduction to ordinary differential equations; John-Wiley, New York, 1989.
- [2] G. F. Simmons; Differential equations with applications and historical notes; Tata McGraw Hill, New Delhi, 1976.
- [3] W. E. Boyce & R. C. Diprima; Elementary differential equations and boundary value problems; John Wiley & Sons, New York, 1977.
- [4] P. Hartman; Ordinary Differential Equations; John Wiley and sons, New York, 1964.

# Linear Algebra

Semester: II Group: A
Course ID: PM2/05 Full Marks: 25

Minimum number of classes required: 35

Index Elective I Elective II

- Vector space of infinite dimension. Existence of bases. Extension of a linearly independent subset to a basis. Reduction of a generating subset to a basis. Characterizations of basis as a maximal linearly independent subset and as a minimal generating subset. Equality of cardinalities of any two bases.
- Duality and transposition. Linear forms or linear functionals. Dual space  $V^d$ , Bidual space  $V^{dd}$ , Dual basis. Natural isomorphism between  $V^d$  and  $V^{dd}$ . Annihilaters  $W^{\square}$  of a nonempty subset W of a vector space V. dim  $W^{\square} = \dim V \dim W$ .  $(W^{\square})^{\square} = W$ . Transpose  $T^t$  of a linear transformation T.  $T^{tt} = T$ .  $(T^t)^{-1} = (T^{-1})^t$  if T is an isomorphism.  $(ImT)^{\square} = ImT^t$ ; dim  $ImT = \dim ImT^t$ ; dim  $\ker T = \dim \ker T^t$ .
- Eigen values and eigen vectors. Characteristic polynomial of a linear transformation. Eigen values and eigen vectors of a linear transformation. Diagonalisation, canonical forms. Characteristic values. Annihilating polynomials. Invariant subspace. Simultaneous trangulisation and diagonalization. Direct seem decomposition. Invariant direct seem. The primary decomposition theorem. Rational canonical forms. Cyclic subspaces of annihilators. Cyclic decompositions. The Rational form. The Jordan canonical form.

### References

- [1] Friedberg, Insel and Spence; Linear Algebra; PHI.
- [2] S. Kumaresan; Linear Algebras, a geometric approach; PHI. 2001.
- [3] Hoffman and Kunze; Linear Algebra; PHI. New Delhi.
- [4] Liptschutz; Linear Algebra; McGraw Hill.
- [5] M. Artin; Algebra; PHI. 1991.

# Complex Analysis - II

Semester: II Group: B
Course ID: PM2/05 Full Marks: 25

Minimum number of classes required: 35

Index | Elective I | Elective II

- Various kinds of singularities of complex valued functions, removable singularity, pole, essential singularity, classification of singularities using Laurent series development, Casorati-Weierstrass theorem concerning the nature of a function having an essential singularity, meromorphic functions, Residue theorem, contour integration and some applications, Argument Principle, Rouche's theorem, fundamental theorem of algebra as a corollary to Rouche's theorem.
- Space of continuous functions, space of analytic functions and space of meromorphic functions defined over open connected domain in  $\mathbb{C}$  a few interesting properties : Arzela-Ascoli theorem, Hurwit'z theorem, Montel's theorem, a subspace of analytic functions is compact iff it is closed and locally bounded, Riemann mapping theorem.
- Analytic continuation and some basic properties.

### References

- [1] J. B. Conway; Functions of One Complex Variable; Narosa Publishing; New Delhi, 1973.
- [2] R. P. Agarwal, K. Perera and S. Pinelas; An Introduction To Complex Analysis; Springer-Verlag.
- [3] S. Lang; Complex Analysis, Fourth edition; Springer-Verlag, 1999.
- [4] L. V. Ahlfors; Complex Analysis: an introduction to the theory of analytic functions of one complex variable; McGraw-Hill; New York, 1966.
- [5] A. I. Markushivich; Theory of Functions of Complex Variables, Vol-I, II; Prentice-Hall, 1965.
- [6] S. Ponnusamy; Foundations of Complex Analysis; Narosa Publishing; New Delhi, 1973.
- [7] R. V. Churchill and J. W. Brown; Complex Variables and Applications; McGraw-Hill; New York, 1996.

# Real Analysis - II

Semester: II Group: A
Course ID: PM2/06 Full Marks: 25

Minimum number of classes required: 35

Index | Elective I | Elective II |

- Metric Spaces: Metric spaces Revisited; Convergence: Convergent sequence and cluster points, Cauchy sequence, completeness: Functions: Continuity, uniform continuity, Isometries. Completeness: Examples of complete Metric spaces, subspace of a complete Metric space, Baire Category theorem, completion of Metric spaces, Banach contraction principle and some of its applications. Compactness: Total boundedness, characterization of compactness for arbitrary Metric spaces; Arzella-Ascoli theorem, Stone-Weierstrass theorem.
- Integrations: Lebesgue's criterion of Riemann integrability over a bounded closed interval [a, b] and its consequence, length of a rectifiable curve in a plane, Riemann-Stieltjes integral over [a, b] and its properties, Integrators of bounded variation, Integration by parts, Stieltjes integral as a Riemann integral, Step function as integrator, Riesz theorem.
- Cesaro's Method of Summability and Fourier Series: Cesaro's method of summability of order 1 and order
   Some specific examples, Regularity of Cesaro's method, Definition of Fourier series and some examples,
   Dirichlet's Kernel, Fejer's Kernel, Fejer's theorem, Dini's and Jordan's tests for point wise convergence of Fourier series.

# References

- [1] A. M. Bruckner, J. Bruckner & B. Thomson: Real Analysis, Prentice-Hall, N.Y. 1997.
- [2] R. R. Goldberg: Methods of Real Analysis, Oxford-IBH, New Delhi, 1970.
- [3] I. P. Natanson: Theory of Functions of a Real Variable, Vol-I, F. Ungar, N.Y. 1955.
- [4] E. Hewitt and K. Stromberg: Real and Abstract Analysis, John-Willey, N.Y. 1965.
- [5] J. F. Randolph: Basic Real and Abstract Analysis. Academic Press, N.Y. 1968.
- [6] P. K. Jain and K. Ahmad: Metric Spaces, Narosa Publishing House.
- [7] G. Tolstov: Fourier Series, Dover Publication, N.Y. 1962.

# Measure Theory - II

Semester: II Group: B
Course ID: PM2/06 Full Marks: 25

Minimum number of classes required: 35

Index Elective I Elective II

- Abstract measure spaces, Some familiar examples (revisited). Construction of Lebesgue-Stieltje's measure (using the well known Caratheodory extension theorem). Integration of non negative valued measurable functions. Monotone-Convergence theorem, Fatou's lemma, Integrable functions on measure spaces. Dominated convergence theorem. Special property of Lebesgue integrable functions, their relations with Riemann integrable functions. Approximation of Lebesgue integrable functions by Riemann integrable functions and also by continuous functions. Lebesgue-Stieltje's integrable functions, their relations with Riemann-Stieltje's integrable functions.
- $\bullet$   $L_p$  spaces constructed over abstract measure spaces. Hölder's and Minkowski's inequality, Riesz-Fisher theorem.
- Integration on product measure spaces. Fubini's theorem. Weak fundamental theorem  $\int_a^b f' d\lambda \leq f(b) f(a)$  concerning the derivative f' of a non-decreasing function  $f:[a,b] \longrightarrow \mathbb{R}$ . Absolutely continuous functions —— Banach-Zarecki theorem characterizing such functions. Notion of Lebesgue point.

#### References

- [1] G. De. Barra; Measure Theory & Integration; Wiley Eastern Limited, 1987
- [2] Charles Schwartz; Measure, Integration & Function Spaces; World Scientific 1994.
- [3] Inder Kumar Rana; An Introduction to measure & Integration; Narosa Publishing House, 1997.
- [4] P. R. Halmos; Measure Theory; D.Van Nostrand Co. inc. London, 1962.
- [5] P. K. Jain & V. P. Gupta; Lebesgue Measure & Integration; New Age International(P)limited Publishing Co, New Delhi, 1986.
- [6] H. L. Royden; Real Analysis; Macmillan Pub. Co. inc 4th Edition, New York, 1993.
- [7] Walter Rudin; Real and Complex Analysis; Tata Mcgraw Hill Publishing Co limited, New Delhi, 1966.
- [8] Bruckner, Bruckner and Thomson; Real Analysis; Prentice Hall International, Inc, 1997.

# General Topology - II

Semester: II
Course ID: PM2/07 Full Marks: 50

Minimum number of classes required: 70

Index | Elective I | Elective II

- Compactness: Characterizations and basic properties, Alexander subbase theorem. Compactness and separation axioms, compactness and continuous functions, Sequentially, Frechet and countably compact spaces. Compactness in metric spaces.
- Connectedness: Connected sets and their characterizations, connectedness of the real line, components, totally disconnected space, locally connected space. Path connectedness, path components, locally path connected space.
- Nets and Filters: Convergence and cluster points, Hausdorffness, continuity, limit point of sets and compactness in terms of them. Canonical way of converting nets to filters and vice-versa. Ultrafilter, subnets and ultranet.
- Product Topology: Tychonoff product topology in terms of standard sub-base and its characterizations. Projection maps. Product spaces vis-à-vis separation axioms, 1st and 2nd countability, separability, Lindelöfness, connectedness, local connectedness, path connectedness and compactness (Tychonoff theorem). Embedding lemma and Tychonoff embedding theorem.
- Identification Topology and Quotient Spaces: Definitions and examples of quotient topology and quotient maps; definition of quotient space of a space X determined by an equivalence relation on X. Associated theorems, cones and suspensions as examples, divisible properties.
- Metrizations: The Urysohn metrization theorem, the Nagata-Smirnov metrization theorem.
- Compactification : Local compactness and one-point compactification. Stone-Čech compactification.

# References

- [1] N. Bourbaki; General Topology Part-I (Transl.); Addison Wesley, Reading(1966).
- [2] J. Dugundji; Topology; Allyn and Bacon, Boston, 1966(Reprinted in India by Prentice Hall of India Pvt. Ltd.).
- [3] R. Engelking; General Topology; Polish Scientific Publishers, Warsaw (1977).
- [4] J. G. Hocking and C. S. Young; Topology; Addison-Wesley, Reading (1961).
- [5] S. T. Hu; Elements of General Topology; Holden-Day, San Francisco (1964).
- [6] K. D. Joshi; Introduction to Topology; Wiley Eastern Ltd. (1983).
- [7] J. L. Kelley; General Topology; Van Nostrand, Princeton (1955).
- [8] M. J. Mansfield; Introduction to Topology; D-van Nostrand Co. Inc, Princeton N.Y. (1963).
- [9] B. Mendelson; Introduction to Topology; Allyn and Becon Inc, Boston (1962).
- [10] James R. Munkress; Topology (2nd edit.); Pearson Education (2004).
- [11] W. J. Pervin; Foundations of General Topology; Academic Press, N.Y. (1964).
- [12] George F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill, N.Y.(1963).

- [13] L. Steen and J. Seebach; Counterexamples in Topology; Holt, Rinechart and Winston, N.Y. (1970).
- [14] W. J. Thron; Topological Structures; Holt, Rinehart and Winston, N.Y. (1966).
- [15] Stephen Willard; General Topology; Addison-Wesley, Reading (1970).

# Functional Analysis - I

Semester: II Group: A
Course ID: PM2/08 Full Marks: 30

Minimum number of classes required: 40

Index Elective I Elective II

- Normed linear space (n.l.sp.), Banach space with examples, quotient space.
- Bounded linear transformation, its equivalence with continuity, space of bounded linear transformations, equivalence of two norms in a linear space, equivalence of any two norms in a finite dimensional vector space, other important properties of a finite dimensional n.l.sp.
- Bounded linear functionals on various n.l.sp., Hahn-Banach theorems and consequences, dual and 2nd dual of a n.l.sp., separability and reflexivity of n.l.sp.
- Open mapping theorem, closed graph theorem and uniform boundedness principle, some applications of these theorems, Projection operator on Banach sp. and its boundedness.

### References

- [1] Bachman and Narici; Functional Analysis; Academic Press (1966).
- [2] G. F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill Book Company (1963).
- [3] Goffman and Pedrick; First Course in Functional Analysis; Prentice-Hall, Inc.
- [4] Walter Rudin; Functional Analysis; Tata McGraw-Hill (1974).
- [5] A. E. Taylor; Introduction to Functional Analysis; John Wiley & Sons. (1958).
- [6] B. V. Limaye; Functional Analysis; New Age International Ltd.
- [7] M. Thamban Nair; Functional Analysis; Prentice-Hall of India Pvt. Ltd., New Delhi (2002).
- [8] Jain, Ahuja and Ahmad; Functional Analysis; New Age International (P) Ltd. (1997).

### Discrete Mathematics - I

Semester: II Group: B
Course ID: PM2/08 Full Marks: 20

Minimum number of classes required: 30

Index | Elective I | Elective II |

• Graph Theory: Definition of undirected graph, Walk, Path, Circuit and Cycles. Subgraphs and induced Subgraphs. Degree of a vertex. Connectivity. Complete and Complete Bipartite graphs. Euler theorem on existence of Euler Paths and Circuits. Hamiltonian paths and cycles, Hamiltonian graph. Definition and properties of Trees. Minimal spanning Trees in a weighted graph and Kruskal algorithm. Planar graph and their properties. Euler formula for connected planar graphs. Kuratowski's theorem (proof not required).

### References

- [1] N. Deo; Graph Theory with Application to Engineering and Computer Science; Prentice Hall of India; New Delhi; 1990.
- [2] John Clark and Derek Allan Holton; A First Look at Graph Theory; World Scientific; New Jersey; 1991.
- [3] J. A. Bondy and U. S. R. Murty; Graph theory and related topics; Academic Press; New York; 1979.
- [4] F. Harary; Graph Theory; Narosa Publishing House; New Delhi; 1969.

### Multivariate Calculus - I

Semester: II Group: A
Course ID: PM2/09 Full Marks: 20

Minimum number of classes required: 30

Index Elective I Elective II

- Multivariable Differential Calculus: Introduction. The Directional derivative. Directional derivatives and continuity. The total derivative. The total derivative expressed in terms of partial derivatives. The Jacobian matrix. The chain rule. Matrix form of the chain rule. The mean-value theorem for differentiable functions. A sufficient condition for differentiability. A sufficient condition for equality of mixed partial derivative. Taylor's formula for functions from  $\mathbb{R}^n$  to  $\mathbb{R}^1$ .
- Implicit Functions and Extremum Problems : Introduction. The Inverse function theorem. The Implicit function theorem. Extrema of real-valued functions of one variable. Extrema of real-valued functions of several variables.

### References

- [1] T. M. Apostol: Mathematical Analysis, Narosa Publishing House, New Delhi.
- [2] M. Spivak: Calculus on Manifolds; W. A Benjamin; New York; 1965.
- [3] C. Goffman: Calculus of Several Variables, A Harper International Student reprint; 1965
- [4] W. Rudin: Principles of Mathematical Analysis; McGraw-Hill; New York; 1964.

# Theory of Manifolds

Semester: II Group: B
Course ID: PM2/09 Full Marks: 30

Minimum number of classes required: 40

Index | Elective I | Elective II

• Manifold: Introductory remarks, Smooth functions, Coordinate functions, Differentiable Manifold, Differentiable Mapping, Differentiable Curve, Tangent Vector, Vector Field, Lie Brackets, Lie Algebra of Vector Fields, Integral curves, Jacobian Map (Differential Map), f-related Vector fields, One parameter group of transformations, Local 1-parameter group of transformations, Cotangent Space, r-form, Exterior Product, Exterior Differentiation, Pull-Back Differential Form.

### References

- [1] Kobayashi & Nomizu: Foundations of Differential Geometry, Vol-I, Interscience Publishers, 1963.
- [2] K. Yano and M. Kon: Structure on Manifolds, World Scientific, 1984.
- [3] S. Helgason: Differential Geometry, Lie Groups and Symmetric Spaces, Academic Press, 1978.
- [4] W. M. Boothby: An Introduction to Differentiable Manifolds and Riemanian Geometry, Academic Press, Revised, 2003.
- [5] W. D. Curtis and F. R. Miller: Differential Manifolds and Theoretical Physics, Academic Press, 1985.

# Algebra - II

Semester: III Group: A
Course ID: PM3/10 Full Marks: 25

Minimum number of classes required: 35

Index | Elective I | Elective II

- Field Extensions: Algebraic extensions, Transcendental extensions, Degree of extensions, Simple extensions, Finite extensions, Simple algebraic extensions, Minimal polynomial of an algebraic element, Isomorphism extension theorem, Splitting fields: fundamental theorem of general algebra (Kronekar theorem), Existence theorem, Isomorphism theorem, Algebraically closed field, Existence of algebraically closed field, Algebraic closures, Existence and uniqueness (up to isomorphism) of algebraic closures of a field, field of algebraic members.
- Separable and inseparable polynomials, Separable and inseparable extensions, Perfect field, Artin's theorem.
- Finite Field: The structure of finite field, existence of  $GF(p^n)$ , Construction of finite fields, field of order  $p^n$ , primitive elements.
- Normal extensions, automorphisms of field extensions, Galois extensions, Fundamental theorem of Galois theory. Solutions of polynomial equations by radicals, insolvability of general polynomial equation of order 5 by radicals. Roots of unity, primitive roots of unity, Cyclotomic fields, Cyclotomic polynomial, Wedderburn's theorem. Geometric constructions by straightedge and compass, only.

#### References

- [1] Malik, Mordeson and Sen: Fundamentals of Abstract Algebra, McGraw Hill (1997).
- [2] J. N. Herstein: Topics in Algebra. Wiley Eastern Ltd. 1975.
- [3] I. Stewart: Galois Theory. Chapman and Hall 1989.
- [4] J. P. Escofier: Galois Theory. GTM Vol 204. Springer 2001.

# Partial Differential Equation

Semester: III Group: B
Course ID: PM3/10 Full Marks: 25

Minimum number of classes required: 35

Index Elective I Elective II

- Non linear first order PDE : Complete Integrals. General solution of higher order partial differential equation with constant coefficients.
- Second order PDE : classification and reduction to canonical forms.
- Laplace equation : Fundamental solution. Properties of harmonic functions. Mean value theorem. Greens functions.
- Wave equation : Elementary solution, D'Alemberts solution. Solution by spherical mean, Non-homogeneous equation.
- Heat equation: Elementary solution, Fundamental solution. Mean value formula. Properties of solution.
- Solution of wave equation, Laplace equation and heat equation by separation of variables.

### References

- [1] I. N. Sneddon; Elements of Partial Differential Equations; McGraw-Hill; London; 1957.
- [2] L. C. Evans; Partial Differential Equations; American Mathematical Society; Rhode Island; 1998.
- [3] F. John; Partial Differential Equations; Narosa Publishing House; New Delhi; 1979.

# Ordinary Differential Equation - II

Semester: III Group: A
Course ID: PM3/11 Full Marks: 30

Minimum number of classes required: 40

Index | Elective I | Elective II

- System of Homogeneous Linear Differential Equations. Theory of non-homogeneous linear system.
- Sturm Liouville Boundary Value Problem: Sturm Liouville (SL) problem, finding of eigen values and eigen function of SL problem, orthogonality of eigen functions, the expansion of a function in a series of orthonormal eigen functions.
- Non linear Differential Equations: Phase Plane, Paths and critical points, Critical points and paths of linear system, Limit cycles and periodic solutions.

### References

- [1] S. L. Ross; Introduction to ordinary differential equations; John-Wiley, New York, 1989.
- [2] G. F. Simmons; Differential equations with applications and historical notes; Tata McGraw Hill, New Delhi, 1976.
- [3] W. E. Boyce & R. C. Diprima; Elementary differential equations and boundary value problems; John Wiley & Sons, New York, 1977.
- [4] P. Hartman; Ordinary Differential Equations; John Wiley and sons, New York, 1964.

# Lie Groups and Lie Algebras

Semester: III Group: B
Course ID: PM3/11 Full Marks: 20

Minimum number of classes required: 30

Index | Elective I | Elective II |

• Lie Groups and Lie Algebras: Lie Group, General Linear Groups, Left translation, Right translation, Invariant Vector Field, Invariant Differential form, Automorphism, One parameter subgroup of a Lie group, Lie Transformation Group (Action of a Lie Group on a Manifold), Exponential Maps.

### References

- [1] S. Helgason; Differential Geometry, Lie Groups and Symmetric Spaces, Academic Press, 1978.
- [2] P. M. Cohn; Lie groups; Cambridge University Press, London, 1957.
- [3] V. S. Varadarajan; Lie groups, Lie algebras and their Representations; Springer (2002).
- [4] Alexander Kirillov, Jr.; An Introduction to Lie Groups and Lie Algebras; Cambridge University Press (2008).
- [5] M. Postnikov; Lectures in Geometry Semester V: Lie Groups and Lie Algebras; Mir Publishers (1986).
- [6] Brian C. Hall; Lie groups, Lie algebras and representations: An Elementary Introduction; Springer (2004).
- [7] Robert Gilmore; Lie groups, Lie algebras and some of their applications; A Wiley Interscience Publication (John Wiley & Sons.) (1941).

# Algebraic Topology - I

Semester: III Group: A
Course ID: PM3/12 Full Marks: 30

Minimum number of classes required: 40

Index Elective I Elective II

- Homotopy Theory: Homotopy between continuous maps, homotopy relative to a subset, homotopy class, null homotopy, contractibility of spaces. Homotopy equivalent spaces, homotopy properties.
- Deformability, deformation retracts, strong deformation retracts. Homotopy between paths, product of paths, fundamental group  $\Pi(X,x)$  of a space X based at the point  $x \in X$ , induced homomorphism and related properties. Simply connected space, special Van Kampan theorem and fundamental group of  $S^n$   $(n \ge 2)$ .
- Fundamental Group of  $S^1$ , Fundamental group of the product and of torus.  $\mathbb{R}^2$  and  $\mathbb{R}^n$  (n > 2) are not homeomorphic.
- Fundamental theorem of Algebra and Brouwer fixed point theorem. Covering projection, covering spaces, lifting of paths and homotopies, the fundamental group of a covering space. The Monodromy theorem, The Borsuk-Ulam theorem and ham-sandwich theorem.

### References

- [1] A. Dold; Lectures on Algebraic Topology; Springer-Verlag (1972).
- [2] W. Fulton; Algebraic Topology: A First Course; Springer-Verlag (1995).
- [3] M. Greenberg; Lectures on Algebraic Topology; W.D.Benjamin, N.Y. (1967).
- [4] Allen Hatcher; Algebraic Topology; Cambridge Univ. Press (2002).
- [5] C. Kosniowski; A First Course in Algebraic Topology; Cambridge University Press (1980).
- [6] W. S. Massey; Algebraic Topology: An Introduction; Springer-Verlag, N.Y. (1990).
- [7] James R. Munkres; Topology (2nd Edit.); Pearson Education Inc. (2004).
- [8] E. H. Spanier; Algebraic Topology; McGraw Hill Book Co. N.Y. (1966).
- [9] C. T. C. Wall; A Geometric Introduction to Topology; Addison-Wesley Publ. Co. Inc(1972).

# Functional Analysis - II

Semester: III Group: B
Course ID: PM3/12 Full Marks: 20

Minimum number of classes required: 30

Index | Elective I | Elective II

- Inner product space, Hilbert space, Orthonormality, Orthogonal complement, Orthonormal basis, Bessel's inequality, Parseval's equation, Gram-Schmidt orthonomalisation process, Riesz representation theorem, reflexivity of Hilbert space., separable and non-separable Hilbert space, General model of all Hilbert spaces (up to isometric isomorphism).
- Introduction to operator theory on Hilbert spaces. Special emphasis on compact, self-adjoint, normal and unitary operators.

### References

- [1] Bachman and Narici; Functional Analysis; Academic Press (1966).
- [2] G. F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill Book Company (1963).
- [3] Goffman and Pedrick; First Course in Functional Analysis; Prentice-Hall, Inc.
- [4] Walter Rudin; Functional Analysis; Tata McGraw-Hill (1974).
- [5] A. E. Taylor; Introduction to Functional Analysis; John Wiley & Sons. (1958).
- [6] B. V. Limaye; Functional Analysis; New Age International Ltd.
- [7] M. Thamban Nair; Functional Analysis; Prentice-Hall of India Pvt. Ltd., New Delhi (2002).
- [8] Jain, Ahuja and Ahmad; Functional Analysis; New Age International (P) Ltd. (1997).

# Algebraic Topology - II

Semester: IV Group: A
Course ID: PM4/13 Full Marks: 20

Minimum number of classes required: 30

Index | Elective I | Elective II |

• Elements of Simplicial Homology Theory: Barycentric coordinates, simplex, geometric complexes and polytopes, barycentric subdivision, simplicial mappings and the simplicial approximation theorem. Oriented complexes; incidence numbers, chains, cycles and boundaries. Homology groups, the decomposition theorems for abelian groups. Betti numbers and torsion coefficients. The structure of Homology groups. Euler characteristic. The Euler-Poincare theorem. Simplicial approximation again; induced homomorphism on Homology groups. The Brouwer fixed point theorem, Hopf classification theorem, Brouwer no-retraction theorem and related results.

### References

- [1] A. Dold, Lectures on Algebraic Topology, Springer-Verlag (1972).
- [2] W. Fulton, Algebraic Topology: A First Course, Springer-Verlag (1995).
- [3] M. Greenberg, Lectures on Algebraic Topology, W.D.Benjamin, N.Y. (1967).
- [4] Allen Hatcher, Algebraic Topology, Cambridge Univ. Press (2002).
- [5] C. Kosniowski, A First Course in Algebraic Topology, Cambridge University Press (1980).
- [6] W. S. Massey, Algebraic Topology: An Introduction, Springer-Verlag, N.Y. (1990).
- [7] James R. Munkres: Topology (2nd Edit.), Pearson Education Inc. (2004).
- [8] E. H. Spanier, Algebraic Topology, McGraw Hill Book Co. N.Y. (1966).
- [9] C.T.C.Wall, A Geometric Introduction to Topology, Addison-Wesley Publ. Co. Inc(1972).

# Calculus of variations and Integral Equations

Semester: IV Group: B
Course ID: PM4/13 Full Marks: 30

Minimum number of classes required: 40

Index Elective I Elective II

- Calculus of variations: Variational problems with fixed boundaries, the fundamental lemma of the calculus of variations, Euler's equation. Functionals dependent on several independent variables, on higher order derivatives etc.
- Fredholm Integral Equation: Solution by method of successive approximation, solution by method of successive substitution, direct substitution method.
- Volterra Integral Equation: Solution by method of successive approximation, solution by method of successive substitution, converting to initial value problem, Volterra Integral Equation of first kind.
- Solutions of Integral Equations with separable kernel. Resolvent kernel, Symmetric kernel, Hilbert Schmidt theory.

### References

- [1] L. Elsgolts; Differential equations and the calculus of variations; Mir Publishers, Moscow, 1973.
- [2] I. M. Gelfand & S. V. Fomin; Calculus of variations; Prentice-Hall, Englewood Cliff, New Jersey, 1963.
- [3] M. L. Krasnov, G. I. Makarenko & A. I. Kiselev; Problems and exercises in the calculus of variations; Mir Publishers, Moscow, 1975.
- [4] R. P. Kanwal; Linear Integral Equations; Birkhauser, Boston, 1997.
- [5] A. M. Wazwaz; A First Course in Integral Equations; World Scientific, Singapore, 1997.
- [6] S. G. Mikhin; Linear Integral Equations; Hindustan Book Agency, Delhi, 1960.

# Computational Mathematics (Theory)

Semester: IV
Course ID: PM4/14/Th | Full Marks: 30

Minimum number of classes required: 40

Index Elective I Elective II

- An overview of theoretical computers, history of computers, overview of architecture of computer, compiler, assembler, machine language, high level language, object oriented language, programming language and importance of C programming.
- Constants, Variables and Data type of C-Program : Character set. Constants and variables data types, expression, assignment statements, declaration.
- Operation and Expressions: Arithmetic operators, relational operators, logical operators, Bitwise operators, one's complement operators, right and left shift operators, Bitwise AND operator, Bitwise OR/XOR operator, Bitwise assignment operators, conditional operators.
- Decision Making and Branching: decision making with if statement, if-else statement, Nesting if statement, switch statement, break and continue statement, the Goto statement.
- Control Statement: While statement, do-while statement, for statement.
- Arrays: One-dimension, two-dimension and multidimensional arrays, declaration of arrays, initialization of one and multi-dimensional arrays.
- User-defined Functions: Definition of functions, return values and their types, function declaration, function call by value, Nesting of functions, passing of arrays to functions.
- Pointers: Pointer operators, address operation, pointer expression pointers and functions, pointers and array, function call by reference, dynamic memory allocations.
- File Management : Defining a file, opening a file, closing a file, input/output operations on file, random access to files.

#### References

- [1] B. W. Kernighan and D. M. Ritchi: The C-Programming Language, 2nd Edi.(ANSI Refresher), Prentice Hall, 1977.
- [2] E. Balagurnsamy: Programming in ANSI 'C', Tata McGraw Hill, 2004.
- [3] Y. Kanetkar: Let Us C; BPB Publication, 1999.
- [4] C. Xavier: C-Language and Numerical Methods, New Age International.
- [5] V. Rajaraman: Computer Oriented Numerical Methods, Prentice Hall of India, 1980.

# Computational Mathematics (Practical)

Semester: IV Course ID: PM4/14/Pr | Full Marks: 20

Index | Elective I | Elective II

• Computational techniques in Some Mathematical Applications Using C-Programming : Bisection, Trapizoidal, fixed point iteration, regular falsi, some operations on matrices, eigen values.

• Computations of some number theoretic functions such as  $\varphi, \tau, \sigma$  etc. Some techniques for primality testing.

### References

- [1] B. W. Kernighan and D. M. Ritchi: The C-Programming Language, 2nd Edi.(ANSI Refresher), Prentice Hall, 1977.
- [2] E. Balagurnsamy: Programming in ANSI 'C', Tata McGraw Hill, 2004.
- [3] Y. Kanetkar: Let Us C; BPB Publication, 1999.
- [4] C. Xavier: C-Language and Numerical Methods, New Age International.
- [5] V. Rajaraman: Computer Oriented Numerical Methods, Prentice Hall of India, 1980.

### Discrete Mathematics - II

Semester: IV Group: A
Course ID: PM4/15 Full Marks: 25

Minimum number of classes required: 35

Index Elective I Elective II

• Graph Theory: Directed graphs (digraphs), Digraphs and binary relations. Coloring of graphs, Chromatic number of a graph and its properties. Euler digraphs. Tournaments, Matrix representation of graphs, Adjacency matrix of a graph and digraph and their properties, Path Matrix, Incidence matrix of graphs and digraphs and their properties.

• Partially Ordered Sets and Lattices: Hasse diagrams of partially ordered set, Linear orders, Linear extension of a partially ordered set, Realizer and Dimension of a poset. Lattices and their properties. Lattice as a partially ordered set, Bounded Lattice. Distributive Lattice. Complements, completed lattices.

### References

- [1] N. Deo: Graph Theory with Application to Engineering and Computer Science; Prentice Hall of India; New Delhi; 1990.
- [2] John Clark and Derek Allan Holton: A First Look at Graph Theory; World Scientific; New Jersey; 1991.
- [3] D. S. Malik and M. K. Sen: Discrete mathematical structures: theory and applications; Thomson; Australia; 2004.
- [4] Edward R. Scheinerman: Mathematics A Discrete Introduction; Thomson Asia Ltd.; Singapore; 2001.

# Mathematical Logic

Semester: IV Group: B (OP1)
Course ID: PM4/15 Full Marks: 25

Minimum number of classes required: 35

Index | Elective I | Elective II

- General Notions : Formal Language, Object and Meta language, General definition of a Formal Theory / Formal Logic.
- Propositional Logic: Formal theory for propositional calculus, derivation, proof, theorem, Deduction theorem, conjunctive and disjunctive normal forms, Semantics, truth tables, tautology, adequate set of connectives, applications to switching circuits, Logical consequence. Consistency, maximal consistency, Leindenbaum lemma. Soundness and completeness theorems. Algebraic semantics.
- Predicate Logic: First order language, symbolizing ordinary sentences into first order formulae, Free and bound variables, interpretation and satisfiability, models, logical validity, Formal theory for predicate calculus, theorems and derivations, deduction theorem, equivalence theorem, replacement theorem, choice rule, Prenex normal form. Soundness theorem, completeness theorem, compactness theorem. First order theory with equality. Examples of First order theories (groups, rings, fields etc.).

### References

- [1] Elliott Mendelson; Introduction to mathematical logic; Chapman & Hall; London (1997)
- [2] Angelo Margaris; First order mathematical logic; Dover publications, Inc., New York (1990).
- [3] S.C.Kleene; Introduction to Metamathematics; Amsterdam; Elsevier (1952).
- [4] J.H.Gallier; Logic for Computer Science; John. Wiley & Sons (1987).
- [5] H.B.Enderton; A mathematical introduction to logic; Academic Press; New York (1972).

# **Number Theory**

Semester: IV Group: B (OP2)
Course ID: PM4/15 Full Marks: 25

Minimum number of classes required: 35

Index Elective I Elective II

- The Arithmetic of  $\mathbb{Z}_p$ , p a prime, pseudo prime and Carmichael Numbers, solving congruence's  $\mod(p^e)$ , Euler's function.
- Group of units, primitive roots, the group  $\mathcal{U}^{p^e}$ , where p is an odd prime, the group  $\mathcal{U}^{p^2}$ , the existence of primitive roots, applications of primitive roots, the algebraic structure of  $\mathcal{U}$ .
- $\bullet \mathcal{U}^{p^e}$ , and non quadratic residues, Legendre symbol, proof of the law of quadratic reciprocity, Jacobi symbols.
- Arithmetic functions, definitions and examples, perfect numbers, the Mobius Inversion formula, properties of Mobius function.
- Sum of two squares, the sum of three squares and the sum of four squares.

### References

- [1] Gareth A Jones and J Mary Jones: Elementary Number Theory, Springer International Edition.
- [2] Neal Koblitz: A course in number theory and cryptography, Springer-Verlag, 2nd edition.
- [3] D. M. Burton: Elementary Number Theory, Wm. C. Brown Publishers, Dulreque, Lowa, 1989.
- [4] Kenneth. H. Rosen: Elementary Number Theory & Its Applications, AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.
- [5] Kenneth Ireland & Michael Rosen: A Classical Introduction to Modern Number Theory, 2nd edition, Springer-verlag.
- [6] Richard A Mollin: Advanced Number Theory with Applications, CRC Press, A Chapman & Hall Book.
- [7] Saban Alaca, Kenneth S Williams: Introduction to Algebraic Number Theory, Cambridge University Press.

# Algebraic Geometry

Semester: IV Group: B (OP3)
Course ID: PM4/15 Full Marks: 25

Minimum number of classes required: 35

Index | Elective I | Elective II

- Introduction : Definition and examples.
- Affine varieties: Algebraic sets, Zariski topology, Hilbert's Nullstellensatz theorem, Irreducibility and dimension.
- Functions, morphisms and varieties: Functions on affine varieties, sheaves, morphisms between affine varieties, Prevarieties, Varieties.
- Projective varieties: Projective spaces and projective varieties, Cones and the projective Nullstellensatz theorem, Projective varieties on a ringed spaces, The main theorem on projective varieties.
- Dimension: The dimension of projective varieties, The dimension of varieties, Blowing up, Smooth varieties.

### References

- [1] Robin Hartshorne; Algebraic Geometry; Springer-Verlag (1977).
- [2] Joe Harris; Algebraic Geometry: a first Course; Springer-Verlag, New York (1992).
- [3] William Fulton; Algebraic Curves: an Introduction to algebraic geometry; W. A. Benjamin, London (1969).

### Multivariate Calculus - II

Semester: IV Group: B (OP4)
Course ID: PM4/15 Full Marks: 25

Minimum number of classes required: 35

Index Elective I Elective II

- Line Integrals: Paths and Line Integrals, Basic Properties and Examples; Work Done as a line integral, Line Integrals with respect to arc length; Independence of the path, The 2nd Fundamental Theorem and the 1st Fundamental Theorem; Gradient vector field, Necessary and Sufficient conditions for a vector field to be gradient; Potential functions.
- Multiple Integrals: Partitions of rectangles and step functions; Double integral upper and lower double integrals; double integral as volume; Integrability of functions; Applications to Area and Volume, Pappus's Theorem; Green's Theorem and its Applications; Change of Variables and Transformation formula.
- Surface Integrals: Surface, Fundamental vector Product; Area of a parametric Surface; Surface Integrals; Stoke's Theorem; Curl and Divergence of a vector field; Divergence Theorem.

### References

- [1] Walter Rudin; Principles of Mathematical Analysis (3e); McGraw-Hill International Editions, (1976).
- [2] Tom M. Apostol; Calculus Volume 2 (2e); John Wiley and Sons.
- [3] James R. Munkres; Analysis on Manifolds; Westview Press.
- [4] Michael Spivak; Calculus on Manifolds; Westview Press.

# Automata Theory

Semester: IV Group: B (OP5)
Course ID: PM4/15 Full Marks: 25

Minimum number of classes required: 35

Index | Elective I | Elective II

- Introduction to Computability Theory: Finite state mechanics, Finite state automata, Turing mechanics.
- Language and Grammar: Operation on Languages, Regular Language, Language generated by a Grammar, Content-sensitive grammar, Pumping lemma and Kleene theorem.

### References

- [1] D. S. Malik and M. K. Sen: Discrete mathematical structures: theory and applications; Thomson; Australia; 2004.
- [2] K. P. L. Mishra and N. Chandrasekaran; Theory of Computer Science; Prentice Hall of India, New Delhi, 2001.
- [3] J. E. Hopcropt and J. D. Ullman; Introduction to Automata Theory, Language, and Computation; Narosa Publishing, New Delhi, 1979.

### Abstract Harmonic Analysis - I

Semester: III Subject Code: 101
Course ID: PM3/E1/101 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Banach Algebra: Normed algebra, Banach algebra, examples of Banach algebra, algebra with involution, C\*-algebra, unitization of Banach algebra, weak-\*-topology, weak-\*-Compactness of the closed unit ball in a dual space, Vector-valued analytic functions, resolvent function and its analyticity, spectrum of a point, spectral radius, ideal and maximal ideal of a Gelfand algebra, character space, maximal ideal space with Gelfand topology, Gelfand representation theorem, theory of non-unital Banach algebras.
- Measure theory on locally Compact Hausdörff space: Abstract measure, positive Borel measure, Riesz representation theorem, Regularity properties of Borel measures, continuity properties of measurable functions, approximation by continuous functions, complex measure, total variation, absolute continuity, Radon-Nikodym theorem and its consequences, bounded linear functional on  $L^p$ , the Riesz representation theorem.
- Fourier Analysis on Euclidean Spaces: Fourier transform on  $L^1(\mathbb{R}^n)$  and its various properties, inversion of Fourier transform, Fourier transform on  $L^2(\mathbb{R}^n)$ , Plancherel theorem.

### References

- [1] M. Stein and G. Weiss; Introduction to Fourier Analysis on Euclidean Spaces; Princeton University Press (1971).
- [2] Bachman and Narici; Functional Analysis; Academic Press (1966).
- [3] C. E. Rickart; General Theory of Banach Algebras; D.Van Nostrand Company, Inc.
- [4] G. F. Simmons; Introduction to Topology and Modern Analysis; McGraw-Hill Book Company (1963)
- [5] Bachman, Narici and Beckenstein; Fourier and Wavelet Analysis; Springer.
- [6] Walter Rudin; Real and Complex Analysis; McGraw-Hill Book Company (1921).
- [7] R. R. Goldberg; Fourier Transforms; Cambridge, N.Y. (1961).

### Abstract Harmonic Analysis - II

Semester: IV Subject Code: 101
Course ID: PM4/E1/101 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Topological Group: Basic definition and facts, subgroups, quotient groups, some special locally compact Abelian groups.
- Haar measure on locally compact group: Construction of Haar measure, properties of Haar measure, uniqueness of Haar measure (up to multiplicative constant).
- Basic Representation Theory: Unitary representations, Schur's lemma, representations of a group and its group algebra, Gelfand-Raikov theorem.
- Fourier analysis on locally compact Abelian group: The dual group, the Fourier transform, Fourier-Stieltjes transforms, Positive-definite functions, Bochner's theorem, the inversion theorem, the Plancherel theorem, Pontryagin duality theorem, representations of locally compact Abelian groups, Closed ideals in  $L^1(G)$ , Wiener's theorem.

#### References

- [1] Hewitt and Ross; Abstract Harmonic analysis (Vol. I & II); Springer-Verlag (1963).
- [2] Walter Rudin; Fourier Analysis on Groups; Interscience Publishers (1962).
- [3] G. B. Folland; A Course in Abstract Harmonic Analysis; CRC Press (1995).
- [4] Bachman and Narici; Elements of Abstract Harmonic Analysis; Academic Press, New York (1964).
- [5] L. H. Loomis; An Introduction to Abstract Harmonic Analysis; D. Van Nostrand Company Inc. (1953).
- [6] Y. Katznelson; An Introduction to Harmonic Analysis; Dover Publications, Inc. (1976).

# Algebraic Aspects of Cryptology - I

Semester: III Subject Code: 102 Course ID: PM3/E1/102 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II

### Theory (Full Marks: 40):

- Probability Theory: Basic laws, Bernoulli and Binomial random variables, the geometric distribution, Markov's inequality, Chebyshev's inequality, Chernoff's bound.
- Complexity Theory: P, NP, P vs NP question, polynomial time reductions (emphasis on oracle machines), NP-Complete problems, Randomized algorithms, Probabilistic polynomial time, Non-uniform polynomial time.
- Basic Algorithmic Number Theory: Faster integer multiplication, Extended Euclid's algorithm, Quadratic Residues, Legendre Symbols, Jacobi Symbols, Chinese Remainder theorem, fast modular exponentiation, choosing a random group element, finding a generator of a cyclic group, finding square roots modulo a prime p, polynomial arithmetic, arithmetic in finite fields, factoring polynomials over finite fields, isomorphisms between finite fields, computing order of an element, computing primitive roots, fast evaluation of polynomials at multiple points, primality testing, Miller-Rabin Test, Generating random primes, primality certificates. algorithms for factorizing, algorithm for computing discrete logarithms.
- Public Key Cryptography: Diffie-Hellman key exchange, RSA, El-Gamal, Rabin.
- Algebraic Geometry: Affine Algebraic Sets, Parametrizations of affine varieties, ordering of the monomials in  $K[X_1, X_2, \ldots, X_n]$ , a division algorithm in  $K[X_1, X_2, \ldots, X_n]$ , Monomial ideals and Dickson's Lemma, Hilbert Basis Theorem, Gröbner basis, properties, Buchberger's Algorithm.

## Practical (Full Marks: 10):

• C implementation of various primitives for cryptographic schemes.

#### References

- [1] Steven D. Galbraith: Mathematics of Public Key Cryptography, Cambridge university press.
- [2] D. R. Stinson, Cryptography: Theory & Practice, CRC Press Company, 2002.
- [3] Jeffery Hoffstein, Jill Pipher, J.H.Silverman: An Introduction to Mathematical Cryptography, Springer.
- [4] Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography, Chapman & Hall/CRC.
- [5] Neal Koblitz: A course in number theory and cryptography, Springer-Verlag, 2nd edition.
- [6] D. M. Burton: Elementary Number Theory, Wm. C. Brown Publishers, Dulreque, Lowa, 1989.
- [7] Kenneth. H. Rosen: Elementary Number Theory & Its Applications, AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.
- [8] Kenneth Ireland & Michael Rosen: A Classical Introduction to Modern Number Theory, 2nd edition, Springer-verlag.
- [9] Richard A Mollin: Advanced Number Theory with Applications, CRC Press, A Chapman & Hall Book.
- [10] Saban Alaca, Kenneth S Williams: Introduction to Algebraic Number Theory, Cambridge University Press.

[11] Jay R Goldman : The Queen of Mathematics: a historically motivated guide to number theory, A K Peters Ltd.

### Algebraic Aspects of Cryptology - II

Semester : IV Subject Code : 102 Course ID : PM4/E1/102 Full Marks : 50

Minimum number of classes required : 55

Index | Elective I | Elective II

### Theory (Full Marks: 40):

- Private Key Cryptography: Private key encryption, perfectly secure encryption and its limitations, semantic security, pseudo-random number generator.
- Computational approach to cryptography: Basic ideas of computational security, efficient algorithms and negligible success probability, proof by reduction, security notions: CPA, CCA, CCA2, Security for multiple encryptions.
- More PKCs: Goldwasser-Micali, Paillier.
- Hash functions: Security properties of Hash functions, birthday attack, MAC, Construction of Hash functions, Number theoretic hash functions, Merkle-Damgard construction.
- Stream Cipher: Boolean function, LFSR, non-linear combiner model, linear complexity, Walsh transformation, Hadamard matrix, Correlation immunity, attacks on Boolean functions, S-Box, Some stream ciphers such as RC4, Attack on RC4.
- Secret Sharing Schemes: Shamir's Secret Sharing Scheme, multiparty computation, visual cryptography, DNA secret sharing scheme.
- Elliptic curves: properties of elliptic curves, elliptic curve over real and modulo a prime, torsion points, secret sharing scheme based on elliptic curve.
- Lattices: Basic nations, Hermite and Minkowski's bounds, computational problems in Lattices, LLL-reduced basis, the LLL Algorithm, Babai's Nearest Plane Algorithm, low exponent attack on RSA using lattices, GGH, NTRU.

#### Practical (Full Marks: 10):

• Sage implementation of various primitives for cryptographic schemes.

#### References

- [1] Steven D. Galbraith: Mathematics of Public Key Cryptography, Cambridge university press.
- [2] D. R. Stinson, Cryptography: Theory & Practice, CRC Press Company, 2002.
- [3] Jeffery Hoffstein, Jill Pipher, J.H.Silverman: An Introduction to Mathematical Cryptography, Springer.
- [4] Jonathan Katz, Yehuda Lindell: Introduction to Modern Cryptography, Chapman & Hall/CRC.
- [5] Neal Koblitz: A course in number theory and cryptography, Springer-Verlag, 2nd edition.
- [6] D. M. Burton: Elementary Number Theory, Wm. C. Brown Publishers, Dulreque, Lowa, 1989.
- [7] Kenneth. H. Rosen: Elementary Number Theory & Its Applications, AT&T Bell Laboratories, Addition-Wesley Publishing Company, 3rd Edition.
- [8] Kenneth Ireland & Michael Rosen: A Classical Introduction to Modern Number Theory, 2nd edition, Springer-verlag.

- [9] Richard A Mollin : Advanced Number Theory with Applications, CRC Press, A Chapman & Hall Book.
- [10] Saban Alaca, Kenneth S Williams: Introduction to Algebraic Number Theory, Cambridge University Press.
- [11] Jay R Goldman : The Queen of Mathematics: a historically motivated guide to number theory, A K Peters Ltd.

### Advanced Real Analysis - I

Semester: III Subject Code: 103
Course ID: PM3/E1/103 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Ordinal numbers: Order types, well-ordered sets, Transfinite induction, ordinal numbers, comparability of ordinal numbers, arithmetic of ordinal numbers, First uncountable ordinal  $\Omega$ .

- Descriptive properties of sets: Perfect sets, decomposition of a closed set in terms of perfect, sets of first category, 2nd category and residual sets, characterization of a residual set in a compete metric space, Borel sets of class  $\alpha$ , ordinal  $\alpha < \Omega$ . Density point of a set in  $\mathbb{R}$ , Lebesgue density theorem.
- Functions of some special classes: Borel measurable functions of class  $\alpha$ , ( $\alpha < \Omega$ ) and its basic properties, comparison of Baire and Borel functions, Darboux functions of Baire class one.
- Continuity: The nature of the sets of points of discontinuity of Baire one functions, Approximate continuity and its fundamental properties, characterization of approximate continuous functions.
- Henstock integration on the real line: Concepts of  $\delta$ -fine partition of the closed interval [a,b] where  $\delta$  is a positive function on [a,b], Cousin's lemma. Definition of Henstock integral of a function over the interval [a,b] and its basic properties. Saks-Henstock lemmas and its applications, continuity of the indefinite integral, Fundamental theorem, Convergence theorems, absolute Henstock integrability, characterization of Lebesgue integral by absolute Henstock integral.

#### References

- [1] A.M.Bruckner, J.B.Bruckner & B.S.Thomson: Real Analysis; Prentice-Hall, N.Y.1997.
- [2] I.P.Natanson: Theory of Functions of Real Variable, Vol.I & II, Frederic Ungar Publishing 1955.
- [3] C.Goffman: Real Functions, Rinehart Company, N.Y, 1953.
- [4] P.Y.Lee: Lanzhou Lectures on Henstock Integration, World Scientific Press, 1989.
- [5] J.F. Randolph: Basic Real and Abstract Analysis, Academic Press, N.Y, 1968.
- [6] S.M.Srivastava: A Course on Borel Sets, Springer, N.Y, 1998.

### Advanced Real Analysis - II

Semester: IV
Course ID: PM4/E1/103 | Subject Code: 103
Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Derivative: Banach-Zarecki theorem, Derivative and integrability of absolutely continuous functions, Lebesgue point of a function, Determining a function by its derivative.
- General measure and Integration: Additive set functions, measure and signed measures, Limit theorems, Jordan and Hahn Decomposition theorems, complete measures, Integrals of non-negative functions, Integrable functions, Absolute continuous and singular measures, Radon-Nikodym theorem, Radon-Nikodym-derivative in a measure space.
- Fourier Series : Fourier series of functions of class L, Fejer-Lebesgue theorem, Integration of Fourier series, Cantor-Lebesgue theorem on trigonometric series, Riemann's theorem on trigonometric series, Uniqueness of trigonometric series.
- Distribution Theory: Test functions, compact support functions, Distributions, operation on distributions, Local properties of distributions, Convergence of distributions, Differentiation of distributions and some examples, Derivative of locally integrable functions, Distribution of compact support, Direct product of distributions and its properties, Convolution and properties of convolutions.

#### References

- [1] A.M.Bruckner, J.B.Bruckner & B.S.Thomson: Real Analysis; Prentice-Hall, N.Y.1997.
- [2] R.L.Jeffery: The Theory of Functions of a Real Variable, Toronto University Press, 1953.
- [3] I.P.Natanson: Theory of Functions of Real Variable, Vol.I & II, Frederic Ungar Publishing 1955.
- [4] F.G.Friedlander: Introduction to the Theory of Distributions, Cambridge Univ Press, 1982
- [5] H.L.Royden: Real Analysis, Macmillan, N.Y, 1963.
- [6] S. Kesavan: Topics in Functional Analysis and its Applications, Wiley Eastern Ltd, New Delhi, 1989.

# Rings of Continuous Functions - I

Semester: III Subject Code: 104
Course ID: PM3/E1/104 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II |

• C(X) and  $C^*(X)$ . C-embedded and  $C^*$ -embedded sets in X, Urysohn's extension theorem. Pseudo-compact spaces — their characterizations. Adequacy of Tychonoff X for consideration of C(X),  $C^*(X)$  – M.H. Stones theorem. Z-filters, Z-ultrafilters on X, their duality with ideals, Z-ideals and maximal ideals of C(X). Structure spaces of C(X),  $C^*(X)$ , hull-kernel topology. Banach-Stone theorem. Wall man compactification, the partially ordered set K(X) of all compactifications of X, its lattice structure. Gelfand-Kolmogoroff theorem. Variant constructions of  $\beta X$  achieved as, (i) The structure space of C(X), (ii) The structure space of  $C^*(X)$ , (iii) The spaces  $\beta \mathbb{N}$ ,  $\beta \mathbb{Q}$  and  $\beta \mathbb{R}$ .

### References

- [1] Gillman and Jerison, Rings of continuous functions; Springer-verlag, N.Y. Heidelberg, Berlin, 1976.
- [2] Charles E. Aull, Rings of continuous functions; Marcel Dekker. Inc. 1985.
- [3] R. C. Walker, The Stone-Čech compactification; Springer, N.Y. 1974.
- [4] R. E. Chandler, Hausdorff compactifications; Marcel Dekker, Inc. N.Y. 1976.
- [5] J. Dugundji, Topology; Boston, allyn and Bacon, 1966.
- [6] Porter and Woods, Extensions and Absolutes of Hausdorff spaces; Springer, 1988.
- [7] Franklin Mendivil, Function Algebras & The Lattice of Compactifications; Proceedings of the American Mathematical Society, Vol-127, No. 6, Pages: 1863-1871, 1999.
- [8] Gillman and Kohls, Convex and pseudo-prime ideals in rings of continuous functions; Math-zeitschr, 72, 399-409, 1960.

# Rings of Continuous Functions - II

Semester: IV
Course ID: PM4/E1/104 | Subject Code: 104
Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II

• Quotient rings of C(X), real and hyper real maximal ideals, convex and absolutely convex ideals, Archimedean and non Archimedean quotient fields of C(X). Real compact spaces, restoration of real compact spaces X from C(X), Hewitt's isomorphism theorem, Hewitt real compactification vX of X, properties of real compact spaces. Cardinals of closed sets in  $\beta X$ , nondiscrete closed sets in  $\beta X \setminus vX$ , restoration of 1st countable spaces X from  $C^*(X)$ , unique determination of metric spaces X from X from

#### References

- [1] Gillman and Jerison, Rings of continuous functions; Springer-verlag, N.Y. Heidelberg, Berlin, 1976.
- [2] Charles E. Aull, Rings of continuous functions; Marcel Dekker. Inc. 1985.
- [3] R. C. Walker, The Stone-Čech compactification; Springer, N.Y. 1974.
- [4] R. E. Chandler, Hausdorff compactifications; Marcel Dekker, Inc. N.Y. 1976.
- [5] J. Dugundji, Topology; Boston, allyn and Bacon, 1966.
- [6] Porter and Woods, Extensions and Absolutes of Hausdorff spaces; Springer, 1988.
- [7] Franklin Mendivil, Function Algebras & The Lattice of Compactifications; Proceedings of the American Mathematical Society, Vol-127, No. 6, Pages: 1863-1871, 1999.
- [8] Gillman and Kohls, Convex and pseudo-prime ideals in rings of continuous functions; Math-zeitschr, 72, 399-409, 1960.

#### Structures on Manifolds - I

Semester: III Subject Code: 105
Course ID: PM3/E1/105 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II

- Almost Complex Manifolds: Introduction, algebraic Preliminaries, Nijenhuis tensor, Eigen values of the complex structure, Existence theorem and Integrability condition of an almost complex structure, Contravariant and covariant almost analytic vector field, Complex manifold.
- Almost Hermite Manifolds: Introduction, Nijenhuis tensor, curvature tensor, Holomorphic sectional curvature, Linear connection in an almost Hermite manifold.
- Kähler Manifolds: Introduction, Holomorphic sectional curvature, Bochner curvature tensor, Affine connection in Kähler manifolds, Conformally flat Kähler manifolds, Projective correspondence between two Kähler manifolds.
- Nearly Kähler Manifolds : Definition, curvature identities.
- Para Kähler Manifolds: Introduction, curvature identities, conformal flatness of para Kähler manifolds.
- Submanifolds of Kähler manifolds: Kaehlerian submanifolds, Anti-invariant submanifolds of Kaehlerian manifolds, CR-submanifolds of Kaehlenian manifolds.

#### References

- [1] R.S.Mishra: Structures on a Differentiable Manifold and Their Applications, Chandrama Prakashan, Allahabad, 1984.
- [2] K. Yano and M.Kon: Structures on Manifolds, World Scientific, 1984.

#### Structures on Manifolds - II

Semester: IV Subject Code: 105 Course ID: PM4/E1/105 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II |

- Contact Manifolds: Contact manifold, contact metric manifold, almost contact manifold, Torsion tensor of almost contact metric manifold, Killing vector field, properties of  $\varphi$ , the tensor field h, some curvature properties of contact metric manifold.
- K-contact Manifolds: Characterizations of K-contact manifolds, some curvature properties of K-contact manifolds, sectional curvature of K-contact manifolds, Locally symmetric and Ricci symmetric K-contact manifolds, semi-symmetric and Ricci semisymmetric K-contact manifolds.
- Sasakian manifolds : Introduction, some curvature properties,  $\varphi$  sectional curvature of a Sasakian manifold, semi-symmetric and Weyl semi-symmetric Sasakian manifolds, C-Bochner curvature tensor, D-Homothetic Deformation.
- N(k)-Contact Metric Manifolds : k-nullity distribution,  $\eta$ -Einstein N(k)-Contact Metric manifolds, Conformally flat N(k)-contact metric manifolds, some curvature properties.
- Para-contact Structure : Almost para-contact structure, Torsion tensor fields, Examples of paracontact manifolds, P-Sasakian manifolds.
- Submanifolds of Sasakian Manifolds: Invariant submanifolds of Sasakian manifolds, Anti-invariant submanifolds tangent to the structure vector field of Sasakian manifolds, Anti-invariant submanifolds normal to the structure vector field of Sasakian manifolds.

#### References

- [1] R.S.Mishra: Structure on a Differentiable manifold and their Applications, Chandrama Prakashani, Allahabad, 1984.
- [2] K. Yano and M. Kon: Structures on Manifolds, World Scientific, 1984.

# Advanced Algebraic Topology - I

Semester: III Subject Code: 106
Course ID: PM3/E1/106 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Fundamental concepts of affine geometry, convex sets, affine simplexes, simplicial complexes, simplicial scheme, normal subdivision, simplicial approximation.
- Cell complexes : Generalities, subcomplexes, products, homotopy extension, cellular approximation, local contractibility.
- Group: Fundamental concepts, direct products, generator and relations, further relations, altering the group presentation; free products.
- Singular homology theory: Simplicial homology groups; singular homology groups; chain complexes; relative homology, exact homology sequence; five lemma; direct sums; reduced 0-th homology groups
- Chain homotopy; the cone construction; the induced chain homotopy; barycentric subdivisions; small chains; Excision theorem; free chain complexes.
- Homology groups of spheres; oriented spheres and balls.

#### References

- [1] G.E.Bredon: Topology and Geometry, Springer-Verlag GTM 139 (1993).
- [2] A.Dold: Lectures on Algebraic Topology, Springer-Verlag (1980).
- [3] W.Fulton: Algebraic Topology, A First Course, Springer-Verlag (1995).
- [4] A.Hatcher: Algebraic Topology, Cambridge University Press (2002).
- [5] William S.Massey: A Basic Course in Algebraic Topology, Springer-Verlag, New York Inc. (1993).
- [6] C.R.F.Maunder: Algebraic Topology, Dover Pub. N.Y. (1996).
- [7] J.J.Rotman: An Introduction to Algebraic Topology, Springer-Verlag, N.Y. (1988).
- [8] H.Schubert: Topology, Macdonald Technical and Scientific, London (1964).
- [9] James W. Vick, Homology Theory: An introduction to Algebraic Topology, Springer-Verlag, N. Y. (1994).

# Advanced Algebraic Topology - II

Semester : IV Subject Code : 106 Course ID : PM4/E1/106 Full Marks : 50

Minimum number of classes required : 55

Index Elective I Elective II

• Singular homology continued: Local homology, cell chains, determination of incidence numbers; finitely generated abelian groups; trace formula; relation between  $H_1(x)$  and the fundamental group.

- Cohomology: The group hom(A, G); cohomology groups, singular cohomology, cohomology ring, examples, cap product, modules, tensor products.
- Homotopy: The higher homotopy groups; equivalent definitions of  $\Pi_n(X, x_0)$ , basic properties and examples; homotopy equivalences, homotopy groups of spheres, the relation between  $H_n(K)$  and  $\Pi_n(|K|)$ .
- Siefert and Van Kampen theorem on the fundamental group of the union of two spaces-applications.
- Structure of the fundamental group of a compact surface.
- The fundamental group and covering spaces of a Graph. Applications to Graph theory. Definition, examples and basic properties of a graph, trees, the fundamental group of a graph; the Euler characteristics of a finite graph; covering spaces of a graph.
- Manifolds: The classification of triangulable two manifolds, examples.

#### References

- [1] G.E.Bredon: Topology and Geometry, Springer-Verlag GTM 139 (1993).
- [2] A.Dold: Lectures on Algebraic Topology, Springer-Verlag (1980).
- [3] W.Fulton: Algebraic Topology, A First Course, Springer-Verlag (1995).
- [4] A.Hatcher: Algebraic Topology, Cambridge University Press (2002).
- [5] William S.Massey: A Basic Course in Algebraic Topology, Springer-Verlag, New York Inc. (1993).
- [6] C.R.F.Maunder: Algebraic Topology, Dover Pub. N.Y. (1996).
- [7] J.J.Rotman: An Introduction to Algebraic Topology, Springer-Verlag, N.Y. (1988).
- [8] H.Schubert: Topology, Macdonald Technical and Scientific, London (1964).
- [9] James W. Vick, Homology Theory: An introduction to Algebraic Topology, Springer-Verlag, N. Y. (1994).

# Universal Algebra, Category Theory & Lattice Theory - I

Semester: III Subject Code: 107
Course ID: PM3/E1/107 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II |

• Universal algebra, examples, subalgebra, subuniverse, congruences and quotient algebra. Homomorphisms of universal algebra, kernel of a homomorphism. First Isomorphism Theorem, Second Isomorphism Theorem, Third Isomorphism Theorem. Semigroup of endormorphisms of a universal Algebra. Birkhoff's theorem. Special faithful representation of universal algebras into semigroups. Theorem of Cohn-Rebane. Free algebra. Direct and subdirect product of universal algebras. Subdirectly irreducible universal algebras. Class operators. Turski's theorem. Identities. Term algebra, universal mapping property, k-free universal algebra. Variety, Birkhoff's theorem on variety. Macev's conditions.

#### References

[1] Stanley Burries and H. P. Sankappanavan: Universal Algebra, Springer-Verlag.

# Universal Algebra, Category Theory & Lattice Theory - II

Semester: IV Subject Code: 107
Course ID: PM4/E1/107 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Category, examples. Dual category, special morphisms. Monic and epic. Retraction and coretraction. Functor, forgetful functor, faithful functor. Product category, bifunctor, natural transformation, representable functor, embedding. Yoneda's lemma and its applications. Adjoint functor. Initial object, terminal object, zero object. Limit and colimit. Pull back diagram and push out diagram.

• Lattice, sublattice, generators of a sublattice, ideal, dual ideal. Homomorphisms and congruences. Distributive lattice. Characterization theorems and representation theorems on distributive lattice, structure of the lattice of congruences of lattices. Semi modular lattices. Characterization theorems and representation theorems. Stone's representation theorem on Boolean Algebra. Algebraic lattice – characterization theorems and representation theorems. One-to-one correspondence between ideals and congruences of Boolean Algebra and generalized Boolean Algebra.

#### References

[1] S. Maclane: Category theory for working mathematician, Springer-Verlag.

[2] B. Mitchel: Theory of categories, Academic Press (1969).

[3] Gratzer: Lattice theory, Verlag, Basel.

[4] Birkhoff: Lattice theory, AMS.

# Advanced Graph Theory - I

Semester: III Subject Code: 108
Course ID: PM3/E1/108 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Revisited: Graphs, walk, path, circuits and cycle. Subgraphs and induced subgraphs. Degree of a vertex. Complete graph and complete bipartite graphs. Properties of a tree, minimal spanning tree and Kruskal algorithm. Distance, center, radius and diameter of a tree.
- Degree sequence: Havel-Hakimi theorem, Statement of Erdos and Gallai theorem. Harary graphs.
- Connectivity: Vertex and edge connectivity, Blocks, Menger's theorem.
- Planar graph: Kuratowski's theorem, Dual of a planar graph, Combinatorial dual, outer planar graphs. Its forbidden subgraph characterization.
- Colouring: Vertex colouring, chromatic number, chromatic polynomial, Brooks theorem, Mycielski construction, edge colouring, chromatic index, map colouring, Five colour theorem, The Four colour theorem (statement and brief history).

Eulerian graphs and Hamiltonian graphs, Ore's theorem, Dirac' theorem, Closure of a Graph, Bondy and Chavtal theorem. The traveling salesman problem.

### References

- [1] J. A. Bondy and U. S. R. Murty; Graph theory with application; Academic Press; 1979.
- [2] G. Chartrand, L. Lesniak and P. Zhang; Graphs and Digraphs; Chapman and Hall, 2011.
- [3] D. B. West; Introduction to Graph Theory; Prentice Hall of India, New Delhi; 2001.
- [4] Holton and Clerk; Graph Theory; Allied Publishers; 1991.
- [5] Robin J. Wilson; Graph Theory; Pearson Education Asia; 2002.

# Advanced Graph Theory - II

Semester: IV
Course ID: PM4/E1/108 | Subject Code: 108
Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Directed graphs: Directed graphs and binary relation, directed paths and connectedness, Euler digraphs, Trees with directed edges, adjacency matrix and incidence matrix of graphs and digraphs and their properties. Acyclic digraphs and decyclization. Tournaments and their properties. Topological sorting.
- Line graph: Definition and characterization of line graphs. Forbidden subgraphs of line graph.
- Clique and stable set: Clique number, clique cover number and stability number of a graph. Definition of a perfect graph.
- Triangulated graphs: Characterization of triangulated graphs with perfect scheme and minimal vertex separator. Transitive orientation and comparability graphs.
- Interval graphs: Intersection graph, definition of an interval graph. Some characterization and application of interval graphs.
- Ramsey Theory: Pigeon-hole principle, A party problem, Ramsey number and Ramsey Graph.

#### References

- [1] J. A. Bondy and U. S. R. Murty; Graph theory with application; Academic Press; 1979.
- [2] G. Chartrand, L. Lesniak and P. Zhang; Graphs and Digraphs; Chapman and Hall, 2011.
- [3] D. B. West; Introduction to Graph Theory; Prentice Hall of India, New Delhi; 2001.
- [4] Holton and Clerk; Graph Theory; Allied Publishers; 1991.
- [5] Robin J. Wilson; Graph Theory; Pearson Education Asia; 2002.

# Theory of Linear Operators - I

Semester: III Subject Code: 109
Course ID: PM3/E1/109 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Spectral theorem in normed linear spaces, resolvent set and spectrum. Spectral properties of bounded linear operators. Properties of resolvent and spectrum. Spectral mapping theorem for polynomials. Spectral radius of a bounded linear operator on a complex Banach space. Certain concepts of the theory of Banach Algebras.

- General properties of compact like operators. Spectral properties of compact linear operators on normed spaces. Behaviours of compact linear operators with respect to solvability of operator equations.
- Spectral properties of bounded self-adjoint operators on a complex Hilbert space. Positive operators. Monotone sequence theorem for bounded self-adjoint operators on a complex Hilbert space. Square root of a positive operator. Projection operators. Spectral family of a bounded self-adjoint linear operator and its properties. Spectral representation of bounded self adjoint linear operators. Spectral theorem.

# References

- [1] E.Kreyszig, Introductory Functional Analysis with applications, John-Wiley and sons, N.Y. (1978)
- [2] P.R.Halmos, Introduction to Hilbert spaces and the theory of Spectral Multiplicity, Cheilsea Publishing co., N.Y. (1957).
- [3] P.R. Halmos, A Hilbert space Problem Book, D. Van Nostrand Co. Inc. (1967).
- [4] N.Dunford and J.T.Schwartz, Linear Operators, 3 Vols., Interscience Wiley, N.Y. (1958).
- [5] G.Bachman and L.Narici, Functional Analysis, Academic Press, N.Y. (1966).
- [6] N.I.Akhiezer and M.Glazman, Theory of Linear Operators in Hilbert Spaces, Fredelick Ungar Pub. Co. Vol-I (1961), Vol-II (1963).

### Theory of Linear Operators - II

Semester : IV Subject Code : 109 Course ID : PM4/E1/109 Full Marks : 50

Minimum number of classes required : 55

Index Elective I Elective II

• Spectral measures. Spectral Integrals. Description of the spectral subspaces. Characterizations of the spectral subspaces. The spectral theorem for bounded normal operators. Unbounded linear operators in Hilbert spaces. Hellinger-Toeplitz theorem. Hilbert adjoint operator. Symmetric and self-adjoint linear operators. Inverse of the Hilbert adjoint operator. Closed linear operators and closures. Hilbert adjoint of the closures. Spectrum of an unbounded self-adjoint operator. Spectral theorems for unitary and self-adjoint linear operators. Multiplication and differentiation operators.

### References

- [1] E.Kreyszig, Introductory Functional Analysis with applications, John-Wiley and sons, N.Y. (1978)
- [2] P.R.Halmos, Introduction to Hilbert spaces and the theory of Spectral Multiplicity, Cheilsea Publishing co., N.Y. (1957).
- [3] P.R.Halmos, A Hilbert space Problem Book, D.Van Nostrand Co. Inc. (1967).
- [4] N.Dunford and J.T.Schwartz, Linear Operators, 3 Vols., Interscience Wiley, N.Y. (1958).
- [5] G.Bachman and L.Narici, Functional Analysis, Academic Press, N.Y.(1966).
- [6] N.I.Akhiezer and M.Glazman, Theory of Linear Operators in Hilbert Spaces, Fredelick Ungar Pub. Co. Vol-I (1961), Vol-II (1963).

# Banach Algebra - I

Semester: III Subject Code: 110
Course ID: PM3/E1/110 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II |

- General preliminaries on Banach Algebras. Definitions and some examples. Regular and singular elements. Topological divisors of zero. The Spectrum. The formula for the Spectral radius.
- The radical, semi-simplicity, ideals, maximal ideals space, structure of semisimple Banach algebras.
- The carrier space and the Gelfand representation theorem, algebras of functions, The Silov boundary, representation of the carrier space, homomorphisms of certain function algebras into a Banach algebra, direct-sum decomposition and related results.
- Involution in Banach algebras, the Gelfand-Neumark theorem.

### References

- [1] C.E.Rickart; General theory of Banach algebras; D.Van Nostrand Company, INC.
- [2] G.F.Simmons; Topology and Modern Analysis, McGraw-Hill book company (1963)
- [3] S.Sakai ;  $C^*$ -Algebras &  $W^*$ -Algebras ; Springer-Verlag, 1971.

### Banach Algebra - II

Semester: IV
Course ID: PM4/E1/110 | Subject Code: 110
Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Commutative-\*-algebras, Self-dual vector spaces and \*-representations, positive functionals and \*-representations on Hilbert space, General properties of  $B^*$ -algebras, structure of ideals and representations of  $B^*$ -algebras.
- Algebras of operators: Elements of algebras of compact operators,  $C^*$ -algebra,  $W^*$ -algebra, positive elements and positive linear functionals on  $C^*$ -algebra, weak topology and various topologies on  $W^*$ -algebra, ideals in  $W^*$ -algebra, spectral resolution of self- adjoint elements in a  $W^*$ -algebra.

#### References

- [1] C.E.Rickart; General theory of Banach algebras; D.Van Nostrand Company, INC.
- [2] G.F.Simmons; Topology and Modern Analysis, McGraw-Hill book company (1963)
- [3] S.Sakai ;  $C^*$ -Algebras &  $W^*$ -Algebras ; Springer-Verlag, 1971.

# Module Theory and Non-commutative Rings - I

Semester: III Subject Code: 201
Course ID: PM3/E2/201 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II |

• Modules [Every ring is assumed to be a unitary ring and every module is assumed to be a unitary module]. Modules. Submodules. Quotient modules. Morphisms. Exact sequences. The three lemma. The four lemma. The Five lemma. Theorem (Butterfly of Zausenhauss). Product and co-product of R-modules. Free modules. Projective modules. Injective modules. Direct seem of projective modules. Direct product of injective modules.

• Divisible groups. Embedding of a module in an injective module. Tensor product of modules. Chain conditions on modules. Noetherian and Artinian modules. Finitely generated modules. Jordan-Hölder theorem. Indecomposable modules. Krull-Schmidt theorem. Semi-simple modules. Submodules, homomorphic images and direct sum of semi-simple modules.

#### References

[1] T.S.Blyth: Module Theory, Clarendon Press, London.

[2] T.Y.Lam: Noncommutative Rings, Springer-Verlag, 1991.

[3] I.N.Herstein: Noncommutative Rings, C. Monographs of AMS, 1968.

[4] T.W. Hungerford: Algebras, Holt-Rinechart and Winton Inc.

# Module Theory and Non-commutative Rings - II

Semester: IV Subject Code: 201
Course ID: PM4/E2/201 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II |

- Prime ideals, m-system, prime radical of an ideal, prime radical of a ring. Semiprime ideal, n-system, prime rings, semiprime ring as a subdirect product of prime ring, prime ideals and prime radical of matrix ring.
- Subdirect sum of rings, representation of a ring as a subdirect sum of rings. Subdirectly irreducible ring, Birkhoff theorem on subdirectly irreducible ring. Subdirectly irreducible Boolean ring.
- Local ring, characterizations of local ring, local ring of formal power series.
- Semisimple module, semisimple ring, characterizations of semisimple module and semisimple ring, Wedderburn-Artin theorem on semisimple ring.
- Simple ring, characterization of Artinian simple ring.
- The Jacobson radical, Jacobson radical of matrix ring, Jacobson semisimple ring, relation between Jacobson semisimple ring and semisimple ring, Hopkins-Levitzki theorem, Nakayama's lemma, regular ring, relation among semisimple ring, regular ring and Jacobson semisimple ring. Primitive ring, structure of primitive ring, Jacobson-Chevalley density theorem, Wedderburn-Artin theorem on primitive ring.
- Lower nil radical, upper nil radical, nil radical, Brauer's lemma, Kothe's conjecture, Levitzki theorem.

#### References

- [1] T.Y.Lam: Noncommutative Rings, Springer-Verlag.
- [2] I.N.Herstein: Noncommutative rings. Carus monographs of AMS, 1968.
- [3] N. Jacobson: Structure of Rings, AMS.
- [4] L.H. Rowen: Ring theory (student edition), Academic Press, 1991.
- [5] T.W. Hungerford: Algebra. Halt-Rinehart and Winstn. Inc.

### Advanced Functional Analysis - I

Semester: III Subject Code: 202
Course ID: PM3/E2/202 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Balanced, convex and absorbing sets and their properties, definitions and examples of linear topological space (l.t.s.) and locally convex space (l.c.s.), characterization of local base at  $\theta$  in an l.t.s. and in an l.c.s., basic properties, descriptions of finest linear and locally convex topologies, bounded and totally bounded sets.
- Seminorm, Minkowski functional, seminorm characterizations of l.c.s., seminormable spaces. Uniformity and metrizability of l.t.s. and l.c.s.. Completeness, F-space and Frechet space. Examples:  $L_p(0,1)$  (0 < p < 1), C(X) (where X is a locally compact,  $\sigma$ -compact and noncompact  $T_2$  space),  $K^I$ , C[a,b] with pointwise convergent topology.
- Linear maps and linear functionals, bounded linear maps. Product and Quotient spaces. Finite dimensional l.t.s., Riesz theorem. Banach Steinhans theorem, open mapping and closed graph theorems for F-spaces.

### References

- [1] John Horvath: Topological Vector Spaces and Distributions, Addison-Wesley Publishing Co. (1966).
- [2] .J.L.Kelly and Isaac Nomioka: Linear Topological Spaces, D.Van Nostrand Co.Inc. (1963).
- [3] Albert Wilansky: Modern Methods in Topological Vector Spaces, McGraw Hill Int. Book Co. (1978).
- [4] Charles Swartz: An Introduction to functional Analysis, Marcel Dekker, Inc. (1992).
- [5] John B.Conway: A Course in functional Analysis, Springer International Edition (1990).
- [6] W.Rudin: Functional Analysis, Tata McGraw-Hill, New Delhi, (1987).

### Advanced Functional Analysis - II

Semester : IV Subject Code : 202 Course ID : PM4/E2/202 Full Marks : 50

Minimum number of classes required : 55

Index Elective I Elective II

- Linear manifold, affine hyperplane, Geometric form of Hahn-Banach theorem, separation form of Hahn-Banach theorem and some of its consequences including Extension theorem.
- Algebraic dual and topological dual of a locally convex space, weak topology and weak-\* topology. Polar, bipolar theorem, Banach-Alaoglu theorem, Alaoglu-Bourbaki theorem. Extreme points and extreme sets, Krein-Milman theorem. Strong topology, Polar topology and Mackey topology, Mackey-Arens theorem, Machey's theorem.
- Bornivorous and barrel subset, Banach-Mackey theorem, bornological and barrelled spaces, infrabarrelled spaces, bidual, semi-reflexivity and reflexivity. Montel spaces and Schwarz spaces. Inverse limit and inductive limit of locally convex spaces. Distribution —— an introduction and certain basic results.

#### References

- [1] John Horvath: Topological Vector Spaces and Distributions, Addison-Wesley Publishing Co. (1966).
- [2] J.L.Kelly and Isaac Nomioka: Linear Topological Spaces, D.Van Nostrand Co.Inc. (1963).
- [3] Albert Wilansky: Modern Methods in Topological Vector Spaces, McGraw Hill Int. Book Co. (1978).
- [4] Charles Swartz: An Introduction to functional Analysis, Marcel Dekker, Inc. (1992).
- [5] John B.Conway: A Course in functional Analysis, Springer International Edition (1990).
- [6] W.Rudin: Functional Analysis, Tata McGraw-Hill, New Delhi, (1987).

#### Advanced Riemannian Manifold - I

Semester: III Subject Code: 203 Course ID: PM3/E2/203 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Connections: Affine Connections (Koszul), Torsion and Curvature tensor field on Affine Connection, Covariant Differential.
- Riemannian Manifold: Riemannian Manifold, Riemannian Connection, Riemann Curvature tensor, Gradient Vector Field, Einstein Manifold, Semi-symmetric Metric Connection, Non-metric Connection, Conformal transformation, Weyl Conformal Curvature tensor and properties, Conformally Symmetric Riemannian Manifold, Geodesic in a Riemannian Manifold, Bi-invariant Riemannian metric on a Lie group, Group Manifold.
- Bundle Theory: Principal Fibre Bundles, Linear Frame Bundle, Associated Fibre Bundle, Induced Bundle, Bundle Homomorphism, Linear Connection, Lift of a Vector Field.

#### References

- [1] B.B.Sinha: An Introduction to Modern Differential Geometry, Kalyani Publishers, New Delhi, 1982.
- [2] W.M.Boothby: An Introduction to Differentiable Manifolds and Riemannian Geometry, Academic Press, Revised 2/e, 2003.
- [3] Yano, K, Kon, M: Structures on Manifolds, World Scientific, 1984.
- [4] Eisenhart, L.P: Riemannian Geometry, Princeton University Press, 1949.
- [5] Kumaresan, S: A Course in Riemannian Geometry, T.I.F.R. Mumbai, 1990.

#### Advanced Riemannian Manifold- II

Semester: IV Subject Code: 203
Course ID: PM4/E2/203 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Special Theory of Relativity: Inertial frame of reference, Principles of special theory of relativity, special Lorentz transformations, Minkowski space time, Causality, Time dilation and Fitzgerald contraction, Velocity 4-vector and acceleration 4-vector in a Minkowski space-time, Lorentz transformation law of velocity 4-vector, Mass and Momentum, World-line of a particle in a Minkowski space-time.

• General Theory of Relativity: Principles of general theory of relativity, Energy Momentum tensor, Perfect Fluid, Einstein Field equation, Acceleration of a particle in a weak gravitational field, Schwarzschild Metric, Einstein Universe, De-Sitter's Universe, Manifold of General Theory of Relativity.

#### References

- [1] Eric A.Lord: Tensors, Relativity and Cosmology, Tata McGraw-Hill Pub. New Delhi.
- [2] S.K.Bose: An Introduction to General Relativity, Wiley Eastern Ltd. 1985.
- [3] S. Weinberg: Gravitation and Cosmology: Principles and Applications of the General Theory of Relativity, John Wiley & Sons. Inc. 1972.
- [4] Barrett O' Neill: Semi-Riemannian Geometry with applications to Relativity, Academic Press, 1983.

### Advanced Number Theory - I

Semester: III Subject Code: 204
Course ID: PM3/E2/204 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II

- Arithmetic Functions: The Mobius function, The Euler totinent function, The Dirichlet product of arithmetical functions, Liouville's functions, generalized convolution.
- Distribution of Prime Numbers : Chebyshev's functions, Some equivalent forms of the prime number theorem, Shapiro's Tauberian theorem.
- Dirichlet's theorem on primes in arithmetic progressions.
- Dirichlet series and Euler products, Zeta functions, Prime number theorem, Riemann Hypothesis concerning zeros of zeta function.

#### References

- [1] T. M. Apostol: Introduction to Analytic Number Theory; Narosa Publishing House; Springer International Student Edition.
- [2] G. H. Hardy & E. M. Wright: An Introduction to the Theory of Numbers; 4th edition, Oxford: Clarendon Press (1960).
- [3] J. P. Serre: A Course in Arithmetic; Narosa Publishing House (1973).
- [4] Donald J. Newman: Analytic Number Theory; Springer (1998).

# Advanced Number Theory - II

Semester : IV Subject Code : 204 Course ID : PM4/E2/204 Full Marks : 50

Minimum number of classes required : 55

Index Elective I Elective II

• The development of Algebraic Number theory: Algebraic Integers, Quadratic Fields, Quadratic integers, Geometric representation, Factorization in quadratic fields, non-unique factorization and ideals.

- Ideals in quadratic fields: Arithmetic of ideals, Lattice and Ideals, unique factorization of ideals, application of unique factorization, divisibility of Diophantine equations, factorization of rational primes, class structure and class numbers, finiteness of class numbers, norm of an ideal, bases and discriminants, the correspondence between forms and fields.
- Geometry of Numbers: The motivation of the problem, quadratic forms, Minkowski's fundamental theorem, Minkowski's theorem for lattices, sum of two and four squares, linear forms, sum and product of linear forms, Dirichlet's theorem, LLL-reduced base, LLL algorithm.
- p-adic numbers and valuations: History, the p-adic numbers, an informal introduction, the formal development, convergence, congruences and p-adic numbers, Hasse's principle, Hasses-Minkowski Theorem, Valuation and Algebraic Number theory.
- Algorithmic Number Theory: Lagendre-Jacobi-Kronecker Symbols, Shanks-Tonelli Algorithm, Solving Polynomial equations modulo p, Some primality testing algorithms, Some factorization algorithms.
- Application to Diophantine Equations: Lucas-Lehmer Theory, Generalized Ramanujan-Nagell Equations, Bachet's equation, The fermat equation, catalan and ABC Conjecture.
- Elliptic curves: The basics, Mazur, Siegel and Reduction, Applications: Factoring & Primality testing, Elliptic Curve Cryptography.

#### References

- [1] Kenneth Ireland & Michael Rosen: A Classical Introduction to Modern Number Theory, 2nd edition, Springer-verlag.
- [2] Richard A Mollin, Advanced Number Theory with Applications, CRC Press, A Chapman & Hall Book.
- [3] Introduction to Algebraic Number Theory, Saban Alaca, Kenneth S Williams, Cambridge University Press.
- [4] Jay R Goldman, The Queen of Mathematics: a historically motivated guide to number theory, A K Peters Ltd.
- [5] Henri Cohere: A course in Computational Number Theory; Springer Verlag, 1996.
- [6] Jurgen Neukirch: Algebraic Number Theory; Springer Verlag, 1999.
- [7] Henri Cohur: Number Theory. Vol-1, Tools of Diaphantine equations, Graduate Text in Mathematics, Springer Verlag, 2007.

# Dynamical System and Integral Equations - I

Semester: III Subject Code: 205 Course ID: PM3/E2/205 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Linear Systems: Uncoupled linear systems, Diagonalization, exponentials of operators, The fundamental theorem for linear systems, Linear systems in  $\mathbb{R}^2$ , Complex eigenvalues, Multiple eigenvalues.

- Nonlinear Systems: Local theory, Some preliminary concepts and definitions, The fundamental existence-uniqueness theorem, Dependence on initial conditions and parameters, The maximal interval of existence, The flow defined by a Differential Equation, Linearization, The Stable-Manifold theorem, The Hartman-Grobman theorem, Stability and Liapunov functions, Saddles, nodes, Foci and Centre.
- Nonlinear System : Global theory, Dynamical Systems and Global Existence Theorems, Limit sets and attractors, periodic orbits, Limit cycles and separatrix cycles, The Poincaré map, The stable manifold theorem for periodic orbits.
- Nonlinear Systems: Bifurcation theory, Structural Stability and Peixoto's theorem, Bifurcation at Nonhyperbolic Equilibrium Points, Hopf bifurcation and bifurcations of limit cycles from a multiple focus.

#### References

- [1] Perko L: Differential Equations and Dynamical Systems, Springer.
- [2] Nemytskii, V.V. and Stepanov, V.V: Qualitative Theory of Differential Equations, Princeton University Press, Princeton.

# Dynamical System and Integral Equations - II

Semester: IV Subject Code: 205
Course ID: PM4/E2/205 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Symmetric Kernels: Orthonormal system of functions. Fundamental properties of eigenvalues and eigenvalues and eigenfunctions for symmetric kernels. Expansion in eigenfunction and bilinear form. Hilbert Schmidt theorem and some immediate consequences. Solution of integral equations with symmetric kernels.

- Green's Functions: Approach to reduce BVP of a self-adjoint DE with homogeneous boundary conditions to integral equation forms. Auxiliary problem with more general and inhomogeneous boundary conditions. Modified Green's function.
- Integral Transforms: Fourier Transforms: Fourier's integral theorem, Fourier transforms, Fourier sine and cosine transforms, Fourier transform of derivatives, The calculation of the Fourier transform of some simple functions, The Fourier transforms of rational functions, the convolution integral, Parseval's theorem for cosine and sine transforms, The solution of integral equations of convolution type.
- Laplace Transform: Calculation of the Laplace transform of some elementary functions, Rules of manipulation of the Laplace transform, Laplace transform of derivatives, Relations involving integrals, The convolution of two functions, The inversion formula for the Laplace transform, The solution of ODE: Initial value problems for a linear equation with constant coefficients, Linear Differential equations with variable coefficients, Solution of Integral Equations.

#### References

- [1] R.P.Kanwal: Linear Integral Equations. Theory and Technique, Academic Press Inc.
- [2] W.V.Lovitt: Linear Integral Equations, Dover Publications, N.Y. 1950.
- [3] W.Pogovzelski: Integral equations and their applications, Vol-I, Pergamon Press, Oxford 1966.
- [4] F.G.Tricomi: Integral Equations, Wiley, N.Y. 1957.
- [5] I.N.Sneddon: The use of Integral transforms, TMH Edition.
- [6] L.Debnath: Integral transforms and their applications, CRC Press 1995.
- [7] E.T.Whittacker and G.N.Watson: A Course of Modern Analysis, Cambridge University Press.

### Advanced General Topology - I

Semester: III Subject Code: 206
Course ID: PM3/E2/206 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Uniformity: Uniformity and its uniform topology, neighbourhoods, bases and subbases, uniform continuity, product uniformities, uniform isomorphism, relativization and products.

Characterization of metrizability, uniformity of pseudometric spaces, uniformity generated by a family of pseudometrics, the gauge of uniformity. Completeness, Cauchy net, Cauchy filter, complete spaces, extension of mappings, completion-existence and uniqueness,

Compactness and uniformity: diagonal uniformities, uniformity via uniform covers.

- Proximity Spaces: Topology induced by a proximity, subspaces and products of proximity spaces, elementary proximity, *p*-continuity and *p*-isomorphism, Compactification of proximity spaces-clusters and ultrafilters, Smirnov's theorem.
- Ordinal Numbers and Ordinal Spaces: Definition and properties of ordinal numbers. Cardinal numbers visà-vis ordinal numbers, Ordinal spaces, topological properties of ordinal spaces —  $\omega_1$  and  $\omega_2$  (in particular).

#### References

- [1] J. Dugundji; Topology; Prentice-Hall of India Pvt. Ltd. (1975).
- [2] R. Engelking; Outline of General Topology; North-Holland Publishing Co., Amsterdam (1968).
- [3] Ioan James; Topologies and Uniformities; Springer-Verlag (1999).
- [4] J. L. Kelley; General Topology; D.Van Nostrand Co. Inc. (1955).
- [5] Jun Iti Nagata; Modern General Topology; North-Holland Pub. Amsterdam (1985).
- [6] S. A. Naimpally & B. D. Warrack; Proximity Spaces; Cambridge University Press (1970).
- [7] S. Willard; General Topology; Addison-Wesley Publishing Co. (1970).

# Advanced General Topology - II

Semester : IV Subject Code : 206 Course ID : PM4/E2/206 Full Marks : 50

Minimum number of classes required : 55

Index | Elective I | Elective II

- Paracompactness: Types of refinements, paracompactness in terms of open locally finite refinements, Michael's theorem, fully normal spaces, Stone's coincidence theorem, paracompactness in terms of open delta refinements, cushioned refinements etc. A. H. Stone's theorem —— every metric space is paracompact, partition of unity, properties of paracompact spaces with regard to subspaces, product etc.
- Function Spaces: Pointwise convergence topology and uniformity, compact-open topology, uniqueness of jointly continuous topology, uniform convergence on a family of sets, completeness, uniform convergence on compacta, K-spaces, compactness and equicontinuity. The Ascoli theorem, Even continuity, topological Ascoli theorem, basis for  $Z^Y$ , compact subsets of  $Z^Y$ , sequential convergence in the c-topology, metric topologies —— relation to the c-topology, pointwise convergence, comparison of topologies in  $Z^Y$ . The spaces C(Y) continuity of the algebraic operations, algebras in  $\widehat{C}(Y,C)$ , Stone-Weierstrass theorem, the metric space C(Y), embedding of Y in C(Y), The ring  $\widehat{C}(Y)$ .
- Metrization: Metrization theorems of Nagata-Smirnov, Bing, Smirnov, A.H.Stone, Arhangeliskii etc.
- Elements of Dimension Theory: Menger-Urysohn dimension (the small inductive dimension) of a space, indX and IndX, dim X, associated results, specially in connection with 0-dimensional or totally disconnected spaces and  $\beta X$  etc.

#### References

- [1] J. Dugundji; Topology; Prentice-Hall of India Pvt. Ltd. (1975).
- [2] R. Engelking; Outline of General Topology; North-Holland Publishing Co, Amsterdam. (1968).
- [3] J. L. Kelley; General Topology; D.Van Nostrand Co. Inc. (1955).
- [4] Jun Iti Nagata; Modern General Topology; North-Holland Pub. Amsterdam (1985).
- [5] S.Willard; General Topology; Addison-Wesley Publishing Co. (1970).
- [6] W. Hurewicz and H. Wallman; Dimension Theory; Princeton University Press (1948).

### Advanced Complex Analysis - I

Semester: III Subject Code: 207
Course ID: PM3/E2/207 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Analytic Functions: Convex function, Mean values, the function A(r), Borel-Caratheodory's theorem. Distributions of zeros of analytic functions, the function n(r), Jensen's theorem.

- Harmonic and Subharmonic Functions: Basic properties of harmonic functions, harmonic function on a disk, subharmonic and superharmonic functions, Dirichlet problems for a disk, Green's function.
- Entire Functions: Infinite product of complex number and complex functions. Order, type, Weierstrass's factorization theorem, Jensen's inequality, exponent of convergence, Hadamard's factorization theorem, order and type in terms of Taylor's coefficient.
- Analytic Continuations : Direct analytic continuations, uniqueness of analytic continuation along a curve, Monodromy theorem and its consequence, Analytic continuation via Reflection.

#### References

- [1] J.B.Conway: Functions of one complex variable; Springer-Verlag, Int. student edition, Narosa Publishing House, 1980.
- [2] L.V.Ahlfors: Complex Analysis; McGraw Hill, 1979.
- [3] A.S.B. Holland: Theory of entire functions; Academic Press, 1973.
- [4] E.C.Tichmarsh: Theory of functions; Oxford University Press, London, 1939.
- [5] S.Lang: Complex Analysis, Forth edition, Springer-Verlag, 1999.
- [6] A.I.Marcushevich: Theory of functions of a complex variable, Vol-I,II,III, Prentice--Hall,1965.
- [7] H.A.Priestly: Introduction to complex analysis; Clarendon Press, Oxford, 1990.
- [8] R.V.Churchill & J.W.Brown: Complex variables and applications, McGraw Hill.
- [9] S. Ponnusamy: Foundations of complex analysis, Narosa Publishing House, 1997.

### Advanced Complex Analysis - II

Semester: IV Subject Code: 207
Course ID: PM4/E2/207 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Meromorphic functions: Basic properties of meromorphic functions, Mittag-Leffler's theorem, application of Mittag-Leffler's theorem for simple poles, Gamma functions and its properties, Riemann zeta functions, Riemann's functional equations, Runge's theorem.
- The range of analytic functions: Bloch's theorem, The little Picard theorem, Schottky's theorem, The Great Picard theorem.
- Inverse and Implicit Functions of Complex Variables: Inverse functions the single valued case, the multivalued case, examples of Lagrange's series, functions of two variables, Weierstrass Preparation theorem, the Implicit function theorem.
- Univalent functions: Necessary and sufficient conditions for univalence, basic properties of univalent functions, Area theorem, Distortions theorem.

#### References

- [1] J. B. Conway: Functions of one complex variable; Springer-Verlag, Int. student edition, Narosa Publishing House, 1980.
- [2] L. V. Ahlfors: Complex Analysis; McGraw Hill, 1979.
- [3] A. S. B. Holland: Theory of entire functions; Academic Press, 1973.
- [4] E. C. Tichmarsh: Theory of functions; Oxford University Press, London, 1939.
- [5] S. Lang: Complex Analysis, Forth edition, Springer-Verlag, 1999.
- [6] A. I. Marcushevich: Theory of functions of a complex variable, Vol-I,II,III, Prentice--Hall,1965.
- [7] H. A. Priestly: Introduction to complex analysis; Clarendon Press, Oxford, 1990.
- [8] R. V. Churchill & J. W. Brown: Complex variables and applications, McGraw Hill.
- [9] S. Ponnusamy: Foundations of complex analysis, Narosa Publishing House, 1997.

### Algebraic Coding Theory - I

Semester: III Subject Code: 208
Course ID: PM3/E2/208 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• The Communication channel. The Coding Problem. Types of Codes. Block Codes. Error-Detecting and Error-Correcting Codes. Linear Codes. The Hamming Metric. Description of Linear Block Codes by Matrices. Dual Codes. Standard Array. Syndrome. Step-by-step Decoding Modular Representation. Error-Correction Capabilities of Liner Codes. Bounds on Minimum Distance for Block Codes. Plotkin Bound. Hamming Sphere packing Bound. Varshamov-Gilbert-Sacks Bound. Bounds for Burst-Error Detecting and Correcting Codes. Important Linear Block Codes. Hamming Codes. Golay Codes. Perfect Codes. Quasi—perfect Codes. Reed-Muller Codes. Codes derived from Hadamard Matrices. Product Codes Concatenated Codes. Tree Codes. Convolutional Codes. Description of Linear Tree and Convolutional Codes by Matrics. Standard Array. Bounds on Minimum distance for Convolutional Codes. V.G.S., bound. Bounds for Burst-Error Detecting and Correcting Convolutional Codes. The Lee metric, packing bound for Hamming code w.r.t. Lee metric.

#### References

- [1] Steven Roman, Coding and Information Theory, Springer-Verlag.
- [2] Richard Hamming, Coding and Information Theory, Prentice Hall.
- [3] F.J. MacWilliams, N.J.A. Sloane: The Theory of Error-Correcting Codes, (North-Holland Mathematical Library).
- [4] Norman L Biggs: Codes- An Introduction to Information Communication and Cryptography: Springer Undergraduate Mathematics Series.

### Algebraic Coding Theory - II

Semester: IV Subject Code: 208
Course ID: PM4/E2/208 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II

• The Algebra of polynomial residue classes. Galois Fields. Multiplicative group of a Galois field. Cyclic Codes. Cyclic Codes as Ideals. Matrix Description of Cyclic Codes. Hamming and Golay Codes as Cyclic Codes. Error Detection with cyclic Codes. Error-Connection procedure for Short Cyclic Codes. Shortened Cyclic Codes. Pseudo Cyclic Codes. Code symmetry. Invariance of Codes under transitive group of permutations. Bose-Chaudhuri-Hocquenghem (BCH) Codes. Majority-Logic Decoding. BCH bounds. Reed-Solomon (RS) Codes. Majority-Logic Decodable Codes. Majority-Logic Decoding. Singleton bound. The Griesmer bound. Maximum-distance Separable (MDS) Codes. Generator and Parity-check matrics of MDS Code. Weight Distribution of MDS Code. Necessary and Sufficient conditions for linear code to be an MDS Code. MDS codes from RS codes. Abramson Codes. Close-loop burst-Error correcting codes (Fire codes). Error Locating Codes.

#### References

- [1] Steven Roman, Coding and Information Theory, Springer-Verlag.
- [2] Richard Hamming, Coding and Information Theory, Prentice Hall.
- [3] F.J. MacWilliams, N.J.A. Sloane: The Theory of Error-Correcting Codes, (North-Holland Mathematical Library).
- [4] Norman L Biggs: Codes- An Introduction to Information Communication and Cryptography: Springer Undergraduate Mathematics Series.

# Advanced Measure Theory and Integration - I

Semester: III Subject Code: 209
Course ID: PM3/E2/209 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• General Measure Theory: Measure space, outer measure and measurability, extension of measures, measurable covers, completion of a measure, sets of measure zero and sets of full measure, extension by sets of measure zero, regular outer measure, metric outer measure, distinction between finite and infinite measure, Lebesgue-Stieltjes measures and Lebesgue-Stieltjes integral, Hausdorff measure and its dimension.

- Product Measures: Cartesian product of two measurable spaces, sections, the product of two finite measure spaces, product of two  $\sigma$ -finite measure spaces, iterated integrals, Fubini's theorem.
- Modes of Convergence and  $L_p$ -Spaces: Integration of complex-valued functions convergence point wise, almost everywhere uniform and almost uniform, convergence in measure,  $L_p$ -spaces and convergence in the p-th mean, necessary and sufficient condition for convergence in  $L_p$ , Dense subspace of  $L_p$ , the spaces of essentially bounded functions.

# References

- [1] M.E.Munroe: Introductions to measure and integration, Addison Wesley, 1953.
- [2] S.K.Berberian: Measure and integration, Chelsa Pub. Co. N.Y. 1965.
- [3] H.L.Royden: Real Analysis, Macmillan N.Y. 1963.
- [4] W.Ruddin: Real and Complex Analysis, Tata McGraw Hill, New Delhi, 1974.
- [5] I.K.Rana: An introduction to measure and integration, Narosa Pub. House, 1997.
- [6] A.M.Bruckner, J.B.Bruckner, B.S.Thomson: Real Analysis, Prentice Hall, 1997.

# Advanced Measure Theory and Integration - II

Semester: IV
Course ID: PM4/E2/209 | Subject Code: 209
Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

- Signed Measures and Complex Measures : Signed measures, absolute continuity of finite signed measures, Jordon-Hahn decomposition theorem for finite signed measure, Randon-Nikodym theorem for finite and  $\sigma$ -finite signed measures and its consequences, computation of the Randon-Nikodym derivative, complex measures, Randon-Nikodym theorem for complex measures.
- Integration over locally compact spaces: Continuous functions with compact support Baire sets, Baire measure, regularity of Baire measure, integration of continuous functions with compact support. The Riesz-Markoff representation theorem.
- Integration: Integration over locally compact groups, topological groups, translates, Haar integral, existence of Haar integral, Haar measure, Uniqueness of a Haar integral.

#### References

- [1] M.E.Munroe: Introductions to measure and integration, Addison Wesley, 1953.
- [2] S.K.Berberian: Measure and integration, Chelsa Pub. Co. N.Y. 1965.
- [3] H.L.Royden: Real Analysis, Macmillan N.Y. 1963.
- [4] W.Ruddin: Real and Complex Analysis, Tata McGraw Hill, New Delhi, 1974.
- [5] I.K.Rana: An introduction to measure and integration, Narosa Pub. House, 1997.
- [6] A.M.Bruckner, J.B.Bruckner, B.S.Thomson: Real Analysis, Prentice Hall, 1997.

### Differential Topology - I

Semester: III Subject Code: 210
Course ID: PM3/E2/210 Full Marks: 50

Minimum number of classes required: 55

Index | Elective I | Elective II

- Manifolds and smooth maps. Immersions; subimmersions; level surfaces; transversal maps; Sard's theorem. Morse functions. Embedding manifolds in Euclidean spaces.
- Manifolds with boundary. Brouwer Fixed point theorem. Genericity of transversal maps. Tabular neighbourhood theorem. Transversality homopoty theorem.
- Intersection theory mod 2. Winding number and the Jordan Brower separation theorem. The Borsuk-Ulam theorem.

#### References

- [1] V. Guillemin and A. Pollack; Differential Topology; Prentice-Hall, Innc. Englewood Cliffs, New Jersey (1974).
- [2] M. Hirsch; Differential Topology; Graduate Texts in Mathematics Series, Springer-Verlag (1976).
- [3] A. A. Kosinski; Differential Manifolds; Academic Press (1993).

### Differential Topology - II

Semester: IV Subject Code: 210
Course ID: PM4/E2/210 Full Marks: 50

Minimum number of classes required: 55

Index Elective I Elective II

• Orientation on a manifold. Oriented intersection number, The Fundamental theorem of Algebra. Euler characteristic. Lefschetz Fixed point theorem. Homotopy invariance of Lefschetz number. Splitting proposition. Local computation of the Lefschetz number. Index of vector fields on a manifold. Poincare-Hopf index theorem. The Hopf degree theorem. Isotopy lemma. The Euler characterization and Triangulations. Integration on manifolds. Exterior algebra. Differential Forms. Exterior derivative. Stoke's theorem. Integration and mappings. Degree formula. Gauss map. Gauss-Bonnet theorem.

#### References

- [1] V. Guillemin and A. Pollack; Differential Topology; Prentice-Hall, Innc. Englewood Cliffs, New Jersey (1974).
- [2] M. Hirsch; Differential Topology; Graduate Texts in Mathematics Series, Springer-Verlag (1976).
- [3] A. A. Kosinski; Differential Manifolds; Academic Press (1993).