# UNIT 3 EXPERIMENTAL DESIGNS

**Objectives**

After reading this unit, you should be able to:

- Discuss the various experimental designs as powerful tools to study the cause and effect relationships amongst variables in research.
- Explain the assumptions embodied in the design models.
- Choose the appropriate design model for a specific research problem.

**Structure**



## 3.1 INTRODUCTION

As you may recall, we have pointed out in unit 1 of this block that experiments are much more effective than descriptive techniques in establishing the casual relationships. First, the units to be studied are selected by the researcher and each unit is assigned to the group determined by the researcher. The units do not select their groups, thus avoiding the self-selection bias. Second, a necessary consequence of the first, the researcher administers the predetermined treatment or treatments to the units with in each group.

The use of a control group is almost mandatory in experimental designs. The inclusion of a control group permits a better isolation of the treatment component through a proper design like a simple cross sectional design.

A major contribution that the statisticians have made to experimental design is the development of randomization concept which enables the researcher to reduce the effect of the uncontrolled variables on comparative measures of response to the variables that are under the experimenter's control. Randomization is a useful device for ensuring on the average, that uncontrolled variables do not favour one treatment versus others.

In this unit, we will be, discussing some of the major experimental designs which include:

1) Completely Randomized Design
2) Randomized Complete Block Design
3) Latin Square Design
4) Factorial Design
5) Analysis. of Covariance

We will describe each of these experimental designs in detail in terms of role, the model, and the assumptions embodied in the model with few illustrations. We will not dwell into the computation aspect and instead focus on interpretation of the results. It is strongly suggested here that you use computer software packages like SPSS, STAT GRAPHICS, and BMD for getting the relevant ANOVA tables as output which can be interpreted by you. The interpretation of the results is much more important than the drudgery of complex computations.

It may be mentioned in the passing that in MS-61, block 5, we have given complete details regarding calculations of the relevant sum of squares and F ratio for hypothesis testing in the case of one way and two way analysis of variance. You please go through the same for understanding the principles of breaking down the total variation into meaningful components of variations. This methodology and principle remains the same in all the designs and therefore with a little more effort you should be able to understand and work out the details. We again reiterate that you use the software packages which have lots of options and flexibilities. You will really enjoy the subject in this way and will be able to understand the intricacies of the models which in turn will enable you to choose the right design for your research problem-be it in medicine, management, social science, etc.

## 3.2 COMPLETLY RANDOMIZED DESIGN

Frequently an investigator wishes to compare three or more treatments in a single experiment. In a survey, too, he may wish to study several populations; for example, he may be interested in IQ scores from a standard test for students at five schools, Such comparisons could be accomplished by looking at the samples two at a time and comparing the means. Although feasible, this is an inefficient method of comparison for more than two populations.

One reason for its inefficiency is that the standard deviation for the difference between the two, sample means is not calculated from all the samples but instead uses samples only from the two populations under immediate consideration. Second, we feel intuitively that we shall almost find a significant difference between at least one pair of means (the extreme ones, e.g.) if we consider enough identical populations. We can no longer trust our level of significance.

Therefore, instead of using two samples at a time, we wish to make a single test to find out whether the students from the five schools are from five populations having the same population mean. The null hypothesis we wish to test is:

HO: = μ2 = P3 = P4 = μ5. Our reason for making such a test is not that we think five population means may be equal. They probably are unequal. However, if a preliminary test fails to show that the means are unequal, we may feel that the differences are rather small and do not warrant further investigation.

Completely randomized design is primarily concerned with tests for population means. To study the means, it is necessary to "analyze the variance".

Let us consider a particular example. An investigator wished to study the effect of fertilizers on the yield of corn. He divided the field into 24 rectangular plots of the same size and shape. His four treatments consisted of (1) no fertilizer, (2) $K_20$ + N, 3) $K_20+P_20_5$, and (4) $N+P_20_5$. He assigned

**Table 1: Yields of Corn Under Four Treatments**

| Observation | 1 | 2 | 3 | 4 | 5 | 6 | $\sum_{j=1}^{Y} ij$ | $\overline{Y}_{i.}$ |
|---|---|---|---|---|---|---|---|---|
| Treatments | | | | | | | | |
| 1. Control | 99 | 40 | 61 | 72 | 76 | 84 | 432 | 72 |
| 2. $K_2O + N$ | 96 | 84 | 82 | 104 | 99 | 105 | 570 | 95 |
| 3. $K_2O + P_2O_5$ | 63 | 57 | 81 | 59 | 64 | 72 | 396 | 66 |
| 4. $N + P_2O_5$ | 79 | 92 | 91 | 87 | 78 | 71 | 498 | 83 |

$\sum_{i=1}^{4} \sum_{j=1}^{6} Y_{ij} = 1,896$ , $\overline{Y}.. = 79$

each treatment at random to six of the 24 plots. The yields in bushels per acre, are presented in table 1. Here we use i to denote the number of the treatment and j to refer to the number of the observation. The sample means are designated by Y;., where the dot in the second subscript position indicates that we have averaged over the second subscript j; in other words Y;., is the mean of all the observations of the ith sample**.** The overall mean is denoted by Y.., the two dots indicate that the mean is obtained by summing over both subscripts and then dividing by the total number.

The investigator's purpose is usually to learn to something about the populations from which the samples are drawn. To accomplish this, he needs an underlying model.
For this Model, we assume that our four samples, each consisting of six corn yields, are independent random samples from four populations, that each of the four populations has a normal distribution, and, finally, that the variance of the four populations are equal. The investigator should consider these assumptions carefully.
The four populations means may be designated $\mu_1, \mu_2, \mu_3,$ and $\mu_4$, We arbitrarily divide each of these four means into two parts. The first part is the mean of the four population means, which we call the "overall mean," and the second part is the difference between the mean of each population and the overall mean. In symbols, the means are written as
$\mu_1 = \mu+\alpha_1, ....., \mu+\alpha_4,$ where $\mu$ denotes the overall mean and $\alpha_i$ is the difference $\mu_i - \mu$. The overall mean has been chosen in such a way that $\sum_{i=1}^{a} \alpha_i = 0$, where a is the number of treatments (in this case, 4). If in our example the four populations means were $\mu_1$ = 70, $\mu_2$ = 100, $\mu_3$ = 70, $\mu_4$ = 80, we would have $\mu$ = (70 + 100 + 70 + 80)/4 = 80. The population means could then be written.

$\mu_1 = \mu+\alpha_1 = 80 + (-10)$
$\mu_2 = \mu+\alpha_2 = 80 + (+20)$
$\mu_3 = \mu+\alpha_3 = 80 + (-10)$
$\mu_4 = \mu+\alpha_4 = 80 + (+0).$

The difference $\alpha_i = \mu_i - \mu$ is often called the effect of the particular treatment. It should not be confused with $\alpha$, the probability of rejecting a null hypothesis which is true. In the example, the $\alpha_i$ sum to zero and this is always the case.
We think, then, of a population mean as the sum of two parts : an overall mean (which, as an average of the four population means, may be of little interest to us) and the part that we attribute to the particular treatment. Using this notation, we can summerize the model just described by saying that each observation $Y_{ij}$ is an independent observation from a normally distributed population whose mean is $\mu+\alpha_i$ and whose variance is denoted by $\sigma^2_e$. This can be written

$$Y_{ij} \text{ IND } (\mu + \alpha_i, \sigma_e^2), \sum_{i=1}^{a} \alpha_i = 0, \; i = 1,.....,a; \; j = 1,.....n,$$

Where a is the number of treatments, n is the number of observations on each treatment, and IND is read as "independently normally distributed." In this model, we are studying only these particular a populations.
An equivalent way of writing down the model which is often convenient is

$$Y_{ij} = \mu + \alpha_i + \in_{ij}$$
$$\sum_{i=1}^{a} \alpha_i = 0; \in_{ij} \text{ IND } (O, \sigma_e^2); = i = 1,.....,a; \; j = 1,.....,n$$

Here we think of a particular corn yield $Y_{ij}$ as made up of its population mean $\mu+\alpha_i$ plus whatever is left over, which we call $\in_{ij}$. In our data, for example, $Y_{23}$= 82=80+20+(-18); therefore, if the mean yield for the second population is 100, $\in_{23}$ = -18. These 24 deviations form a random sample of 24 $\in_{ij}$, all from a normal population with zero mean and variance $\sigma^2_e$. Note that because in practice we do not know the population means, we cannot know the values of the 24 $\in_{ij}$'s. They can however, be estimated from the sample data.

**Format for Analysis of Variance Table for Completely Randomized Design**

| Source of Variation | Sum of Squares | D F | Mean Square | Computed F value | Table F value |
|---|---|---|---|---|---|
| Due to Treatment | $SS_a = n\sum_{i=1}^{ai} (\bar{Y}_{i\cdot} - \bar{Y}_{\cdot\cdot})^2$ | $a - 1$ | $MS_a = SS_a/(a-1)$ | $MS_a/MS_r$ | Read at α% from table |
| Residual | $SS_r = \Sigma\Sigma(Y_{ij} - \bar{Y}_{i\cdot})^2$ | $a(n-1)$ | $MS_r = SS_r/a(n-1)$ | | |
| Total | $SS_t = \Sigma\Sigma(Y_{ij} - \bar{Y}_{\cdot\cdot})^2$ | $an - 1$ | | | |

**Analysis of Variance Table For Corn Yields (Our Illustration)**

| Source of Variation | Sum of Squares | D F | Mean Squares | Computed F value | Table F value |
|---|---|---|---|---|---|
| Due to treatment | 2940 | 3 | 980 | 5.99 | 3.10(at 5%) |
| Residual | 3272 | 20 | 163.6 | | |
| Total | 6212 | 23 | | | |

It may be pointed out here that the format table above is the "Analysis of Variance" Table (often called as ANOVA table)-the general table for a completely randomized design with equal number of observations on each treatment.

The second one is the ANOVA table for our illustration on the yield of corn as influenzed by the four treatments.

We proceed finally to an F test of the null hypothesis that all the four population means are the same. This amounts to:

Ho: $\mu_1=\mu_2=\mu_3=\mu_4$

Since the calculated F value exceeds the table F value at 5% level, we reject Ho and conclude that the four population means are not the same and that the treatment levels are different.

**Activity 1**

Suppose an advertising firm wants to test three different themes-Theme A, Theme B, Theme C, for a brand on a sample target audience of 60 people. The firm measures the response on a 0-11 scale. (0 denotes no interest and 11 denote very high interest in purchasing the advertised brand). How will you go about evaluating the three themes by performing a completely randomized design model? State the null hypothesis. You may assign equal number of observations to each treatment (theme). State clearly the assumptions of this model.

## 3.3 RANDOMIZED COMPLETE BLOCK DESIGN

In the completely randomized design, treatments are assigned at random. For example, if the treatments are three drugs and there are 24 patients, eight patients are assigned at random to each of the three treatments.

The 24 patients may vary widely in initial condition, and their initial condition may affect their response to the drugs. In the completely randomized design, we try to take care of these differences among the patients by assigning them at random into groups of eight patients. Unfortunately, it is possible that all the patients receiving drug 1 may be comparatively healthy and all those receiving drug 2 may be comparatively unhealthy, even though the assignment was randomly made. By randomization, however, at least we have given each drug an equal chance with respect to the initial condition of the groups. Further more, we can expect that if the experiment is large enough, randomization will roughly equalize the initial condition of the three groups. Besides initial condition, the experimenter may feel that other factors might influence the response to the drugs (e.g., age or weight).

A block design is a much used method for dealing with factors that are known to be important and which the investigator wishes to eliminate rather than to study.

In the randomized complete block design, still with three treatments and 24 patients, the patients are divided into eight blocks, each consisting of three patients. These blocks are farmed so that each block is as homogeneous as possible. Each block consists of as many experimental units as there are treatments-three, in this case. The blocks might be easily formed on the basis of age, for example, with blocks 1 and 8 consisting of the three youngest and the three oldest patients, respectively. The individuals in a particular block are as alike as possible. On the other hand, there may be wide differences between the individuals for different blocks.

After the blocks are formed, the three drugs are assigned at random to the three patients within each block. If the blocking has been done on a factor such as initial condition, and if initial condition is important in determining the level of the response, the responses to the drugs will differ widely from block to block. However, because each drug is used exactly once in each block, the design is balanced and the mean treatment responses to the three drugs will be comparable. The differences observed among the drugs should be largely unaffected by initial condition. Below are the eight blocks with a possible treatment assignment.

Block Number

| Patient Number | 1 | 2 | .... | 8 |
|---|---|---|---|---|
| 1 | Drug 3 | Drug 2 | .... | Drug 1 |
| 2 | Drug 1 | Drug 1 | .... | Drug 3 |
| 3 | Drug 2 | Drug 3 | .... | Drug 2 |

When the data are gathered, they are arranged in rows according to treatment (drug), and we have

Block Number

| Drug Number | 1 | 2 | .... | 8 |
|---|---|---|---|---|
| 1 | $Y_{11}$ | $Y_{12}$ | .... | $Y_{18}$ |
| 2 | $Y_{21}$ | $Y_{22}$ | .... | $Y_{28}$ |
| 3 | $Y_{31}$ | $Y_{32}$ | .... | $Y_{38}$ |

Note that this is a balanced design-each treatment occurs once in each block; thus if we obtain the mean response for drug 1 over the eight blocks, it will be comparable to the mean response for drugs 2 or 3.

This type of design is widely used. For example, industrial material frequently arrives in batches that tend to be homogeneous; thus a batch may be used as a block. In laboratories, to take another case, results often differ from day to day, and therefore days frequently serve as blocks. A common practice is to block out technician effect. In agricultural experiments, the blocks are sometimes separate plots of land. The technique of randomized blocks is a very useful one for removing unwanted variation. Frequently the investigator can obtain significant differences among treatment effects using a smaller sample size with the randomized complete block design than with a completely randomized design.

In planning an experiment it is important to identify in advance the factors that may introduce unwanted variation in the response (i.e., variation not due to the treatment effect) and to block accordingly. If results differ from day to day, days become blocks, and the design should be balanced within days. Each treatment must occur exactly the same number of times within each day, and it must be assigned at random within the day.

**The model**

We assume that each observation can be described as follows:

$$Y_{ij} = \mu + \alpha_i + \beta_j + \epsilon_{ij} \quad i = 1,.....,a; \quad j = 1,.....b,$$

with

$$\sum_{i=1}^{a} \alpha_i = 0, \sum_{j=1}^{b} \beta_j = 0 \qquad \text{and} \qquad \epsilon_{ij} \text{ IND } (0, \sigma_e^2).$$

Here $\alpha_i$ is the effect of the ith treatment and $\beta_j$ is the effect of the jth block. Note that the treatments correspond to rows and the blocks correspond to columns. Thus $Y_{ii}$ with i=2 and j=3 denotes the second treatment and the third block.

From the model as just given, we can read four assumptions:

1) The response to the ith treatment in the jth block $Y_{ii}$ is from a normal distribution. (There are ab distributions.)
2) The means of the ab normal distributions can be expressed in the form $\mu+\alpha_i+\beta_j$. This property is often called additivity, or alternatively, no interaction.
3) The variances of the ab populations are equal. This property is known as homoscedasticity.

4) The $\in_{ij}$ (deviations from the means) are statistically independent. If we know that $\in_{11}$ is large, we have no reason to expect $E_{12}$ to be small (or large, for that matter).

As an example of a randomized complete block design, we use the following data.

Randomized complete block design with three treatments (3 coupon plans and four blocks (Store sizes)

| Data represents campa cola sales in number of cases. | | | | | |
|---|---|---|---|---|---|
| | | Block | | | |
| Treatment | Store 1 | Store 2 | Store 3 | Store 4 | $\overline{Y}_{i\cdot}$ |
| Coupon plan 1 | 20 | 18 | 15 | 11 | 16 |
| Coupon plan 2 | 17 | 14 | 13 | 8 | 13 |
| Coupon plan 3 | 14 | 10 | 7 | 5 | 9 |
| $\overline{Y}_{\cdot j}$ | 17 | 14 | 11.7 | 8 | $Y_{\cdot\cdot} = 12.7$ |

Stores are blocked on the basis of the size because we expect some variation in cola sales due to the size of the stores. Store 1 is the largest size, followed by store 2 next largest, and so on. Treatments (coupon plan 1, coupon plan 2, and coupon plan 3) are randomly assigned to test units within each block.

Hypothesis of primary interest-There is no difference amongst treatment effects.

Hypothesis of secondary interest-There is no difference amongst block effects.

Format of Analysis of Variance Table for Randomized Complete Block Design

**Format of Analysis of Variance Table for Latin Square Design**

| Source of Variation | Sums of Squares | D F | Mean Squares | Computed F value | Table F value |
|---|---|---|---|---|---|
| Due to Treatments | $SS_a = b\sum_{i=1}^{a} (\overline{Y}_{i\cdot} - \overline{Y}_{\cdot\cdot})^2$ | $a-1$ | $MS_a = SS_a/(a-1)$ | $MS_a/MS_n$ | Read at |
| Due to Blocks | $SS_b = a\sum_{j=1}^{b} (\overline{Y}_{\cdot j} - \overline{Y}_{\cdot\cdot})^2$ | $b-1$ | $MS_b = SS_b/(b-1)$ | $MS_b/MS_n$ | $\alpha$ % from |
| Residual | $SS_n = \sum_{i=1}^{a}\sum_{j=1}^{b} (Y_{ij} - \overline{Y}_{i\cdot} + \overline{Y}_{\cdot j} + \overline{Y}_{\cdot\cdot})^2$ | $(a-1)(b-1)$ | $MS_n = SS_n/(a-1)(b-2)$ | | table |
| Total | | $ab-1$ | | | |

**ANOVA Table for Our Illustration**
**Coupon Experiment with Blocking for Store Size**

| Source of Variation | Sum of Squares | D F | Mean Squares | Computed F value | Table F value |
|---|---|---|---|---|---|
| Due to Treatments | 98.7 | 2 | 49.4 | 70.6 | 5.14 (at 5%) |
| Due to Blocks | 129.8 | 3 | 43.3 | 61.9 | 4.76 (at 5%) |
| Residual | 4.2 | 6 | 0.7 | | |
| Total | 232.7 | 11 | | | |

Since the calculated F value exceeds the table F value at 5% level of significance both for treatment effect and block effect, we reject the null hypotheses and conclude that treatment effect is significant and also the block effect is significant.

We conclude that after blocking for store size, the coupon plans do make a difference in sales of cola.

**Activity 2**

What are the improvements made in the randomized complete block design over the completely randomized design?

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## 3.4 LATIN SQUARE DESIGN

In the randomized complete block design, the effect of a single factor was removed. It is occasionally possible to remove .the effects of two factors simultaneously in the same experiment by using the Latin Square design. In order to use the Latin square design, however, it is necessary to assume that no interaction exists between the treatment effect and either block effect. In addition, the number of treatments must be equal to the number of categories for each of the two factors. We might, for instance, wish to test four detergents, using four methods of application, at four hospitals. A 4X4 Latin square design could then be employed, using each detergent exactly once with each method and exactly once in each hospital. The assignment of detergent could be made as shown in the following table; the roman numeral in the ith row and jth column indicates the detergent that will be used by the ith application method in the jth hospital. As assigned in the Table 2, the first detergent is used in hospital 1 by method 1, in hospital 2 by method 4, in hospital 3 by method 3, and in hospital 4 by method 2. Only 16 observations are needed because of the balanced arrangement used and because of the assumption of no interaction.

**Table 2: Latin Square Design Hospital Number**

| | 1 | 2 | 3 | 4 |
|---|---|---|---|---|
| 1) | I | II | III | IV |
| 2) | II | III | IV | I |
| 3) | III | IV | I | II |
| 4) | IV | I | II | III |

**Table 3: Data and Prelimitary Calculations on Four Detergents**

| | 1 | 2 | 3 | 4 | $\bar{Y}_{i..}$ |
|---|---|---|---|---|---|
| 1) | 8.7(I) | 9.2(II) | 11.6(III) | 9.1(IV) | 9.650 |
| 2) | 7.5(II) | 12.7(III) | 4.6(IV) | 7.3(I) | 8.025 |
| 3) | 14.0(III) | 9.2(IV) | 5.1(I) | 6.7(II) | 8.750 |
| 4) | 11.3(V) | 8.7(I) | 4.0(II) | 12.9(III) | 9.225 |
| $\bar{Y}_{.j.}$ | 10.375 | 9.950 | 6.325 | 9.000 | $\bar{Y}_{...}$=8.9125 |

$\bar{Y}_{.1} = (8.7+8.7+5.1+7.3)/4 = 7.45$
$\bar{Y}_{.2} = (7.5+9.2+4.0+6.7)/4 = 6.85$
$\bar{Y}_{.3} = (14.0+12.7+11.6+12.9)/4 = 12.80$
$\bar{Y}_{.4} = (11.3+9.2+4.6+9.1)/4 = 8.55$

The above data table gives the measurements obtained from using the four detergents.

**The Model**

It is assumed that each observation $Y_{ijk}$ can be expressed as follows:

$$Y_{ijk} = \mu + \alpha_i + \beta_j + \gamma_k + \epsilon_{ijk},\ i = 1,....,p;\ j = 1,....,p;\ k = 1,.....,p$$

Where

$$\sum_{i=1}^{p} \alpha_i = \sum_{j=1}^{p} \beta_j = \sum_{k=1}^{p} \gamma_k = 0 \text{ and } \epsilon_{ijk} \text{ IND } (0, \sigma_e^2)$$

Here $\mu$ denotes the overall mean response for all p treatments using all $p^2$ combinations of the two factors; there are thus $p^3$ population means altogether. In the detergent example. $\mu$ is the average of $p^3 = 64$ population means. Each $\alpha_i$ is the part of the mean that is due to the ith row. (method); $\beta_j$ is the part of the mean that is due to the jth column (hospital); $y_k$ is the part of the mean due to the kth treatment (detergent).

There are $p^3$ populations, but we have economized by making an observation on only $p^2$ populations. In the detergent example, instead of 64 observations, we have only 16.

The assumptions implied by the model are as follows:

1) The $p^3$ populations are normally distributed.
2) They have equal variances.
3) There is no interaction.

4) The $\in$'s are independent of one another.

The assumptions of no interaction is implicit in the statement of the model. We have stated that the mean of $Y_{ijk}$ is $\mu+\alpha_i+\beta_j+Y_k$; this indicates that the first detergent has the same effect no matter which hospital is involved and no matter which method is used. Each hospital performs equally well with each method.

The assumption of independent $\in$'s would be violated if, for example, half the experiment were conducted at one time and half were conducted six months later.

**Format of Analysis of Variance Table for Latin Square Design**

| Source of Variation (1) | Sums of Squares (2) | DF (3) | Mean Squares (4) | F Computed (6) | F Tabled (7) |
|---|---|---|---|---|---|
| Due Rows | $SS_a=p\sum_{i=1}^{p}(\overline{Y}_{i..}-\overline{Y}...)^2$ | $p-1$ | $MS_a=SS_a/(p-1)$ | $MS_a/MS_r$ | Read at |
| Due Columns | $SS_b=p\sum_{j=1}^{p}(\overline{Y}_{.j.}-\overline{Y}...)^2$ | $p-1$ | $MS_b=SS_b/(p-1)$ | $MS_b/MS_r$ | $\alpha-1$ from |
| Due Treatments | $SS_c=p\sum_{k=1}^{p}(\overline{Y}_{..k}-\overline{Y}...)^2$ | $p-1$ | $MS_c=SS_c/(p-1)$ | $MS_c/MS_r$ | table |
| Residual. | $SS_n=\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}Y_{ijk}$ $-\overline{Y}_{i..}-\overline{Y}_{j..}-\overline{Y}_{k..}+2\overline{Y}...)^2$ | $(p-1)(p-2)$ | $MS_r=SS_r/(p-1)(p-2)$ | | |
| Total | $\sum_{i=1}^{p}\sum_{j=1}^{p}\sum_{k=1}^{p}(Y_{ijk}-\overline{Y}...)^2$ | $p^2-1$ | | | |

**Analysis of Variance Table for Detergent Data**

| Source of Variation (1) | Sums of Squares (2) | DF (3) | Mean Squares (4) | F Computed (6) | F Tabled (7) |
|---|---|---|---|---|---|
| Due Method | 5,822 | 3 | 1,941 | 1.44 | 4.76 (at 5%) |
| Due Hospital | 39,672 | 3 | 13,224 | 9.78 | 4.76 (at 5%) |
| Due Detergent | 86,548 | 3 | 28,849 | 42.68 | 4.76 (at 5%) |
| Residual | 4,055 | 6 | 1,676 | | |
| Total | 136,097 | 15 | | | |

The hypotheses to be tested are:

$H_0$: There are no differences among the row means

$H_0$: There are no differences among the column means

$H_0$ : There are no differences among the treatment means

In our example, the rows are methods, columns are hospitals, and the treatments are the detergents.

The calculated F exceeds the table F at 5% level for hospitals and detergents. Reject $H_0$ and conclude that there are differences in performance among the four detergents; there are differences among the hospitals.

The calculated F is less than the table F at 5% level in the case of the methods. Do not reject $H_o$ and infer that there may be no differences among the four application methods.

The advantage of a Latin square design over the randomized complete block design is that the effect of a second factor is eliminated without increasing the size of the experiment, provided always that no interactions exist.

**Activity 3**

State the Latin square model with the assumptions clearly embodied in the model.

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## 3.5 FACTORIAL DESIGN

Often a researcher can use a single experiment advantageously to study two or more different kinds of treatments. For example, in investigating performance of two types of seeds, he may wish to vary the level of fertilizer used during the experiment. If he chose three levels of fertilizer-low, medium, and high-one factor would be "type of seed", the second factor "level of fertilizer". A factorial design, with two factors, would consist of employing all six treatments formed by using each type of seed with each level of fertilizer. Factorial designs can involve more than two factors; however, we consider here the case of two factors only.

A factorial design can also be used in a survey. For example, we might wish to compare three methods of teaching operations research, and at the same time compare the fast four grades. We might have records on standardized tests for two classes in each grade taught by each method. The class mean improvement from initial test to final test could be the measure of success. Our data would then consist of two observations on each of 12 (3 x 4) different treatment combinations.

The characteristic of the factorial design is that every level of one factor is used in combination with every level of the other factor. The design is effective for studying the two factors in combination. This implies that factorial designs are appropriate in finding out whether interactions exist between factors.

Some factors can be measured quantitatively, and different levels for them are chosen on an ordered scale; level of fertilizer, dosage level; and temperature are all factors of this type. Other factors involve no obvious underlying continuum and can be said to be qualitative; drug and type of seed are factors of the second type.

As an example of a two-factor design, let us take a study on rye yields involving two types of seed, each used at three fertilizer levels-low, medium, and high. There were available 24 small plots of ground, and the six treatment combinations were assigned at random to 24 plots, 4 plots receiving each treatment. Two tables (Tables 3 & 4) are given below. Table 3 represents the notation and the second gives the actual observations replacing their symbols.

**Table 3: Notation for Two Factor Design**

| | Fertilizer Level | | | |
|---|---|---|---|---|
| | Low (1) | Medium (2) | High (3) | |
| Seed Type 1 | $Y_{111}$<br>$Y_{112}$<br>$Y_{113}$<br>$Y_{114}$ | $Y_{121}$<br>$Y_{122}$<br>$Y_{123}$<br>$Y_{124}$ | $Y_{131}$<br>$Y_{132}$<br>$Y_{133}$<br>$Y_{134}$ | |
| | $\overline{Y}_{11\cdot}$ | $\overline{Y}_{12\cdot}$ | $\overline{Y}_{13\cdot}$ | $\overline{Y}_{1\cdot\cdot}$ |
| 2 | $Y_{211}$<br>$Y_{212}$<br>$Y_{213}$<br>$Y_{214}$ | $Y_{221}$<br>$Y_{222}$<br>$Y_{223}$<br>$Y_{224}$ | $Y_{231}$<br>$Y_{232}$<br>$Y_{233}$<br>$Y_{234}$ | |
| | $\overline{Y}_{21\cdot\cdot}$ | $\overline{Y}_{22\cdot}$ | $\overline{Y}_{23\cdot}$ | $\overline{Y}_{2\cdot\cdot}$ |
| | $\overline{Y}_{\cdot 1\cdot}$ | $\overline{Y}_{\cdot 2\cdot}$ | $\overline{Y}_{\cdot 3\cdot}$ | $\overline{Y}_{\cdots}$ |

**Table 4: Yields of Rye and Their Means (Bushels/Acre)**

| | Fertilizer Level | | | |
|---|---|---|---|---|
| | Low | Medium | High | |
| Seed Type 1 | 14.3 | 18.1 | 17.6 | |
| | 14.5 | 17.6 | 18.2 | |
| | 11.5 | 17.1 | 18.9 | |
| | 13.6 | 17.6 | 18.2 | |
| | $\overline{Y}_{11\cdot}=13.475$ | $\overline{Y}_{12\cdot}=17.600$ | $\overline{Y}_{13\cdot}=18.225$ | $\overline{Y}_{1\cdot\cdot}=16.433$ |
| 2 | 12.6 | 10.5 | 15.7 | |
| | 11.2 | 12.8 | 17.5 | |
| | 11.0 | 8.3 | 16.7 | |
| | 12.1 | 9.1 | 16.6 | |
| | $\overline{Y}_{21\cdot}=11.725$ | $\overline{Y}_{22\cdot}=10.175$ | $\overline{Y}_{23\cdot}=16.625$ | $\overline{Y}_{2\cdot\cdot}=12.842$ |
| | $\overline{Y}_{\cdot 1\cdot}=12.600$ | $\overline{Y}_{\cdot 2\cdot}=13.888$ | $\overline{Y}_{\cdot 3\cdot}=17.425$ | $\overline{Y}_{\cdots}=14.638$ |

A response (in this case yield) is denoted by $Y_{ijk,}$ where i indicates the seed type, j

indicates the fertilizer level, and k is the observation number. For example, $Y_{213}$, is the yield in the third of the four plots that used seed type 2 and a low fertilizer level. The cell means, denoted by $\overline{Y}_{ij.}$ are the means for each treatment combination. The mean of all 12 observations on the i th seed type is $\overline{Y}_{i..}$; the mean of all 8 observations on the jth fertilizer is $Y_{.j.}$; the overall mean of the 24 observations is $\overline{Y}_{...}$.

**The Model**

Where

$\mu$ = Overall means response;

$\alpha_i$ = effect of the ith level of the first factor

$\beta_j$ = effect of the jth level of the second factor

$(\alpha\beta)_{ij}$ = interaction between the ith level of the first factor and the jth level of the second factor.

$\in_{ijk}$ = deviation of $Y_{ijk}$ from the population mean response for the ij th population.

$$\sum_{i=1}^{a} \alpha_i = \sum_{j=1}^{b} \beta_j = \sum_{i=1}^{a} (\alpha\beta)_{ij} = \sum_{j=1}^{b} (\alpha\beta)_{ij} = 0 \text{ and } \in_{ijk} \text{ IND } (0, \sigma_e^2)$$

Format Of Analysis Of Variance For Two Factor Factorial Design

| Source of Variation (1) | Sums of Squares (2) | DF (3) | Mean Squares (4) | F Computed (6) | F Tabled (7) |
|---|---|---|---|---|---|
| Due A | $SS_a = bn\sum_{i=1}^{a} (\overline{Y}_{i..} - \overline{Y}_{...})^2$ | a−1 | $MS_a = SS_a/(a-1)$ | $MS_a/MS_r$ | Read at α% from table |
| Due B | $SS_b = an\sum_{j=1}^{b} (\overline{Y}_{.j.} - \overline{Y}_{...})^2$ | b−1 | $MS_b = SS_b/(b-1)$ | $MS_b/MS_r$ | |
| Due AB | $SS_{ab} = n\sum_{i=1}^{a}\sum_{j=1}^{b} (\overline{Y}_{ij.} - \overline{Y}_{i..} - \overline{Y}_{.j.} + \overline{Y}_{...})^2$ | (a−1) (b−1) | $MS_{ab} = SS_{ab}/(a-1)(b-1)$ | $MS_{ab}/MS_r$ | |
| Residual | $SS_r = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} Y_{ijk} - \overline{Y}_{ij})^2$ | ab(n−1) | $MS_r = SS_r/ab(n-1)$ | | |
| Total | $SS_t = \sum_{i=1}^{a}\sum_{j=1}^{b}\sum_{k=1}^{n} (Y_{ijk} - \overline{Y}_{...})^2$ | abn−1 | | | |

ANOVA Table For Our Illustration Problem On Two Types Of Seed With Three Fertilizer Levels

| Source of Variation (1) | Sums of Squares (2) | DF (3) | Mean Squares (4) | F Computed (6) | F Tabled (7) |
|---|---|---|---|---|---|
| Due Seeds | 77.4 | 1 | 77.4 | 63.3 | 4.41 (at 5%) |
| Due Fertilizer Levels | 99.9 | 2 | 49.9 | 40.9 | 3.55 (at 5%) |
| Seed Fertilizer Interaction | 44.1 | 2 | 22.1 | 18.0 | 3.55 (at 5%) |
| Residual | 22.0 | 18 | 1.2 | | |
| Total | 243.4 | 23 | | | |

From the last two columns of the table, we conclude (by comparing the computed F value with table F value) the following:

1) Differences exist between yields from the two seed types.
2) Differences exist among yields from the three levels of fertilizer.
3) Interactions exist between seed type and fertilizer level.

* computed F exceeds table IF at 5% significance level for all the three above, leading to rejection of null hypothesis of no difference.

**Activity 4**

What are the specialities of the factorial designs?

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## 3.6 ANALYSIS OF COVARIANCE

Analysis of covariance is a combination of the two techniques-analysis of variance and regression. It is the simultaneous study of several regressions.

The purpose of analysis of covariance is to remove the effect of one or more unwanted factors in an analysis of variance. For example, in studying the heights of three populations of children (cyanotic heart disease children, sibs of heart-disease children, and "well children"), we may wish to eliminate the effect of age. A variable whose effect one wishes to eliminate by means of a covariance analysis is called a covariate or a concomitant variable.

The analysis of covariance has the following uses:

1) To increase precision in randomized experiments. In such applications the covariate X is a measurement taken on each experimental unit before the treatments are applied that predicts to some degree the final response Y on the unit. By adjusting the treatment means after giving regard to the concomitant variable, we obtain a lower experimental error and more precise comparisons among the treatments. This is probably the commonest use of covariance.

2) To adjust for sources of bias in observational studies. An investigator is studying the relationship between obesity in workers and the physical activity required in their occupations. If obesity is linearly related to age, differences found in obesity among different occupations may be due in part to these age differences. Consequently he adjusts for a possible source of bias in his comparison among occupations.

3) To throw light on the nature of treatment effects in randomized experiments. In an experiment on the effects of soil fumigants on nematodes, which attack some farm crops, significant differences between fumigants were found both in the numbers of nematode cysts and in the yields of crop. This raises the question: can the differences in yields be ascribed to the differences in numbers of nematodes? Analysis of covariance can provide answer to this question.

4) To study regressions in multiple classifications. For example, an investigator is studying the relationship between expenditure per student in schools (Y) and percapita income (X) in large cities. If he has data for a large numbers of cities for each of four years, he may want to examine whether the relationship is same in different sections of the country, or whether it remains the same from year to year. Sometimes the question is whether the relationship is straight or curved.

Because analysis of covariance techniques are complicated, we confine ourselves mainly to the purpose and uses as mentioned above and highlight a practical situation where the covariance analysis can be employed. Those who would like to go deeper, may refer to the book on "Statistical Methods" by Snedecor & Cochran. It is also strongly suggested that you should use SPSS, or Biomedical Programmes for analysis and interpretation of covariance. These packages provide a complete and flexible analysis.

As an example consider the following data on language scores (Y) for students taught by three different methods. Measurements on IQ(X) are also available. Since the students are not assigned at random to the three teaching methods, there may easily be differences in IQ scores among the three groups.

Our objective is to examine differences in language scores among the three methods after the effect of IQ has been eliminated. Otherwise, if we claim that method I is superior to method II, we may not be able to refute the statement that the observed difference between the methods occurs because the IQs of the students using method I were higher than using method II.

**Table 5: Data on language scores (Y) using three teaching methods and IQ scores (X)**

| | | Student | | | | | | | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|
| Method | | 1 | 2 | 3 | 4 | 5 | 6 | 7 | 8 | 9 | 10 |
| 1 | Y | 72 | 75 | 85 | 70 | 73 | 86 | 92 | 68 | 91 | 75 |
| | X | 87 | 119 | 121 | 112 | 100 | 133 | 135 | 109 | 139 | 105 |
| 2 | Y | 90 | 98 | 73 | 88 | 83 | 90 | 98 | 81 | 84 | 79 |
| | X | 110 | 128 | 117 | 94 | 107 | 125 | 111 | 80 | 123 | 95 |
| 3 | Y | 59 | 65 | 67 | 71 | 59 | 61 | 58 | 70 | 59 | 48 |
| | X | 95 | 120 | 125 | 107 | 85 | 98 | 100 | 138 | 112 | 90 |

**The Model**

$Y_{ij} = \alpha_{io} + \beta_i X_{ij} + \epsilon_{ij}$, i=1,.....a; j=1,.....$n_i$,

Where $\epsilon_{ij}$ IND $(0, \sigma^2_e)$. $Y_{ij}$ and $X_{ij}$ denote the Y and X values for the jth individual on the ith treatment. The number of treatment is a and the number of individuals in the ith sample is $n_i$. In our example, a=3, $n_1$=10, $n_2$=10, $n_3$=10. The model expressed above is clearly that of a separate linear regressions. The assumptions implied are exactly same as in a linear regression.

For our example, the standard F test of the analysis of covariance leads to the rejection of the null hypothesis of no difference among the three teaching methods. Therefore we infer that the three teaching methods do make difference on the language score after eliminating the effect of IQ. For the complete analysis, use SPSS, SAS, or BIO-medical programmes.

**Activity 5**

Explain briefly why do you need analysis of covariance in experimental designs?

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## 3.7 SUMMARY

We began the discussion with a brief introduction on the usefulness of the experimental designs in studying the causal relationship effectively. We have then introduced the simplest of the experimental designs-completely randomized design. The F test of the null hypothesis that no differences exist among the treatment means has been illustrated with a suitable example.

Then the randomized complete block design has been explained as a suitable design for removing an unwanted effect by blocking. The design has been illustrated through a problem on studying the effect of different coupon plans on sales of cola after blocking for store size.

It may be mentioned in the passing that the case of unequal sample size has not been separately dealt with. The procedure remains the same except for some minor modifications in the computations and assumption. Those interested may please go through MS-61 Block 5 for the procedure. Also the random effect and mixed effect models have not been discussed separately and it is suggested that you use the software package which has all these options built into the programme. It may also be added here that the underlying principal of ANOVA (breaking the total variance into meaningful component variance and residual variance) remains unaltered irrespective of any design model. That is why every alternative model has not been individually focused and instead the typical one has been illustrated.

The next in order we have brought out is Latin square design. This is introduced as a powerful design that can be used to remove the effects of two unwanted factors simultaneously when effects are additive and no interaction exists.

We have then moved on to the factorial design which we have conceptualised in case of two factors. We have emphasised that this design is very effective for student the two factors in combination from the angle of finding whether the interaction exist between factors.

The last model in this unit we have discussed is the analysis of covariance. This is useful technique in experiments or in surveys when the investigator wishes to red the variance and to remove the effect of a factor (concomitant variable or covariance such as age, which may have an appreciable effect on his response variable.

## 3.8 SELF-ASSESSMENT EXERCISES

1) State clearly the assumptions embodied in all the experimental designs-completely randomized design, randomized complete block design, Latin square design, factorial design, and analysis of covariance.

2) The marketing research department of the Gamma adhesive company is attempting to find some attribute of their gummed labels that can be merchandised as being superior to competitive brands. The manager of the department feels that their strength of their adhesive represents a good promotional point in increasing sales. Accordingly, samples of the company's adhesive and three other brands are tested by an independent research company The "strength indices" of the four products are as follows.

| Trial | Gamma Adhesive | Competitive Adhesive | | |
|---|---|---|---|---|
| | | X | Y | Z |
| 1 | 35 | 32 | 22 | 24 |
| 2 | 11 | 29 | 18 | 19 |
| 3 | 28 | 17 | 23 | 26 |
| 4 | 26 | 24 | 17 | 19 |
| 5 | 32 | 15 | 19 | 22 |

a) Test the null hypothesis that the means of all treatments are equal

b) Assume now that the trials can be treated as blocks, perform the analysis of the randomized complete block design and compare your answer with part (a)

c) Discuss clearly the advantages of the randomized complete block design over the completely randomized design in the context of this example.

3) A researcher has carried out the following Latin square design:

| | $B_1$ | $B_2$ | $B_3$ | $B_4$ |
|---|---|---|---|---|
| $A_1$ | $C_1 = 13$ | $C_2 = 16$ | $C_3 = 16$ | $C_4 = 14$ |
| $A_2$ | $C_4 = 9$ | $C_1 = 17$ | $C_2 = 20$ | $C_3 = 20$ |
| $A_3$ | $C_3 = 14$ | $C_4 = 19$ | $C_1 = 17$ | $C_2 = 21$ |
| $A_4$ | $C_2 = 15$ | $C_3 = 17$ | $C_4 = 18$ | $C_1 = 19$ |

The data above refer to unit sales.

Ai = Shelf height i,j,k = 1,2,3,4,...

Bj = Number of racings

Ck = Shelf fullness

a) Test the hypothesis that no significant differences exist among sales responses due to shelf height, number of facings, and shelf fullness.

b) Write the complete Latin square model for this problem.

4. Consider the following factorial layout:

| Direct Mail | Personal Selling Effort | | |
|---|---|---|---|
| | Level 1 | Level 2 | Level 3 |
| Level 1 | 40;33 | 49;47 | 56;60 |
| Level 2 | 37;40 | 47;51 | 62;56 |
| Level 3 | 51;47 | 51;60 | 73;76 |

a) Test the null hypothesis that there is no difference in sales due to personal selling effort and direct mail

b) Does a significant interaction exist between personal selling effort and direct mail advertising?

5) a) Distinguish clearly between analysis of variance and analysis of covariance.

b) Give one practical situation where analysis of covariance will have to be used.

## 3.9 FURTHER READINGS

Dunn Olive Jean and Virginia A Clarck, *"Applied .Statistics"* John Wiley and Sons.

Green Paul E and Donald S. Tull, *"Research for Marketing Decisions"* Prentice Hall of India, New Delhi.

Snedecor, George W, " *Stanslad MetbocV'* The IOWA State University Press, AMES, IOWA, USA (6th Edition).