ZIMSEC

For Performance Measurement

ZIMBABWE SCHOOL EXAMINATIONS COUNCIL (ZIMSEC)

ZIMBABWE GENERAL CERTIFICATE OF EDUCATION (ZGCE)

For Examination in November 2012 – 2017

O Level Syllabus

MATHEMATICS (4008/4028)

Subjects 4008/4028. MATHEMATICS

4008.This version is for candidates not using calculators

4028.This version is for candidates using calculators in Paper 2

Subjects 4008/4028 MATHEMATICS

1.0 **PREAMBLE**

This syllabus caters for those who intend to study mathematics and/or related subjects up to and beyond `O' level and for the mathematical requirements of a wide range of professions. The syllabus assumes the mastery of the Z.J.C. mathematics syllabus.

The syllabus is in two versions 4008 and 4028. Syllabus 4008 is the non-calculator version and syllabus 4028 is the calculator version.

2.0 **THE SYLLABUS AIMS**

To enable pupils to:

2.1 understand, interpret and communicate mathematical information in everyday life;

2.2 acquire mathematical skills for use in their everyday lives and in national development;

2.3 appreciate the crucial role of mathematics in national development and in the country's socialist ideology;

2.4 acquire a firm mathematical foundation for further studies and/or vocational training;

2.5 develop the ability to apply mathematics in other subjects;

2.6 develop the ability to reason and present arguments logically;

2.7 develop the ability to apply mathematical knowledge and techniques in a wide variety of situations, both familiar and unfamiliar;

2.8 find joy and self-fulfilment in mathematics and related activities, and appreciate the beauty of mathematics;

2.9 develop good habits such as thoroughness and neatness, and positive attitudes such as an enquiring spirit, open-mindedness, self-reliance, resourcefulness, critical and creative thinking, cooperation and persistence;

2.10 appreciate the process of discovery and the historical development of mathematics as an integral part of human culture.

3.0 **ASSESSMENT OBJECTIVES**

Students will be assessed on their ability to:

3.1 recall, recognise and use mathematical symbols, terms and definitions;

3.2 carry out calculations and algebraic and geometric manipulations accurately; check the correctness of solutions;

3.3 estimate, approximate and use appropriate degrees of accuracy;

3.4 read, interpret and use tables, charts and graphs accurately;

3.5 draw graphs, diagrams and constructions to given appropriate specifications and measure to a suitable degree of accuracy;

3.6 translate mathematical information from one form into another (e.g. from a verbal form to a symbolic or diagrammatic form);

3.7 predict, draw inferences, make generalisations and establish mathematical relationships from provided data;

3.8 give steps and/or information necessary to solve a problem;

3.9 choose and use appropriate formulae, algorithms and strategies to solve a wide variety of problems (e.g. agriculture, technology, science and purely mathematical contexts);

3.10 apply and interpret mathematics in daily life situations.

4.0 NOTES

4.1 MATHEMATICAL TABLES AND ELECTRONIC CALCULATORS

Mathematical tables and electronic calculators are prohibited in 4008/1 and 4028/1. However, the efficient use of mathematical tables is expected in 4008/2 and the efficient use of electronic calculators is expected in 4028/2. In 4028/2 mathematical tables may be used to supplement the use of the calculator.

Mathematical tables will be provided in the examination. A scientific calculator with trigonometric functions is strongly recommended.

4.2 MATHEMATICAL INSTRUMENTS

Candidates are expected to bring their own mathematical instruments to the examination. Flexi curves are not allowed.

UNITS

4.3.1. Sl units will be used in questions involving mass and measures; the use of the centimetre will continue.

4.3.2. The time of day may be quoted by using either the 12-hour or the 24-hour clock,

e.g. quarter past three in the morning may be stated as either 3.15 a.m. or 03 15;
quarter past three in the afternoon may be stated as either 3.15 p.m. or 15 15.

4.3.3. Candidates will be expected to be familiar with the solidus notation for the expression of compound units e.g. 5 cm/s for 5 centimetres per second, 13/gcm^3 for 13 grams per cubic centimetre.

5.0 METHODOLOGY

In this syllabus, teaching approaches in which mathematics is seen as a process and which build an interest and confidence in tackling problems both in familiar and unfamiliar contexts are recommended.

It is suggested that:

5.1 concepts be developed starting from concrete situations (in the immediate environment) and moving to abstract ones;

5.2 principles be based on sound understanding of related concepts; and whenever possible, be learnt through activity based and/or guided discovery;

5.3 skills be learnt only after relevant concepts and principles have been mastered;

5.4 the human element in the process of mathematical discoveries be emphasised;

5.5 an effort be made to reinforce relevant skills taught in other subjects;

5.6 pupils be taught to check and criticise their own and one another's work;

5.7 group work be organised regularly;

5.8 a deliberate attempt be made to teach problem-solving as a skill, with pupils being exposed to non-routine problem solving situations;

5.9 pupils be taught to identify problems in their environment, put them in a mathematical form and solve them e.g. through project work.

6.0 CONTENT/TEACHING OBJECTIVES

TOPIC		OBJECTIVES All pupils should be able to:
6.1 **NUMBER**		
6.1.1.1 Number concepts and operations.		
number types (including: directed numbers, fractions and percentages)	-	demonstrate familiarity with the notion of odd, even, prime, natural, integer, rational and irrational numbers, including surds,
	-	use of the number line;
	-	recognise equivalence between common/decimal fractions and percentages, convert from one to the other and use these three forms in appropriate contexts;
	-	use directed numbers in practical situations (e.g. temperature, financial loss/gain);
factors, multiples, HCF, LCM	-	find and use common factors/multiples, HCFs and LCMs of given natural numbers;
the four operations (+, −, ×, ÷) and rules of precedence	-	apply the four operations and rules of precedence on natural numbers, common/ decimal fractions, percentages, integers, surds and directed numbers (including use of brackets);
6.1.2. Approximations and estimates	-	use the approximation sign (≏, ≃ or ≈) appropriately
	-	make estimates of numbers and quantities, and of results in calculations;
	-	give approximations to a specified number of significant figures and decimal places;
	-	round off to a given accuracy;
	-	round off to a reasonable accuracy in the context of a given problem;

6.1.3. Limits of accuracy	-	obtain appropriate upper and lower bounds to solutions of simple problems given data to a specified accuracy(e.g. calculation of area of a rectangle).
6.1.4. Standard form	-	express in, and use the standard form A x 10^n where is an integer (including zero) and $1 \leq A < 10$;
6.1.5. Number bases, - bases 2,3,4,5,6,7,8, 9 and 10	-	do the following: - state and use place value; - add and subtract; - convert from one base to another;
6.1.6. Ratio, proportion and rates	-	use ratio, direct and inverse proportion (including use of unitary method) and rates (e.g. speed, cost per unit area);
6.1.7. Scales and simple map problems	-	find scales from given information;
	-	use given scales to calculate distances and areas;

6.2 **SETS**

6.2.1. Language and notation

• definition of a set	-	define sets by listing and describing e.g.,V={a,e,i,o,u} or V={vowels};
	-	define sets using the set builder notation e.g.A={x:x is a natural number}, B={(x,y): y=mx+c} , C={x:a < x < b}
• notation	-	correctly use symbols as follows: - is an element $\in$, - is not an element of, $\notin$, - number of elements in set A,n(A), - complement of set A, A´, - the universal set, $\in$, - the null set,{} or $\varnothing$, - A is a proper subset of B, $A \subset B$, - A is contained in B, $A \subset B$,

- B contains A, $B \supset A$

- A is a subset of A, $A \subseteq A$,

- Φ is a subset of A, $\Phi \subseteq A$,

- union of A and B, $A \cup B$

- intersection of A and B, $A \cap B$;

- use the idea of complement of a union or an intersection;

- use the following symbols $\subseteq$,$\not\subset$, $\not\supset$, and $\not\subseteq$,

- use sets and Venn diagrams to solve problems involving no more than three sets and the universal set;

6.3 **CONSUMER ARITHMETIC**

6.3.1.

- interpret data (including data on real life documents like water/electricity bills, bank statements, mortgages and information in the media);

- solve problems on budgets (e.g. household, cooperative and state budgets), rates (including foreign exchange and household rates), insurance premiums, wages, simple interest, discount, commission, depreciation, sales/income tax, hire purchase and bank accounts (savings and current accounts);

- read, interpret and use data presented in charts, tables, maps and graphs (e.g. ready reckoners, road maps, charts and graphs in newspapers);

6.4 **MEASURES AND MENSURATION**

6.4.1. Measures

- time
  - read time on both the 12 and 24 hour clock(e.g. 7.35 p.m or 19 35).

- SI units
  - use Sl units of mass, temperature in degrees celsius length/ distance, area, volume/capacity and density in practical situations,
  - express quantities in terms of larger or smaller units;

6.4.2. Mensuration

- perimeter
- density
- area
- volume/capacity

- carry out calculations involving:
  - the perimeter and area of a rectangle, triangle, parallelogram and trapezium;
  - density
  - the circumference of a circle and the length of a circular arc;
  - the area of a (circle including sector and segment); rectangle, triangle, parallelogram and trapezium.
  - the surface area and volume of a cylinder, cuboid, prism of uniform cross-section, pyramid, cone and sphere;

(formulae for surface areas and volumes of pyramid, cone and sphere will be provided);

(units of area to include the hectare);

6.5 GRAPHS AND VARIATION

6.5.1. Coordinates	-	use Cartesian coordinates in two dimensions to interpret and infer from graphs and to draw graphs from given data;
6.5.2. Kinematics		
• travel graphs • speed/velocity • distance/displacement • acceleration	-	draw and interpret displacement-time and velocity-time graphs and solve problems involving acceleration, velocity and distance.
6.5.3. Variation		
• direct • inverse • joint	-	express direct, inverse, joint and partial variation in algebraic terms and hence solve problems in variation;
• partial	-	draw and interpret graphs showing direct, inverse and partial variation;
6.5.4. Functional graphs	-	construct tables of values, draw and interpret given functions which include graphs of the form $ax + by + c = 0$, $y = mx + c$, $y = ax^2 + bx + c$ and $y = ax^n$ where $n = -2,-1,0,1,2$, and 3 and simple sums of these;
	-	use the f(x) notation;
• solution of equations	-	solve linear simultaneous equations graphically;
	-	solve equations using points of intersection of graphs (e.g. drawing $y = 1/x$ and $y = 2x + 3$ to solve $2x^2 + 3x - 1 = 0$);
• gradients and rates of change	-	estimate gradients of curves by drawing tangents and hence estimate rates of change (e.g. speed, acceleration);
	-	find turning points (maxima and minima) of graphs (calculus methods not required);
	-	calculate the gradient of a straight line from the coordinates of points on it, interpret and obtain the equation of a straight line in the form $y = mx + c$;
	-	identify parallel straight lines using gradients;
• area under a curve	-	estimate area under a curve by counting squares and by dividing into trapezia (trapezium rule not to be used);

6.6 **ALGEBRAIC CONCEPTS AND TECHNIQUES**

6.6.1. Symbolic expression	-	express basic arithmetic processes in letter symbols;
• formulae	-	substitute numbers for words and letters in algebraic expressions (including formulae);

- change of subject - change the subject of a formula and substitute in formulae including those from other subjects (e.g. science);

6.6.2. Algebraic manipulation

- operations - use the four operations and rules of precedence to manipulate:
  - directed numbers,
  - monomials (including use of like and unlike terms),
  - simple algebraic fractions;

- factors, multiples, HCF, LCM - find and use common factors, common multiples, HCF and LCM;

- expansion - expand expressions of the forms a(x + y), (ax + by)(cx + d), (ax + by) (cx + dy); etc where a, b, c and d are rational numbers;

- factors factorise expressions of the form
  ax + bx, ax + bx + ay + by,
  $ka^2 - kb^2$,
  $ax^2 + bx + c$; where a, b, c and k are intergers

6.6.3. Indices

- laws of indices - use the following laws of indices (where m and n are rational other than zero):
  - $a^m \times a^n = a^{m+n}$
  - $a^m \div a^n = a^{m-n}$;
  - $a^o = 1$;
  - $(a^m)^n = a^{mn}$;
  - $a^{1/n} = \sqrt[n]{a}$;
  - $a^{-n} = 1/a^n$;
  - $a^{m/n} = \sqrt[n]{a}^{\,m} = (\sqrt[n]{a})^m$;

squares/square roots - calculate squares and use factors to find
cubes/cube roots roots and cube roots;

6.6.4. Equations - solve the following:

- linear equations - simple linear equations (including those involving algebraic fractions);

- simultaneous equations - simple linear simultaneous equations (by graphs, by substitution and by elimination);

- quadratic equations - quadratic equations of the form $ax^2 + bx + c = 0$ (by factorisation by graphs and by formula);

6.6.5. Logarithms - use the following basic ideas of the theory of logarithms:

$\log_b MN = \mathrm{Log}_b M + \log N$, $\log_b \left(\frac{M}{N}\right) = \log_b M - \log_b N$

and $\log_b M^p = p\log_b M$ where b and p are rational numbers and M and N are greater than zero.

		-	use common logarithms in calculations (including finding powers and roots);
6.6.6.	Inequalities		
	• signs	-	use the following in appropriate situations: $=, >, <, \geq, \leq$ $\neq, \ngtr, \nless$;
	• linear inequalities	-	solve linear inequalities e.g. of the form $ax + b > c$, $ax + b < cx < dx$, $ax + b < cx + d < ex + f$, $c < ax + b < d$ etc where a, b, c, d, e and f are rational;
	• linear programming	-	represent inequalities and their solutions on a number line;
		-	use simple linear programming methods to solve problems (unwanted regions to be shaded, with inequality boundaries shown by broken lines);

6.7 GEOMETRIC CONCEPTS AND TECHNIQUES

6.7.1.	Points, lines and angles	-	identify interpret and apply the following concepts: point, line, parallel, perpendicular;
	• types of angles	-	right angle, acute, obtuse, reflex, complementary, supplementary, vertically opposite angles, angles at a point, angles on a straight line;
	• parallel lines	-	transversal, allied or co-interior angles, corresponding angles, interior opposite or alternative angles;
	• angles of elevation and depression	-	angles of elevation and depression;
6.7.2.	Bearings	-	interpret and use three-figure bearings measured clockwise from north, (i.e. from 000° to 360°) and compass bearings (e.g. N 47° E or 47°E of N);
6.7.3.	Polygons		
	• triangles • quadrilaterals	-	use properties of: triangles (including isosceles and equilateral), quadrilaterals (including kites, parallelograms, rectangles, rhombi, squares, trapezia);
	• n-sided polygons	-	regular and irregular n-sided polygons,
		-	state the special names of n-sided polygons (up to n=10),
	• parallel lines and area	-	use the area property of triangles and parallelograms between the same parallels;
6.7.4.	Circles	-	use the properties: - radius - diameter - chord - tangent - cyclic quadrilateral

	-	use the following circle theorems: - angle subtended at the centre and on the circumference - angle in a semi-circle - angles in the same segment - angle in the alternate segment;
6.7.5. Similarity and Congruency	-	identify similar and congruent figures and solve problems on similar and congruent triangles; solve problems on:- - areas of similar plane figures, - volumes and masses of similar solids;
6.7.6. Constructions	-	construct the following using ruler and compasses only: - angle bisector, perpendicular bisector, angles of 30°, 45°, 60°; and 90°; and single combination of these;
	-	construct a perpendicular: - from a given point to a given line - through a given point on a given line;
• triangles • parallelograms • regular polygons	-	construct triangles, parallelograms and simple n-sided polygons (protractors may be used where necessary);
• scale drawings	-	produce scale drawings using an appropriate/given scale;
6.7.7. Loci	-	construct and use the locus (in two-dimensions) of a point - equidistant from - two given points, - two intersecting lines, - at a given distance from, - a fixed point, - a given straight line;
6.7.8. Symmetry	-	identify line symmetry in two dimensions;
line symmetry	-	balance properties of isosceles triangles, equilateral triangles, regular polygons, parallelograms and circles directly related to their symmetries;
identify symmetry	-	identify rotational symmetry (including order of rotational symmetry) in two dimensions;

6.8 TRIGONOMETRY

6.8.1. Pythagoras theorem and trigonometrical ratios	-	apply Pythagoras theorem, sine, cosine and tangent for acute angles to solve simple problems involving right-angled triangles in two dimensions;
	-	use and interpret sine, cosine and tangent of obtuse angles, use the sine and cosine rules for the solution of triangles (angles in either degrees/minutes or degrees to 1 decimal place);

• three dimensional problems	-	solve three-dimensional problems involving the angle between a line and a plane;
6.8.2. Area of a triangle	-	use the formula Area = ½absin C for the area of a triangle;

6.9 VECTORS AND MATRICES

6.9.1. Vectors in two dimensions

• translation and notation	-	represent a translation by a column vector and by a directed line segment and use the notation $\overrightarrow{AB}$ or AB or $\underline{a}$ or a or $\vec{a}$ or **a**;
• operations	-	add and subtract vectors and multiply by a scalar;
	-	calculate the magnitude of a vector and use the notation $\left\lvert\overrightarrow{AB}\right\rvert$ or $\lvert a\rvert$; etc
• position vectors • equal vectors • parallel vectors	-	identify and use the concepts of - position vectors, - equal vectors, - parallel vectors,

6.9.2. Matrices

• dimension/order	-	use and interpret a matrix as a store of information and show familiarity with the idea of dimension/order of a matrix;
• operations	-	add and subtract matrices (where appropriate) and multiply by a scalar;
	-	multiply matrices (of order 2 x 2 or less) where appropriate;
• identity matrix	-	use the property of identity and zero matrix for 2 x 2 matrices;
• determinant	-	find the determinant of a 2 x 2 matrix and distinguish between singular and non-singular matrices and use the notation determinant A or Det A or $\lvert A\rvert$
• inverse matrix	-	find and use the inverse of a 2 x 2 non-singular matrix; (e.g solving simultaneous linear equations) and use the rotation A^{-1};

6.10 TRANSFORMATIONS

6.10.1.	-	carry out the following transformations in x-y plane:
6.10.2.		
• translation	-	translate (T) simple plane figures;
• reflection	-	reflect(M) simple plane figures in the axes and in any line;
• rotation	-	rotate (R) about any point clockwise or anti-clockwise through 90° and 180°,
• enlargement	-	enlarge(s) about any point using a rational scale factor;

• stretch	-	stretch (S); both one way and two way stretch using the axes as the invariant stretch lines and rational stretch factor,
• shear	-	shear (H), using the axes as the invariant lines and rational shear factor.
	-	apply combinations of the above (e.g. if M(a)=b and R(b)=c then RM(a)=c);
	-	describe transformations fully;
6.10.2 Matrices as operators	-	identify interpret and/or use matrices which represent the above transformations,
	-	describe transformations using coordinates and matrices (singular matrices are excluded);

6.11 STATISTICS AND PROBABILITY

6.11.1. Statistics

• collection and classification	-	collect, classify and tabulate statistical data;
• data representation	-	read, interpret, draw and make simple inferences from bar charts, pie charts, histograms and frequency tables/charts and frequency polygons (see also 6.3.1.);
• measures of central tendency	-	calculate the mean, mode, median from given data and distinguish between the purposes for which they are used;
	-	use an assumed mean where appropriate;
	-	read and interpret data presented in classes and determine the modal class;
• cumulative frequency	-	draw and use a cumulative frequency curve/orgive;

6.11.2. Probability

• terms	-	use the terms: random, certain, impossible event, trial, sample space, equally likely, mutually exclusive, independent events;
• experimental probability	-	distinguish between experimental and
• theoretical probability	-	theoretical probability;
• probability of - single events	-	solve simple problems involving the probability of a single event;
- combined events	-	calculate the probability of and solve simple problems involving combined events e.g. mutually exclusive and independent events (use of tree diagrams and outcome tables is recommended).

7.0 SCHEME OF ASSESSMENT

	PAPER 1	**PAPER 2**
WEIGHTING	50%	50%
TYPE OF PAPER	Approximately 30 short answer questions	Structured Questions Section A (6 compulsory questions) Section B (3 questions out of 6)
TIME ALLOWED	$2\frac{1}{2}$ hours	$2\frac{1}{2}$ hours